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THEORETICAL INVESTIGATION OF THE APPLIVABILITY MANEVICH-PAVLENKO MODEL FOR DETERMINING THE STRENGTH OF EXTRACTION OF A SINGLE TRAVERSE FROM THE BALLAST PRISM OF A MOVABLE RAILROAD

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ABSTARCT
Increasing the strength of movable railroads against their lateral displacement would make the operation of the technologic railroad haulage in the opencast mines much more reliable. This is related to the increasing of strength of extracting a single traverse and determining the stress while doing it.
A model is examined, which is created by Manevich-Pavlenko in the middle of the 80's of 20th century and its applicability to the above task is shown by means of the theoretical mechanics, mathematical physics and the classical mathematical analysis.

INTRODUCTION

There is an observation of a unique phenomenon related to movable railroads - displacement in a horizontal plane during summer. One of the considerable reasons for that is their insufficient strength against lateral movement in a horizontal plane.

Relatively cost-effective opportunity without general raise of cost for the upper construction of the movable railroads is to in-build into it a metal traverse of the shape of a cross in the underrail section.

This engeneering approach, realized in large scale requires immense investments. It is quite natural to question if they are reasonable and if there are any possibilities to estimate quantitatively and presicely enough the increase of strength of movable railroads in a horizontal plane. Actually the problem reduces to finding a sufficiently reliable method for calculating the strength of extracting of a single traverse from the ballast bed. This requires development of an adequate mathematical apparatus, based on a physically grounded model. The process, though, has a dynamical character. Therefore it is necessary to describe the mechanical system of three traverses (the one in the middle is supposed to be released of upholding couplings) and ballast in a moment of limited balance. i.e. a model is needed through which by the same type of equations to describe the processes of limited balance between the ballast and the traverses with a conventional and crossed shape in the under-rail cut, when the system is still at rest, however there is an initial, even though an infinitely small movement of the extracted traverse.

BASIC MODEL

In general there are two approaches. The first one requires the establishing of an intentional model, which describes the discussed physical processes. However, this a very difficult task, which is to be solved only by the efforts of collective teams. The second approach assumes utilizing of a physics-mathematical model, intended for other conditions, but applicable to the specific task. Its implementation demands relevant physical-mathematical arguments.

The second approach is chosen in this study. The model of Manevich and Pavlenko, developed in the beginning of 90's in (*Manevich L. et al.,* 1982) is selected as a basic model. Its essence is illustrated in fig. 1, and the common scheme of thinking is as follows:

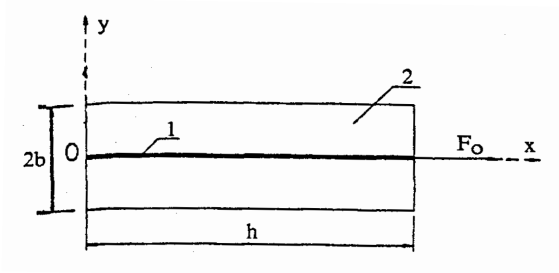

Figure 1. Basic model of Manevich and Pavlenko

Physically, the model represents a plane plate (item 2) with measurements "2.b" and "h", and a fibre is extracted out of it (item 1). The plane plate (2) represents a homogeneous medium. The fiber (1) is allocated in it and it has a strcuture different from ambient structure:

- non-deformable;
- rigid.

After applying an axis force "F_O" on the fiber into the direction of "x" axis, extracting of the fiber (1) from the environmental medium of the plate (2) begins. For conciseness, further on the ambience of the plate will be called matrix. Within the process of extracting (extracting), the behaviour of the fiber (1) and the matrix (2) is shown by the following dependences:

- Equations of equilibrium of the matrix (2) – this a system of partial differential equations (1) ;
- Equation of displacement of points of fiber (1) - differential equation (2).

$$\left\| \begin{aligned} & B_1 \frac{\partial^2 U}{\partial^2 y} + G\frac{\partial^2 U}{\partial x^2} + \left(\nu_2 B_1 + G\right)\frac{\partial^2 V}{\partial x \partial y} = 0 \\ & G\ \frac{\partial^2 V}{\partial y^2} + B_2\frac{\partial^2 V}{\partial x^2} + \left(\nu_1 B_2 + G\right)\frac{\partial^2 U}{\partial x \partial y} = 0 \end{aligned} \right. \qquad (1)$$

$$EA.\frac{\partial^2 W}{\partial x^2} = F_O.\delta\left(x\right) - 2\tau\left(x\right) \qquad (2)$$

where:

z(u, V) – vector of displacing of the matrix;

u, V – vector components of displacing the matrix. In this case the axes x and y coincide;

G – rigidity of matrix to cutting;

F_O – axis strength, applied to the fiber in the boundary point;

EA – geometrical characteristics of fiber when tensile stresses;

B_1, B_2 – rigidity to tensile stress and compression of the matrix;

u_1,u_2 – the coefficients of Poisson;

τ(y) – tension to cutting between fiber and matrix. It is assumed that in the contact area

$$\tau\left(y\right) = G\left.\frac{\partial U}{\partial X}\right|_{x=0} \quad \mathbf{t}\text{ (y) = O}$$

W – extraction of fiber;

δ(y) – delta function of Diraque. It represents integral functional and characterizes the extracting strength.

