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ANALYSES OF ONE ELECTROSTATIC PROBLEM AND ITS APPLICATION IN MEASUREMENT OF THE ELECTROSTATICAL FIELD PARAMETERS

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ABSTRACT
In this report is considered creation of non-uniform field of point charge, in which as a sensor is introduced a conductive sphere Two instances – earthing and isolate sphere are treated and is used popular principle that inducted on the sphere surface charge could be changed with charge-image, located into the sphere volume. Analysis we made proves that two sensors, located on the sphere surface when the sphere is earthed, make measurements.

INTRODUCTION

Most modern device designed for measurement of parameters of electrostatical fields use the effect of electrostatical induction.

Electrostatical induction causes accumulation of charges on the measuring electrode of the converter, which field is superposed on the measured field and deforms it. That is why the measuring instrument designed for measurement of parameters of electrostatical field converts the value of deformed field.

It isknown that measuring instrument for electrostatical field, graduated according to the particular source is inexact when is used fro measurement of the field with other dimensions and configuration [1, 2]. Most often the graduation of the measuring instrument is made in the field of plane capacitor [1]. But in real conditions should be measured parameters of non-uniform field.

It should be mentioned, that in order to use the induction meters correctly it is necessary to earthen them.

Let note the coefficients of intensity deformation as:

$$K_E = \frac{E}{E_o} \qquad (1),$$

where E is the average value of intensity of deformed field on the surface of the measuring electrode and E_o is intensity of measured field when the device is missing.

The indication α of the device is proportional to the field intensity on the surface of measuring electrode: $\alpha = mE$, where m is the coefficient of transmission of the device. Therefore:

$$K_E = \frac{\alpha}{mE_o} \qquad (2)$$

The coefficient K_E depends on field deformation and on distance between source and measuring instrument. In case of ideal measuring instrument, the coefficient K_E is constant for fields with different configuration and intensity.

FIELD OF EARTH SPHERE

We treat solution of classical problem for the field of conducting medium that is in the field of point charge +q (fig.1)

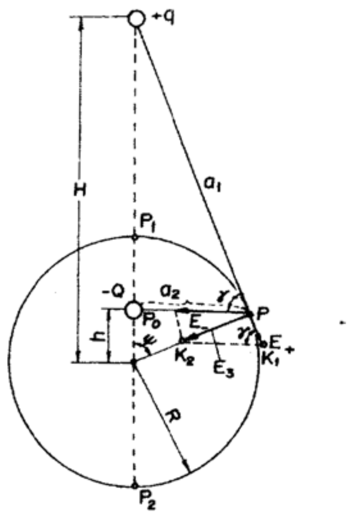

Figure 1.

The point charge +q is at distance H from the centre of earthed conducting sphere, which radius is R. Inducted on the sphere surface charge could be changed with charge-image - Q, which is concentrated in point P_0 inside the sphere at distance h from the centre. Charge -Q and distance h are selected so that the fields of real charge +q and charge image –Q to have a zero equipotential plane, coincided with the sphere surface. For every point of this plane is valid:

$$\varphi = 0 = \frac{q}{4\pi\varepsilon_0 a_1} - \frac{Q}{4\pi\varepsilon_0 a_2} \quad (3)$$

From (3) follows:

$$\frac{a_1}{a_2} = \frac{q}{Q} = k = const \quad (4)$$

In point p1 -

$$\frac{a_{1'}}{a_2} = k = \frac{H-R}{R-h} \quad (5)$$

In point p2 -

$$\frac{a_{1'}}{a_2} = k = \frac{H+R}{R+h} \quad (6)$$

We solve the system of equations (5) and (6) towards the unknown k and h and obtain:

$$h = \frac{R^2}{H} \quad (7)$$

$$k = \frac{H}{R} \quad (8)$$

This way the conducting surface of the sphere with the zero potential could be changed with charge image:

$$Q = -\frac{q}{k} = -q\frac{R}{H} \quad (9)$$

The resultant intensity of the field in anyone point p from the surface of the conducting sphere has two components:

