

## STUDYING SOME OF THE TRIBO-CHARACTERISTICS OF MINERAL GRINDING

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### ABSTRACT

Functional responses of the coefficient of friction depending on the technological regime of machines for mineral grinding were tested. The coefficient of friction depending on the fullness of a drum mill was determined.

Key words: tribo-characteristics, mineral grinding, coefficient of friction.

### TASK:

Typical tribo-characteristics for determining the coefficient of friction  $\mu$  depending on the coefficient of fullness are tested for drum mills for mineral grinding.

### THEORETICAL ANALYSIS

Equation 1 from [1] allows the determination of the most general value of friction forces for mills working with different coefficients of fullness  $\varphi$  and different speed regimes.

$$N_{TP} = 19,7 \frac{\sqrt{D}}{\sqrt[4]{1+K^2}} f(\varphi, K) W_{\partial} \operatorname{tg} \rho \left( \frac{1}{\sqrt{\sin \rho}} - 1 \right) (0,576 + \sin \theta), \text{ kW} \quad (1)$$

where

$D$  is the drum diameter, m;

$W_{\partial}$  - weight of grinding bodies, moving along a circular trajectory;

$\varphi$  - coefficient of fullness;

$\theta$  - angle of deviation of the gravity centre;

$K = \frac{R_1}{R}$  radius of the drum and  $R_1$  (radius of the most

inner layer of grinding bodies);

$f(\varphi, K)$  - part of grinding bodies, moving along a circular trajectory;

$\rho$  - angle of friction.

A certain technologic regime in consideration of the theory of Levenson requires the determination of equations of the type:

$$N_{TP} = 10,1 \sqrt{D} W_{\partial} \operatorname{tg} \rho \left( \frac{1}{\sin \rho} - 1 \right), \text{ kW}. \quad (2)$$

Preliminary considerations reveal that in case of equal  $D$ ,  $\varphi$  и  $W_{\partial}$  the equations (2) have equal coefficients, 10, 1, which shows that their powers are equal, i.e. correctness of accepted equations is confirmed.

It is evident that in case of equal values of  $D$ ,  $D$ ,  $W_{\partial}$ ,  $\theta$ ,  $\varphi$  and  $K$  the power of forces of friction depends on the expression:

$$y = \operatorname{tg} \rho \left( \frac{1}{\sin \rho} - 1 \right), \quad (3)$$

where  $\rho$  is the angle of friction.

In case of  $\rho = \frac{\pi}{2}$ , the expression is undefined.

An analysis of the following function reveals:

$$\begin{aligned} \lim_{\rho \rightarrow \frac{\pi}{2}} \operatorname{tg} \rho \left( \frac{1}{\sqrt{\sin \rho}} - 1 \right) &= \lim_{\rho \rightarrow \frac{\pi}{2}} \frac{\operatorname{tg} \rho (1 - \sqrt{\sin \rho})}{\sqrt{\sin \rho}} = \\ &= \lim_{\rho \rightarrow \frac{\pi}{2}} \frac{\sin \rho}{\sqrt{\sin \rho}} \lim_{\rho \rightarrow \frac{\pi}{2}} \frac{1 - \sqrt{\sin \rho}}{\cos \rho} \end{aligned}$$

$$\text{but } \lim_{\rho \rightarrow \frac{\pi}{2}} \frac{\sin \rho}{\sqrt{\sin \rho}} = 1.$$

Then

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \frac{1 - \sqrt{\sin \rho}}{\cos \rho} = \lim_{\rho \rightarrow \frac{\pi}{2}} \frac{-\frac{1}{2} \frac{\cos \rho}{\sqrt{\sin \rho}}}{-\sin \rho} = \lim_{\rho \rightarrow \frac{\pi}{2}} \left( -\frac{1}{2} \right) \frac{\cos \rho}{(\sin \rho)^{3/2}} = 0 \quad (4)$$

Theoretical considerations reveal that when friction becomes infinitely high there will not be any relative motion between grinding bodies and lining of the mill. Work of the forces of friction acquires the value of zero.

The expression  $\frac{1-\sqrt{\sin \rho}}{\cos \rho}$  can be transformed in the following way:

$$\frac{1-\sqrt{\sin \rho}}{\cos \rho} = \frac{1-\sqrt{\cos(\frac{\pi}{2}-\rho)}}{\sin(\frac{\pi}{2}-\rho)} = \frac{1-\sqrt{\cos \alpha}}{\sin \alpha} = \frac{1-\cos \alpha}{\sin \alpha(1+\sqrt{\cos \alpha})}$$

for  $\frac{\pi}{2} - \rho = \alpha$ , or

$$\frac{1-\cos \alpha}{\sin \alpha(1+\sqrt{\cos \alpha})} = \frac{2 \sin^2 \frac{\alpha}{2}}{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}(1+\sqrt{\cos \alpha})} = \frac{\operatorname{tg} \frac{\alpha}{2}}{1+\sqrt{\cos \alpha}}$$

then

$$\lim_{\rho \rightarrow \frac{\pi}{2}} \frac{1-\sqrt{\sin \rho}}{\cos \rho} = \lim_{\alpha \rightarrow 0} \frac{\operatorname{tg} \frac{\alpha}{2}}{1+\sqrt{\cos \alpha}} = 0 \quad (5)$$

The equation (3) is analyzed for determining the value of the angle  $\rho$ , when the work of the forces of friction acquires maximum value. It is evident that minimum work is acquired when  $\rho = \frac{\pi}{2}$  in the lack of relative sliding of milling bodies along the lining.