The new system of equations [(1) & (2)] has the following boundary conditions (3) – [3.1], [3.2], [3.3], [3.4].

[3.1] $\left.\begin{matrix} U = W \\ V = 0 \end{matrix}\right|_{x=0}$ - its physical meaning is that in

the direction of the fiber "y", the component of solution "U" coincides with "W", which is the solution of the equation (2), i.e., if the value of "W" is known at any point of "y", then "U" (component of the vector of extracting of the matrix) has the same value. Practically that means, that the vector of displacement of the matrix for points situated on the axis "y" has its second component V=0 (the solution Z=U, 0 or Z=W, 0);

[3.2] $U = V = 0\Big|_{y=+b}$ - its physical meaning is that

the matrix is fastened on by its two ends in parallel to the axis "x", i.e., that immobility of the matrix is guaranteed out of points with coordinates "±b";

[3.3] $\left.\frac{\partial U}{\partial y} = V = 0\right|_{x=0}$ - its physical meaning is that

the vector of speed of displacement of the matrix on direction of axis "x" is equal to zero in the point x=0;

[3.4] $U = V = 0\Big|_{x=\infty}$ - its physical meaning is, that

when "x" is very large, than displacements in the matrix are equal to zero.

ANALOGICAL ADAPTATION OF THE BASIC MODEL TO THE SOLUTION OF THE TASK

The peculiarities of the model of Manevich-Pavlenko are:

- The matrix is homogeneous medium;
- The fiber has a structure, different from the structure of the matrix and carries the following characteristics:
  - It is non-deformable;
  - It is rigid;

While extracting of a steel traverse of a crossed shape from the under-rail section, released from connections and dipped to its upper edge into gravel ballast prism, the following should be observed:

- cutting up the gravel ballast along the surfaces, represented by lines 1-1 and 2-2 (fig. 2), i.e. because of the different structure, this system may be treated as follows:
  - the gravel – like a matrix;
  - the traverse – like an extracted fiber;
- cutting up the gravel along the surface, which coincides with the lower base of the traverse, i.e. if assumed that through the height of the traverse the behavior of cutting through all horizontal sections of the extracted traverse is the same, than principally the picture resembles to a level of identification of the theoretical model according to fig. 1.

In case that the traverse is situated in the gravel bed and several traverses are missing around it, from both sides, then cutting of the gravel (according the above mentioned considerations) is realized under an angle φ (fig. 3) from the farther (in relation to the direction of extracting) end of the crossed extensions onto the surface, coinciding with the surface of the lower base. The last is due to the following circumstance:

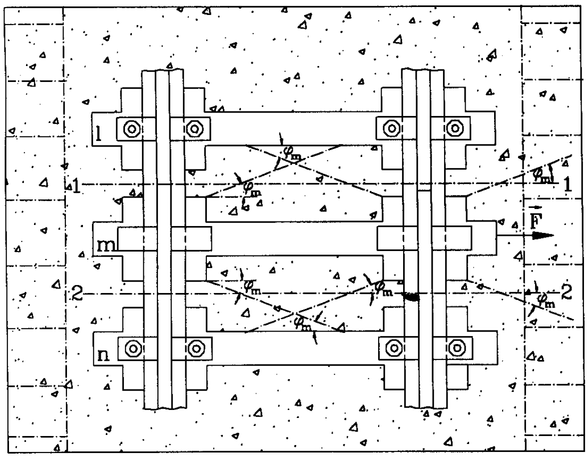

Figure 2. Lines of cutting of the gravel ballast during the extraction of a cross-shaped traverse

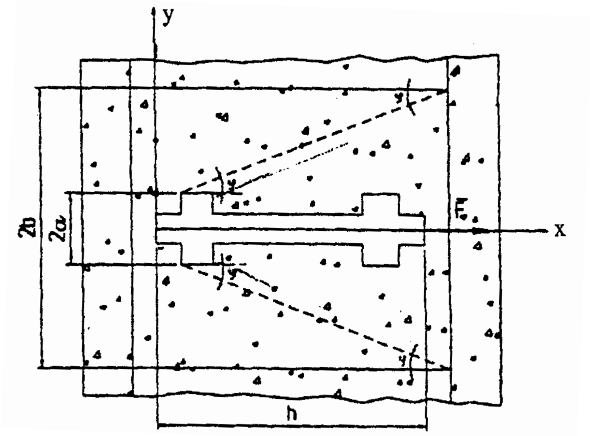

Figure 3. Illustration of cutting of gravel ballast, caused by the cross-shaped traverse

Availability of under-traverse thresholds under and near to the extracted traverse determines the larger strength of shearing of the ballast prism under angle "φ" (of internal friction of gravel) into depth towards the ground base. At the same time, force "F_o" actively operating on the axis of the traverse seeks the zone of the less strength. And it is namely onto the horizontal plane, coinciding with the lower base of the extracted traverse due to the lower degree of consistence compared to compression of gravel in the zone of under-traverse thresholds. The above mentioned considerations are logical and truthful. But they do not report and there is no way to report, that the model of Manevich-Pavlenko is created to be applied to extracting of a fiber from composite medium.