a) from charge +q on the direction of the line a_1 :

$$E_+ = \frac{q}{4\pi\varepsilon_0 a_1^2} \quad (10)$$

б) to charge -Q on the direction of the line a_2

$$E_- = -\frac{Q}{4\pi\varepsilon_0 a_2^2} \quad (11)$$

The ratio of absolute values of these components is: $\frac{E_-}{E_+} = \frac{Qa_1^2}{qa_2^2} = k$. The geometrical sum of E_+ and E_-, i.e. the complete value of the intensity of the field E_3 in point p could be determine from similarity of triangles (+q)p(-Q) and pk_1k_2 :

$\frac{E_3}{E_+} = \frac{H-h}{a_2}$ from where:

$$E_3 = E_+\frac{H-h}{a_2} = \frac{q}{4\pi\varepsilon_0 a_1^2}\frac{H-\frac{R^2}{H}}{\frac{a_1}{H/R}} = \frac{q}{4\pi\varepsilon_0}\frac{H^2-R^2}{a_1^3 R} \quad (12)$$

The intensity of deformed field E_3 on the surface of earth sphere is determined by expression (12). The vector E_3 has direction on normal to spherical surface.

FIELD OF ISOLATED SPHERE

Considered sphere in general could has the arbitrary potential $\varphi_{c\phi}$, which is different form zero when the earthing is removed and the charge $+Q^{'}$is located in the its centre. The charge is determined by the formula:

$$+Q^{'} = \varphi_{c\phi} 4\pi\varepsilon_0 R \quad (13)$$

Complete charge of the sphere is equal to: $Q_{c\phi} = Q^{'} - Q$, where –Q is the charge image of the external charge +q.

If the sphere is isolated than its complete charge is equal to zero, therefore:

$$Q^{'} = Q = q\frac{R}{H} \quad (14)$$

Intensity of the electrostactical field on the sphere surface in this case will have an additional component. It could be determine by charge $+Q^{'}$ and arithmetically is extracted from the value E_3 in case of earting sphere. i.e.

$$E_{II} = \frac{q}{4\pi\varepsilon_0}\left(\frac{H^2-R^2}{a_1^3 R} - \frac{1}{RH}\right) \quad (15)$$

The value E_{II} is zero when: $\frac{H^2-R^2}{q_1^3 R} - \frac{1}{RH} = 0$, from where:

$a_1 = \sqrt[3]{H(H^2-R^2)}$.

According to fig.1 $a_1^2 = H^2 + R^2 - 2HR\cos\varphi$, from where obtain, that the intensity E_{II} is zero when:

$$\cos\varphi = \frac{1}{2}\left[\frac{H}{R} + \frac{R}{H} - \sqrt[3]{\frac{H}{R}\left(\frac{H}{R} - \frac{R}{H}\right)^2}\right]$$

We subtract (15) from (12) and obtain the component of intensity of deformed field $E_3^{'}$, conditioned by the effect of earthing of the sphere

$$E_3^{'} = E_3 - E_{II} = \frac{q}{4\pi\varepsilon_0}\frac{1}{RH} = \varphi_q\frac{1}{R} \quad (16)$$

where $\varphi_q = \dfrac{q}{4\pi\varepsilon_0 H}$ is the potential of the field in the centre of the sphere, created by the charge q. In expression (16) is replaced $a=a_1$. This way the intensity of earthed sphere is sum of two components:

$$E_3 = E_H + E_3^{'} \qquad (17).$$

Expression (16) is transformed:

$$E_3^{'} = E_0\left(\frac{H}{R} - 2 + \frac{R}{H}\right) \qquad (18)$$

where E_0 is intensity of the external field created by the charge q in point p_1 form the sphere surface.

