

κ – equivalent hardness of the system;

Q – generalized force of the weight (+ Q for lowering, - Q for hoisting);

M – moment of the engine (brakes) reduced to the drum axle;

R – drum radius;

d – coefficient of damping, resp. for progressively moving and rotating masses [2].

In the bi-mass model, the generalized force Q is assumed as equal to the maximal one for dimensioning of the hoist force, determined in [1], and the el.motor moment M as proportional to the angular speed of the rotor.

The system of linear differential equations (2) is solved for different initial conditions whereby the specific peculiarities of the physical process are introduced in the mathematical model. These peculiarities in the cage hoists must reflect the characteristic periods of the technological cycle, and namely:

First period t_1 – equi-accelerating movement with speed increase from 0 up to V_{max} by linear law, with constant acceleration a ;

Second period t_2 – movement with constant speed V_{max} for acceleration $a = 0$;

Third period t_3 – equi-retarding movement with speed decrease from V_{max} down to 0 for constant acceleration a .

During the period of movement of hoist vessels with constant speed $V_{max} = \text{const.}$, only static forces act in the hoist system, determined by the weight of the hoisted loads, the weight of the rope with the engaging units and the friction forces. In the periods of equi-accelerating and equi-retarding movement, besides the static forces, there also act inertia forces of the moving masses. Consequently, in this specific case, of interest are the dynamic loads upon acceleration and stopping of the load Q . If in the theoretical investigations it is assumed that the starting torque of the engine and the brakes torque are constant, and also that the washer block is undeformable, the solution of system (2) is of type [2]:

$$\begin{cases} x = \frac{A_1}{2} t^2 + B_1 t + C_1 + D_1 e^{-\lambda t} \left(\cos \omega t + \frac{E_1 - D_1 \lambda}{\omega D_1} \sin \omega t \right) \\ \varphi = \frac{A_2}{2} t^2 + B_2 t + C_2 + D_2 e^{-\lambda t} \left(\cos \omega t + \frac{E_2 - D_2 \lambda}{\omega D_2} \sin \omega t \right) \end{cases} \quad (3)$$

where:

ω is the frequency of vibrations, which depends on the parameters of the bi-mass model and is determined by:

$$\omega = \sqrt{k \left(\frac{1}{m} + \frac{R^2}{J i^2} \right) - \frac{d^2}{4} \left(\frac{1}{m} + \frac{R^2}{J i^2} \right)^2}$$

λ is coefficient of attenuation, determined by:

$$\lambda = \frac{d}{2} \left(\frac{1}{m} + \frac{R^2}{J i^2} \right)$$

$A_{12}, B_{12}, C_{12}, D_{12}, E_{12}$ are coefficients depending on the system parameters and the initial conditions;

t – time.

The velocities \dot{x} and $\dot{\varphi}$ can be determined through differentiation of system (3).

To solve the problem, it is necessary to determine the constants $A_{12}, B_{12}, C_{12}, D_{12}, E_{12}$, in accordance with [2], for the respective initial conditions, which for the period of load acceleration, are: $t = 0, x_0 = 0, \dot{x}_0 = 0, \varphi_0 = 0, \dot{\varphi}_0 = 0$

$$\begin{cases} A_1 = \left(\frac{M}{m} \pm g \frac{R}{i} \right) \frac{k}{J} \cdot \frac{R}{i} / (\lambda^2 + \omega^2) \\ B_1 = \left[\left(\frac{M}{J} \cdot \frac{d}{m} \pm g \frac{d}{J} \frac{R}{i} \right) \frac{R}{i} - 2A_1 \lambda \right] / (\lambda^2 + \omega^2) \\ C_1 = (\pm g - A_1 - 2B_1 \lambda) / (\lambda^2 + \omega^2) \\ E_1 = -B_1 - 2C_1 \lambda \\ D_1 = -C_1 \end{cases} \quad (4)$$

$$\begin{cases} A_2 = \left[\frac{k}{J} \left(\frac{M}{m} \pm g \frac{R}{i} \right) \right] / (\lambda^2 + \omega^2) \\ B_2 = \left[\frac{M}{J} \frac{d}{m} \pm g \frac{d}{J} \frac{R}{i} - 2A_2 \lambda \right] / (\lambda^2 + \omega^2) \\ C_2 = \left(\frac{M}{J} - A_2 - 2B_2 \lambda \right) / (\lambda^2 + \omega^2) \\ E_2 = -B_2 - 2C_2 \lambda \\ D_2 = -C_2 \end{cases} \quad (5)$$