A comprehensive theoretical is needed in order to prevent suspicions in the adequacy of utilizing the basic model for the purposes of the above task.

BASIC PRECONDITIONS ABOUT APPLICATION OF THE MODEL

The basic preconditions, assuming validity of the theoretical background for applying of the Manevich-Pavlenko model are as follows, according to fig. 4.

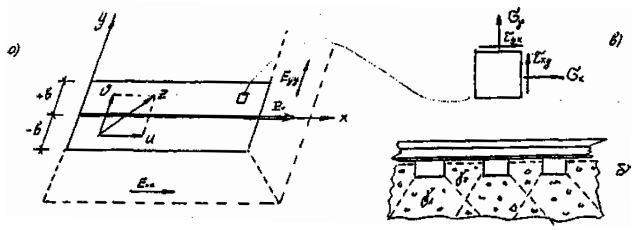

Figure 4. Initial basis for genesis of the Manevich-Pavlenko *model. a) principal scheme of the movable railroad line according the Manevich-Pavlenko model; b) physical behavior of the gravel ballast bed; c) elementary segment of the compacted gravel ballast bed.*

They are as follows:

FIRST: It is assumed that, a plane fragment of the gravel ballast prism and the rail-traverse frame is been treated. It is supposed that, in whatever way (by an approach up to now unknown) the traverses are compressed volumetrically to the transformation into dimensionless fibers and through extracting them from the gravel ballast prism they provoke (by an approach up to now unknown) cutting in the gravel bed onto the axis "x" (fig. 4 a).

SECOND: The components U & V of the vector z of displacement of the matrix are in parallel to the axes x and y (fig.4 a);

THIRD: The elastic modulii of the gravel ballast bed, though close to the values on axes x and y are different, i.e., $E_{xx} \neq E_{yy}$ (fig. 4. a), i.e. the gravel prism is orthotropic. Thus the modulus on the axis of road E_{yy} is higher than the modulus transversely to the road E_{xx}.

FOURTH: The gravel is compacted to a rate of formation of under-traverse thresholds and its compactness as a function of the bulk weight is figured out by the inequality:

$$\gamma_1 > \gamma_2 < \gamma_{cp}^{\Theta} \qquad (\text{по фиг. } 4\,в), \tag{4}$$

where: γ^{θ}_{cp} is the gross weight, after which it is so compressed that its behavior and characteristics are similar to the behavior of an elastic body..

FIFTH: The system is in a dynamical equilibrium, i.e. every one infinitely small element of the matrix is into an equilibrium – on (fig.4.c).

REASONING OF THE COMPACTION OF THE CONCRETE PRISM

In connection to the fourth precondition and dependence (5), it should be notified that, during the development of (*Stoyanov D. et al.* 1998; 2000) by the team of D. Stoyanov, observations were made on the loading of the operating trains on a movable railroad at the “Obrouchishte” dumping area at “Trayanovo” mine of the “Maritza Iztok” Co.

According to *Ivanov G.* (1981), *Kostov T.* (1991) etc. in *Shahonyants G.* (1982) etc., and general administrations of the conventional railroads in many counties assume that the gravel ballast bed of the railroad is compacted enough after passing over on it of 1,000,000 gross tons. Furthermore, it is assumed that the process goes on according to the response in fig.5.

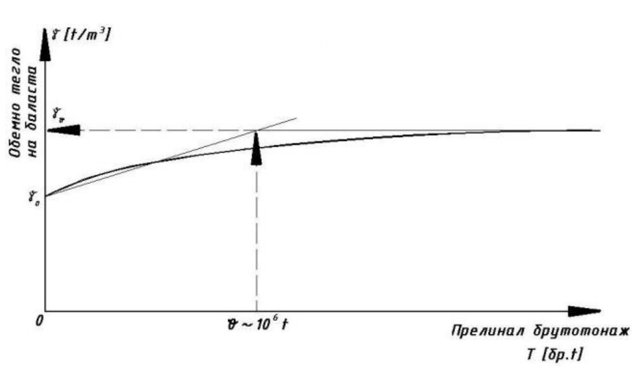

Figure 5. Illustration of the process of compacting of gravel in the gravel prism after passing over a definite gross-tonnage

A constant number of cars of 16 is assumed with the aim of safety;