Or from (3)

$$y' = \frac{1}{\cos^2 \rho} \left( \frac{1}{\sqrt{\sin \rho}} - 1 \right) + \operatorname{tg} \rho \left( -\frac{1}{2} \frac{\cos \rho}{(\sin \rho)^{3/2}} \right) \quad (6)$$

$$y' = \frac{1}{\cos^2 \rho} \left( \frac{1}{\sqrt{\sin \rho}} - 1 \right) - \frac{1}{2} \operatorname{tg} \rho \frac{\cos \rho}{(\sin \rho)^{3/2}} \quad (7)$$

The maximum ( $y_{\max}$ ) is acquired from the expression:

$$\frac{1}{\cos^2 \rho} \left( \frac{1}{\sqrt{\sin \rho}} - 1 \right) - \frac{1}{2} \frac{1}{\sqrt{\sin \rho}} = 0 \quad (8)$$

$$\frac{1}{\sqrt{\sin \rho}} \left( \frac{1}{\cos^2 \rho} - 1 \right) - \frac{1}{2} \frac{1}{\sqrt{\sin \rho}} = 0, \quad (9)$$

iiif  $0 < \rho < \frac{\pi}{2}$  and  $\sqrt{\sin \rho} = t$

$$\frac{1}{\sqrt{\sin \rho}} (2 - \cos^2 \rho) - 2 = 0 \quad (10)$$

$$2 - \cos^2 \rho - 2\sqrt{\sin \rho} = 0 \quad (11)$$

$$1 - \sin^2 \rho - 2\sqrt{\sin \rho} = 0. \quad (12)$$

The equation is reduced to the following:

$$1 + t^4 - 2t = 0$$

$$f(t) = t^4 - 2t + 1 \quad (13)$$

it has two roots  $f(0) = 1$  and  $f(1) = 0$ .

It can be reduced to:

$$\rho(t) = t^3 - t^2 + t - 1 = 0. \quad (14)$$

The above equation has a real root and is solved with an accepted precision

$$t = 0,5437$$

$$t = \sqrt{\sin \rho} = 0,5437$$

$$\rho \approx 17^\circ 12' \text{ and } \mu = \operatorname{tg} \rho = 0,309, \quad (15)$$

i.e. when  $\mu = 0,309$ ,  $N_{mp}$  will possess the maximum value.

## RESULTS AND DISCUSSION

The theoretical analyses and adopted methodic directions allow the determination of some tribo-technologic characteristics of the process of grinding of mineral resources in drum mills. The functional dependence of the power of the coefficient of friction is shown in fig. 1. The figure shows increase of power under values of the coefficient of friction from 0,01 to 0,35 and sharp decrease to  $\mu = 0,4$ , then it remains almost constant in a wide range. It can be considered that this is a result of the integrated action of tested parameters and the change in the composition of grinding and grindable media. Results afford an opportunity for selection of effective technologic regime under given technologic parameters.

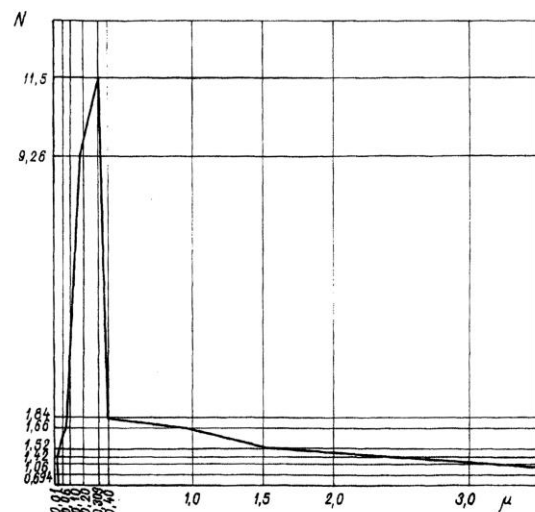


Figure 1.

REFERENCES

- Friction, wearing and lubrication. Handbook, vol. 1, vol. 2.  
Edited by Prof. DSc I. V. Kragelskii and Dr. V. V. Alisina,  
Moscow, Mashinostroenie, 1979 (in Russian).
- Laricov E. A., Viliaevskaya T. I., Units and parts of mechanical  
apparatures. Moscow, Mashinostroenie, 1974 (in Russian).

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