The intensity in p_1 could be expressed by E_0, E_{3p1}:

$$E_{3p1} = \frac{q}{4\pi\varepsilon_0 (H-R)^2}\left(1+\frac{H}{R}\right) = E_0\left(1+\frac{H}{R}\right) \qquad (19)$$

Intensity $E_И$ for the same point is:

$$E_{Иp1} = \frac{q}{4\pi\varepsilon_0 (H-R)^2}\left(3-\frac{H}{R}\right) = E_0\left(3-\frac{R}{H}\right) \qquad (20)$$

Dividing (19) and (20) to E_0, according to (1) we find the coefficients of deformation of the field on the sphere surface in point p_1 respectively of earthing and isolated spheres\;

$$K_{E3p1} = 1 + \frac{H}{R} \qquad (21)$$

$$K_{EИp1} = 3 - \frac{R}{H} \qquad (22)$$

It should be mentioned, that when $a = \sqrt[3]{H(H^2 - R^2)}$ on the line of electrical neutral the intensity $E_И$ is zero, because $E_3 = E_3^{'}$ [3,4]. This way by using of the intensity of line of electrical neutral of earthing sphere could be determined intensity on surface of isolated sphere.

When $a = \sqrt{H^2 + R^2}$ intensity of earthing sphere on the line of geometrical neutral in point p_2 is:

$$E_{3p2} = E_0 \frac{(H-R)^2 (H^2 - R^2)}{R(H^2 + R^2)\sqrt{H^2 + R^2}} \qquad (23)$$

We extract (23) from (19) and obtain the difference of intensities in point p_1 and point p_2: $E_3^{'} = E_{3p1} - E_{3p2}$. Now we find:

$$E_3^{'}/E_0 = 1 + \frac{H}{R} - \frac{(H-R)^2 (H^2 - R^2)}{R(H^2 + R^2)\sqrt{H^2 + R^2}} \qquad (24)$$

CONCLUSION

Calculations results for K_{E3p1}, $K_{EИp1}$ and $E_3^{'}/E_0$ by using the formulas (21), (22) and (24) are given in table:

R	1	2	3	4	5	6
H	1	4	9	16	25	36
H/R	1	2	3	4	5	6
R/H	1	0,50	0,33	0,25	0,20	0,17
$K_{3Иp1}$	2	3	4	5	6	7
$K_{EИp1}$	2	2,5	2,67	2.75	2.80	2.83
$E_3^{''}/E_0$	2	2,73	2,99	3,07	3,10	3,11

On fig.2 are shown the dependences of coefficients of deformation according to (21) and (22) and of the ratio $E_3^{''}/E_0$, according to (24).

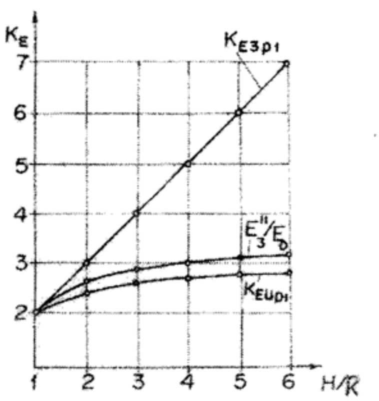

Figure 2

When the sphere is earthed, the coefficient of deformation K_{E3p1} increases unlimitedly when the distance H increases. In the sphere is isolated, the coefficient of deformation $K_{EИp1}$ aims to be constant equal to three, when the distance H increases. A special characteristic of the ratio $E_3^{''}/E_0$ is that it is insignificantly different from the coefficient $K_{EИp1}$, and therefore the intensity $E_3^{''}$ is close to intensity $E_{Иp1}$.

Characteristic for graphics, shown on fig.2, is that when H/R increases, functions $K_{E3p1}, K_{EИp1} u\, E_3^{''}/E_0$ monotonously increase. When the values of H/R are small, the values of shown above functions do not decrease (they stay minimum-extremum), It could be explained with the fact that common capacity of the system conducting sphere-point charge is equal to zero.

Analysing the theoretical conclusions (21), (22) and (24) and graphics, shown on fig.2 is offered a principle of design of device for measurement of intensity of electrostatical field with optional configuration with determine precision. The principle is:

Two converters are used – the first one is located on the surface of conducting body of the converter in the area, turned to the source of field, and another one – on the line of geometrical neutral.

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