- useful car volume is 40 m^3 and own weight is 34 t. The transported overburden consists of different clays with average bulk weight of 2,1 t/m^3 and average coefficient of swelling K_p = 1,45. Twenty nine cars are measured and an average coefficient of filling up of cars is established to $K_н$ = 1,02. ,

In this case the gross weight of a train is:

$$Q_{вл} = P_л + \frac{n_в.V_в.\rho.K_н}{K_p} + n_в.q_T \qquad (5)$$

and the loading stress requires addition of the weight of empty train –

$$P_л = n_в.q_в$$

where:

$P_л$ - weight of the locomotive. For EL2- $P_л$ series = 147 t;

$n_в$ - number of cars in the train. Assumed $n_в$ =16;

$V_в$ - volume of the bucket of the car. For the trains of Russian and Bulgarian manufacture with dumping cars - $V_в$ =40 m^3;

$K_н$ - coefficient of filling the bucket of the car. Average coefficient in the calculation is $K_н$ =1,02;

K_p - coefficient на swelling. Assumed K_p =1,45;

q_T - coefficient of the cars q_T $\approx$34 t.

Therefore, the loading stress of one train is 2 274 t.

For the both mines, using the ¨Obruchishte¨ dumping area there is a two-shifts regime of railroad haulage. For the aim of safety a coefficient of use is assumed as

$K_в$ = 0,75;

- The loaded overburden spreader AS-6 accepts between 3 and 6 trains per hour. For the aims of safety it is assumed 4 trains/hour.

Therefore, for a twenty four hour period the overburden spreader AS-6 accepts:

$$N_{вл.дн} = n_{вл/h}.T.K_в = 4.24.0,75 = 72 \text{ trains}$$

Then the total gross tonnage for a twenty four hour period is:

$$Q_{бр.дн} = N_{вл.дх}.Q_{бр} = 2274.72 = 163728t$$

This means that the gross tonnage of 1,000,000 t on the movable railroad of overburden spreader AS-6 at the ¨Obruchishte¨ dumping area is accumulated for less than 7 days.

Therefore, after 6-7 days of operating on a new track the gravel in the gravel prism of the movable railroad near the overburden spreader AS-6 is thickened through (5) and its behavior and properties are approximating to those of the elastic body.

THEORETICAL PROVE

Since all the above is known, it is accepted that the matrix (gravel bed) is in equilibrium. Then every plane fragment of it (from first precondition) is in equilibrium. This also applies to the infinitely small element of the matrix – fig.-4.c., i.e. the conditions for equilibrium are in validity.

$$\left\| \begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0 \\ &\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \end{aligned} \right. \qquad (6)$$

Theoretical mechanics and strength of materials (*Kisliakov S.,* 1980; *Федосьев В.,* 1965 *etc.*) reveal that, the infinitely small movements of the matrix through components U and V of the vector Z (from the second precondition) may be introduced down by the equations:

$$\left\| \begin{aligned} &\varepsilon_{xx} = \frac{\partial U}{\partial x} \\ &\varepsilon_{xx} = \frac{\partial V}{\partial y} \\ &\varepsilon_{xy} = \gamma_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \end{aligned} \right. \qquad (7)$$

The generalized type of the Hook law is:

$$\left\| \begin{aligned} &\sigma_x = a_{11}.\varepsilon_{xx} + a_{12}.\varepsilon_{yy} + a_{13}.\varepsilon_{xy} \\ &\sigma_y = a_{21}.\varepsilon_{xx} + a_{22}.\varepsilon_{yy} + a_{23}.\varepsilon_{xy} \\ &\tau_{xy} = a_{31}.\varepsilon_{xx} + a_{32}.\varepsilon_{yy} + a_{33}.\varepsilon_{xy} \end{aligned} \right. \qquad (8)^*$$

* *Equations (8) represent the 2 dimensional case. If a 3-dimensional case is treated the equations are 6, and the coefficients a_{in}-36.*

In the system of equations (8) a_{in} are elastic constants, characterizing the matrix (compacted gravel prism). As it is assumed (the third precondition from IV) that the matrix is arthotropic, these two coefficients, based on the theorem for interaction of operations and calculations (*Федосьев В.,* 1965), are two by two equal.