For the period of load stopping for initial conditions $t_0 = T - t_c$, (t_c – time of the braking process), $x_0 = H - h_x$ (from

Fig.1), $\dot{x}_0 = V_{max} = \text{const}$, $\varphi_0 = \frac{2\pi R}{i}$ и $\dot{\varphi}_0 = \dot{\varphi}_{max}$, the constants $A_{12}, B_{12}, C_{12}, D_{12}, E_{12}$, are determined by systems of equations (6) and (7):

$$\begin{cases} A_1 (\lambda^2 + \omega^2) = \left(\frac{M}{m} \pm g \frac{R}{i} \right) \frac{k}{J} \cdot \frac{R}{i} \\ 2A_1 \lambda + B_1 (\lambda^2 + \omega^2) = \left(\dot{x}_0 \frac{k}{J} \frac{R}{i} + \varphi_0 \frac{k}{m} + \frac{M}{J} \frac{d}{m} \pm g \frac{d}{J} \frac{R}{i} \right) \frac{R}{i} \\ A_1 + 2B_1 \lambda + C_1 (\lambda^2 + \omega^2) = \dot{x}_0 \frac{d}{J} \frac{R^2}{i^2} + x_0 \frac{k}{J} \frac{R^2}{i^2} + \dot{\varphi}_0 \frac{d}{m} \frac{R}{i} + \varphi_0 \frac{k}{m} \frac{R}{i} \pm g \\ B_1 + 2C_1 \lambda + E_1 = \dot{x}_0 + x_0 d \left(\frac{1}{m} + \frac{1}{J} \frac{R^2}{i^2} \right) \\ C_1 + D_1 = x_0 \end{cases} \quad (6)$$

$$\begin{cases} A_2 (\lambda^2 + \omega^2) = \frac{k}{J} \left(\frac{M}{m} \pm g \frac{R}{i} \right) \\ 2A_2 \lambda + B_2 (\lambda^2 + \omega^2) = \frac{M}{J} \frac{d}{m} + \dot{\varphi}_0 \frac{k}{m} + \frac{k}{J} \frac{R}{i} x_0 \pm g \frac{d}{J} \frac{R}{i} \\ A_2 + 2B_2 \lambda + C_2 (\lambda^2 + \omega^2) = \frac{M}{J} + \frac{d}{m} \dot{\varphi}_0 + \frac{k}{m} \varphi_0 + \frac{d}{J} \frac{R}{i} \dot{x}_0 + \frac{k}{J} \frac{R}{i} x_0 \\ B_2 + 2C_2 \lambda + E_2 = \dot{\varphi}_0 + \varphi_0 d \left(\frac{1}{m} + \frac{1}{J} \frac{R^2}{i^2} \right) \\ C_2 + D_2 = \varphi_0 \end{cases} \quad (7)$$

The sign (-) refers to a case of IQ load lifting, and the sign (+) for a case of its lowering.

Thus, for known height of the hoist H and duration of the movement of the hoist vessel T , the force in the rope F can be determined as a difference between the coordinates of the moving masses, multiplied by the hardness κ , and namely:

$$F = \left[\frac{R}{i} \varphi(t) - x(t) \right] \kappa \quad (8)$$

The modeling of dynamic processes and the composing of adequate models for investigation of real objects is a theoretical approach where the volume of natural experimental measurements for evaluation of the functional and operational fitness of the machines is optimized.

REFERENCES

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