In the Hook law a substitution is done in the conditions of equilibrium; it is differentiated by "x" and "y" and first and third and second and fourth equations are added.

$$1.\ \frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0$$

$$\frac{\partial\sigma_x}{\partial x}=a_{11}.\frac{\varepsilon_{xx}}{\partial x}+a_{12}.\frac{\varepsilon_{yy}}{\partial x}+a_{13}.\frac{\varepsilon_{xy}}{\partial x}$$
$$\frac{\partial\tau_{xy}}{\partial y}=a_{31}.\frac{\varepsilon_{xx}}{\partial y}+a_{32}.\frac{\varepsilon_{yy}}{\partial y}+a_{33}.\frac{\varepsilon_{xy}}{\partial y}$$

Or

$$\frac{\partial\sigma_x}{\partial x}=a_{11}.\frac{\partial^2 U}{\partial x^2}+a_{12}.\frac{\partial^2 V}{\partial x\partial y}+a_{13}.\frac{\partial^2 U}{\partial x\partial y}+a_{13}.\frac{\partial^2 V}{\partial x^2}$$
$$\frac{\partial\tau_{xy}}{\partial y}=a_{31}.\frac{\partial^2 U}{\partial x\partial y}+a_{32}.\frac{\partial^2 V}{\partial y^2}+a_{33}.\frac{\partial^2 U}{\partial y^2}+a_{33}.\frac{\partial^2 V}{\partial x\partial y}$$

Due to the assumed orthotropic type

$$a_{13}=a_{31}=a_{32}=a_{23}=0$$
$$a_{12}=a_{21}$$

Then the first equation acquires the type by 1'

$$1'.\ \frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=a_{11}.\frac{\partial^2 U}{\partial x^2}+a_{33}.\frac{\partial^2 V}{\partial y^2}+\left(a_{12}+a_{33}\right).\frac{\partial^2 V}{\partial x\partial y}$$

$$2.\ \frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=0$$

Or

Similarly, equation (2) acquires the type by 2'

$$2'.\ \frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}=a_{33}.\frac{\partial^2 V}{\partial x^2}+a_{22}.\frac{\partial^2 V}{\partial y^2}+\left(a_{21}+a_{33}\right).\frac{\partial^2 U}{\partial x\partial y}$$

Or the system of equations for equilibrium (6) acquire the type of (9)

$$\left\|\begin{aligned}&a_{11}\frac{\partial^2 U}{\partial x^2}+a_{11}\frac{\partial^2 U}{\partial x^2}+\left(a_{12}+a_{33}\right)\frac{\partial^2 V}{\partial x\partial y}=0\\&a_{33}\frac{\partial^2 V}{\partial x^2}+a_{22}\frac{\partial^2 V}{\partial y^2}+\left(a_{21}+a_{33}\right)\frac{\partial^2 U}{\partial x\partial y}=0\end{aligned}\right. \qquad (9)$$

On the other side the generalized Law of Hook (*Kisliakov S.,* 1980; *Федосьев В.,* 1965 *etc.*) for the two-dimensional problem acquires the type of (10)

$$\left\|\begin{aligned}&\varepsilon_{xx}=\frac{\sigma_x}{E_{xx}}-\mu_x\frac{\sigma_y}{E_{xx}}\Rightarrow\varepsilon_{xx}=\frac{1}{E_{xx}}\left(\sigma_x-\mu_x.\sigma_y\right)\\&\varepsilon_{yy}=\frac{\sigma_y}{E_{yy}}-\mu_y\frac{\sigma_y}{E_{yy}}\Rightarrow\varepsilon_{yy}=\frac{1}{E_{yy}}\left(\sigma_y-\mu_y.\sigma_y\right)\sigma_y\\&\varepsilon_{xy}=\gamma_x=\frac{\tau_{xy}}{G_{xy}}=\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\Rightarrow\tau_{xy}=G_{xy}\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\end{aligned}\right. \qquad (10)$$

where: μ_x, μ_y – coefficients of Poisson for interrelating transverse and lengthwise deformations;
G_{xy}- second modulus (modulus of Young of shearing.

The first equation of (10) is multiplied by μ_y and is added to the second, and the second is multiplied by μ_x and is added to the first. After certain transformations (6) and (10) acquire the type as shown in (11):

$$\left\|\begin{aligned}&\frac{E_{xx}\sigma_y}{1-\mu_x.\mu_y}.\frac{\partial^2 U}{\partial x^2}+G_{xy}.\frac{\partial^2 U}{\partial y}+\left(\frac{E_{xx}\sigma_y}{1-\mu_x.\mu_y}\mu_x+G_{xy}\right).\frac{\partial^2 V}{\partial x\partial y}=0\\&G_{xy}.\frac{\partial^2\Theta}{\partial x^2}+\frac{E_{yy}\sigma_y}{1-\mu_x.\mu_y}.\frac{\partial^2 V}{\partial y^2}+\left(\frac{E_{yy}\sigma_y}{1-\mu_x.\mu_y}\mu_x+G_{xy}\right).\frac{\partial^2 U}{\partial x\partial y}=0\end{aligned}\right. \qquad (11)$$

The comparison of the system (11) and the system (1) from the basic model of Manevich – Pavlenko (*Manevich L. et al.,* 1982) and the system (9) gives a reason to state, that:

- $a_{11}=\dfrac{E_{xx}}{1-\mu_x.\mu_y}$ in the basic model is indicated by B_1 and expresses the reduced module of elasticity of the matrix on "x";
- $a_{22}=\dfrac{E_{yy}}{1-\mu_x.\mu_y}$ in the basic model is indicated by B_2 and expresses the reduced module of elasticity of the matrix on "y";
- $a_{33}=G_{xy}$ in the basic model is indicated by G and expresses the reduced second modulus (of shearing) of Young;

- $a_{12}=a_{21}=\dfrac{E_{xx}}{1-\mu_x.\mu_y}.\mu_x=\dfrac{E_{yy}}{1-\mu_x.\mu_y}.\mu_y=B_1\mu_x=B_2\mu_y$

Through the accomplished transformations and conclusions it is determined the applicability of the system of equations (1) from the basic model for the task. It remains to reason also the constituition of the third equation – (2) from the basic model. Considerations are as follows:

If accepted that the fiber in the modulus is subordinated to the Law of Hook for uni-dimensional stress state – tensile strength. Then the stress is:

$$\sigma_x=\frac{P}{F} \qquad (12)$$

where F – the surface of the fiber.

The stress according the complete Law of Hook is expressed by (14) and (8)

$$\sigma_x = a_{11}.\varepsilon_{xx} + a_{12}.\varepsilon_{yy} + a_1.\varepsilon_{xy} \tag{13}$$

however

$$a_{12}.\varepsilon_{yy} = 0 \tag{14}$$

(15) comes after the uni-dimensional condition of the fiber

Then

$$a_{13}.\varepsilon_{xy} = 0 \tag{15}$$

(16) following of $a_{13}=a_{31}$ because of the orthotropic type (third basic precondition of VI)

Therefore the tension in the fiber can be expressed yet by:

$$\sigma_x = a_{11}.\varepsilon_{xx} \tag{16}$$

But from the theoretical mechanics and tensile strength (*Kisliakov S.,* 1980; *Федосьев B.,* 1965 *etc.*) and the already applied equations(7), is known that:

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} \tag{17}$$

Then

$$\frac{\sigma_x}{\partial x} = a_{11}\frac{\partial \varepsilon_{xx}}{\partial x} = a_{11}\frac{\partial^2 U}{\partial x^2} \tag{18}$$

On the other hand, the basic model of Manevich – Pavlenko (*Manevich L. et al.,* 1982) treats a condition in which the extracted fiber is still in balance. In the same time by the conclusion (11)

$$a_{11} = \frac{E_{xx}}{1-\mu_x\mu_y} \tag{19}$$

In this case by the reason of single-dimensionality of the extracted fiber μ_y =0, i.e.

$$a_{11} = E_{xx} \tag{20}$$

Therefore

$$\frac{\sigma_x}{\partial x} = E_{xx}\frac{\partial^2 U}{\partial x^2} \tag{21}$$

Considering the third equation (2) from the basic model in a different way and treating the second basic precondition for the components of the vector Z for displacement along axes x and y, and subsitute W (displacement) with its component and comply from (22), that from (13) the surface F should be added, then finally (2) is represented as:

$$F.E_{xx}\frac{\partial^2 U}{\partial x^2} = F_0\delta(x) - 2\tau(x) \tag{22}$$

Therefore, it is assumed for a moment, that the left side of (22) is obvious and the right-handed part should be made clear.

There is the product $F_0\delta(x)$, where F_0 is the extracting force, and δ(x) is delta function of Diraque. The delta function in this case is initiated in order to indicate the characteristics of the force. It [δ(x)] is a generalized function. It does nor have any physical meaning. It shows only, that the force is applied in a point (point force) and that it changes from zero to a certain value. Exactly, that final value brings to extraction. Changing from zero to the final value is subordinated to the law

$$\left.\begin{aligned} \delta(x)\big|_{x=\pm\infty} &= 0 \\ \delta(x)\big|_{x=o} &= \int_{-\infty}^{+\infty}\delta(x)\partial x = 1 \end{aligned}\right\} \tag{23}$$

The geometrical interpretation is shown in fig. 6.

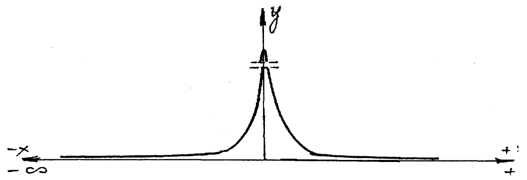

Figure 6. Geometrical interpretation of the integral from delta *function of Diraque*

One of the properties of δ(x) is that its sub-integral surface is always one. This allows the treating of a series of δ(x) of constantly narrowing interval to characterize the change of force from zero to a certain value, in which the extraction takes place. Therefore, the presence of δ(x) in the right-hand side of the equation (22) does not change the force, it only characterizes it.

The physical sense of the last member of the right-hand of the equation (22) needs to be explained. Considerations are as follows:
The equation

$$F.E_{xx}\frac{\partial^2 U}{\partial x^2} = F_0\delta(x) \tag{24}$$

is in fact another type of the Newton's law, usually represented in the type (25).

$$\vec{F} = m.\vec{a} \tag{25}$$

However, from the mathematical physics (*Armanovich J. et al.,* 1969; *Genchev T.,* 1976; *Mechanical engineering* 1982-1994; *Academy of Science of the USSR*, 1971—1982; *Russian Academy of Science,* 1985-1999 etc.) is known, that the completeness of this law includes also the strength of medium, where movement takes place (in this case – the extracted fiber) and its inertness.

If W denotes the coordinate and t the trajectory of the points of the fiber with the time and there is no strength, then the Newton's law is as follows (26)

$$m.W^{''} = F \tag{26}$$

where W is movement of points from extracted fiber.

Since the fiber is moving only along a straight line, the movement of a point from the fiber is a movement along a straight line, in fig. 7.

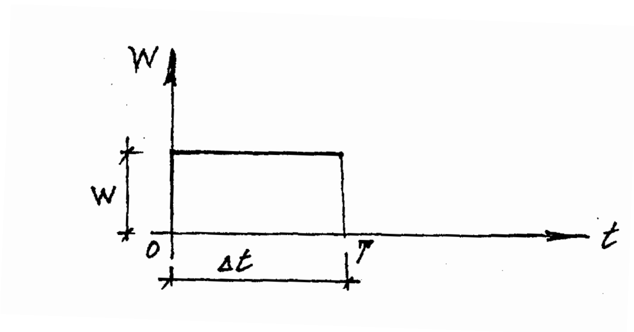

Figure 7.Geometrical representation of a uno-dimensional displacement of points of the fiber for time Δt

In the most general case the law is non-linear, because it depends on the time t, displacement W and its second derivative. A linear solution is needed, because the mathematical physics suggests typical solutions.

Supposing that F depends linearly of displacement W and its first derivative W ‘, a formal mathematical record of this dependence is as follows:

$$F\left(W_1+W_2,W^{'},t\right)=F\left(W_1,W^{'},t\right)+F\left(W_2,W^{'},t\right) \tag{27}$$

That means that

$$W = W_1 + W_2 \tag{28}$$

When a function depends linearly on its first and its second argument, the solution may be presented in the following description:

$$F\left(W,W^{'},t\right)=C_1,W+C_2W^{'}+F\left(t\right) \tag{29}$$

Where C_1 and C_2 are known integral constants.

Then the Newton law in the treated case is:

$$C_1.W^{''}+C_2.W^{'}+C_3.W \;\; =F\left(t\right).\delta\left(x\right) \tag{30}$$

The physical meaning of the constants is as follows:
C_1 – characterizes the mass of the fiber;
C_2 – characterizes the strength (contact strength) of outside medium (matrix), in which fiber moves;
C_3 – characterizes inertness of moving fiber.

The physical meaning of these constants most clearly is illustrated and explained by one of most simple problems in mechanics – the pendulum (fig. 8).

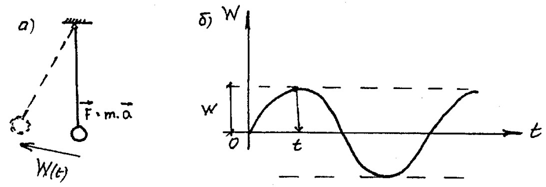

Figure 8. a) Schematic view of a waver, set in motion by a force $\vec{F}$ and displaced at W b) Response of pendulum motion with time

Four cases are possible:

First case: Medium does not have any strength, force is applied only once.

In this case the equation of the pendulum is:

$$W^{''}.\left(t\right)+\omega^2.W.\left(t\right)=0 \tag{31}$$

Solution of the differential equation (31) is:

$$W\left(t\right)=C_1.\cos.\omega t+C_2.\sin.\omega t \tag{32}$$

Here C_i are constants and depend on A (the amplitude) and φ (the phase), and ω is frequency.

Second case: Medium does not have any strength, the force F_0 acts permanently according to a cosine law.

The equation of the pendulum is:

$$W^{''}.\left(t\right)+\omega^2.W.\left(t\right)=F_0.\cos\omega_1 t \tag{33}$$

Solution for the differential equation (33) is:

$$W\left(t\right)=C_1.\cos.\omega t+C_2.\sin.\omega t+\frac{F_0}{\omega^2-\omega_1^2}.\cos\omega_1 t \tag{34}$$

In this case when $\omega_1=\omega$ there is a resonance.

Third case: Medium has a strength, the force is applied only ince.

The equation of the pendulum is:

$$W^{''}.\left(t\right)+a.W^{''}.\left(t\right)+\omega^2.W.\left(t\right)=0 \tag{35}$$

If there is a linear dependence between displacement “W ‘ and time t, then solution depends on the characteristic equation (36):

$$\lambda^2+a\lambda+\omega^2=0 \tag{36}$$

The discriminant D of (36) is:

$$D=a\;\;-4\omega \tag{37}$$

The solution of (35) has a physical sense, when the discriminant D is negative, i.e. D<0. Then solutions of the characteristic equation (36) are:

$$\lambda_{1,2} = \frac{-a \pm i.\sqrt{a^2 - 4\omega}}{2} \tag{38}$$

and it is presented as (39):

$$\lambda_{1,2} = \alpha \pm i.\beta \tag{39}$$

Then solution of the equation of pendulum (35) is:

$$W(t) = C_1.e^{\alpha.t}.\cos.\beta t + C_2..e^{\alpha.t}.\sin.\beta t + \Phi(t) \tag{40}$$

This is the case of gradual attenuation of amplitude and frequency – fig. 9.

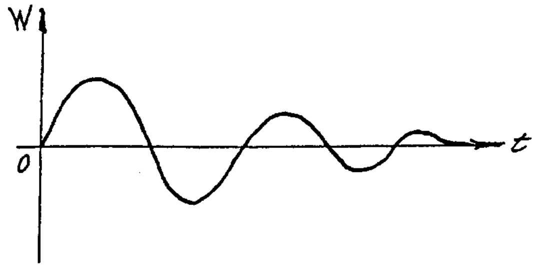

Figure 9. Gradual attenuation of amplitude and frequency of the pendulum

Fourth case: Medium has its own strength, the force F_0 acts according to a cosine law.

The equation of pendulum is:

$$W^{"}.(t) + a.W^{'}.(t) + \omega^2.W.(t) = F_0.\cos\omega_1 t \tag{41}$$

Without paying attention of the solution of (41), and if analyzing it and assuming, that the mass of pendulum is normed to one, then there is a full similarity with the law of Newton and the model of Manevich-Pavlenko (*Manevich L. et al.,* 1982) for the extracted fiber. Furthermore,:

a) the coefficient before the second derivative of displacement W” is one. In the basic model this coefficient is $F.E_{xx}$.

b) the coefficient in front of the first derivative shows the strength of medium, in which the pendulum moves. In the general case this is a strength of friction.

In the basic model of Manevich-Pavlenko this is a contact strength of medium round the fiber. It is indicated by τ(x) and it is transferred to the right-hand side of the equation, because it always counteracts to extracting force. The value of coefficient in front of τ(x) is 2, because it is assumed that it acts simultaneously from the both sides of the fiber – fig. 10.

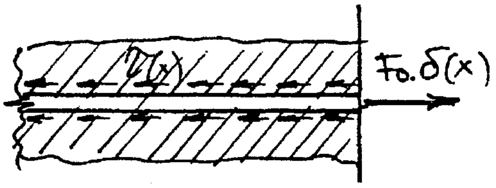

Figure 10. Illustration of contact stress τ(x)

c) The coefficient ω in front of displacement W(t) stands for inertia of the process, which on its own depends on weight of the moving mass.

At the basic model the coefficient C_3 is zero, because the authors Manevich and Pavlenko (*Manevich L. et al.,* 1982) presume, that uni-dimensional fiber is weightless. Finally, the equation in the basic model acquires the type (2), and namely:

$$F.E_{xx}.\frac{\partial^2 U}{\partial x^2} = F_O.\delta(t) - 2\tau(x)$$

This does not comprise all the peculiarities of genesis of the basic model. This is because in order to function, the three equation [system (1) and equation (2)] in one model, it is necessary to associate the system for equilibrium of the matrix (1) and the equation for movement of fiber into a dependence.

This connection of the model (*Manevich L. et al.,* 1982) is formulate by the assigned relation (42).

$$\tau(x) = G\left.\frac{\partial U}{\partial y}\right|_{y=0} \tag{42}$$

It defines, that the contact stress in the direction y=0 is proportional to the reduced second modulus of Young of the matrix. The higher G, the higher τ(x).

The G is just the coefficient a_{33} of the matrix.

$$G = \frac{G_{xy}}{1 - \mu_x \mu_y} \tag{43}$$

In other words, the contact stress is proportional to the elastic modulus of shearing of the matrix and, the inverse proportional to the difference ($1-\mu_x.\mu_y$) , where μ_x. and μ_y are coefficients of Poisson for already compacted gravel bed onto axes x and y.

RESULTS AND CONCLUSIONS

BASIC CONCLUSION is, that the third equation (2) of the basic model does not concern and does not affect by the type of strength, which medium (matrix) effects to the extracted fiber – friction, shearing, shearing of adhesion, shearing of friction etc. The equation only reports on the quantity of strength of medium (matrix) while extracting of the fiber. Just for that, the authors of the model define it as a “contact” and have in mind, that it is manifested and effected simultaneously from both sides along the whole length of extracted fiber.

THE BASIC RESULT is in the proof of the correctness of application of the model, developed by Manevich and Pavlenko for determination of strength, which is applied to an extracted traverse (treated as a fiber) from the side of the gravel bed (treated as a matrix) from a movable railroad.

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