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TRAJECTORY INVESTIGATION OF A HYDRAULICALLY DRIVEN MANIPULATOR

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ABSTARCT
The present work presents an investigation conducted in order to determine the abilities of loading manipulator to move a particular load along desired trajectory through space. Such a trajectory is defined via number of known path points (including initial and final positions). Each path point is represented as position and orientation of the load within the manipulator's working space according to some basic frame. The particular construction under consideration is hydraulically driven, five degree of freedom structure with revolute joints. Given the time elapsed between the path points, the necessary driving torques to counterbalance the system of external static and dynamic forces at each degree of freedom are computed. Such computations are performed using interactive Newton – Euler algorithm. The necessary flow output of the hydraulic pump to provide desired velocities of the manipulator's links is also determined. The computed required forces and flow are compared to the actual parameters of the manipulator in order to evaluate the capabilities of concrete installation to follow predefined trajectory in space.

Trajectory as stated in robotics refer to a time history of position, velocity and acceleration for each degree of freedom These functions are determined by taking into account a number of known positions and orientations of the tool, load or the last link of manipulator, i.e. the basic problem is to move the manipulator from one initial to one final position. Note that, in general, this motion involves change in position, but change in orientation as well relative to some basic frame. Sometimes the trajectory can be specified in more details as sequence of "via points" are given in addition to the starting and ending positions. All the starting, ending locations as well as the "via points" are referred as "trajectory points". We can also include the time factor – a time interval required to complete the motion via the set of trajectory points.

It is obvious, that while in motion, the links of the manipulator are subjected to a system of external forces – both static and dynamic. This is quite true for heavy duty loading equipment, designed to lift and transport considerable loads. In such cases the mass of the links could not be negligible. The set of external forces produces reactions – three dimensional torque and force vectors at each joint. All components of these force and moment vectors are resisted by the structure of the mechanism itself, except for the torque about the joint axis, which is balanced by the hydraulic system actuators

On the other hand, there is constant desire to intensify the working process mainly by increasing link's velocity and thus reducing the working cycle. This leads however to increasing accelerations and escalating dynamic forces and thus – to the greater stress upon the construction.

In general, the universal loading manipulators are intended to perform large scope of activities, including some purely technological tasks. This is why the structure and the driving system are designed having in mind some general requirements, sometimes without regard to any specific applications. When going to specific tasks however, it is necessary to conduct some kind of investigations upon the given construction. Such investigation can be the study of the manipulator's ability to maintain given trajectory in space. This involves not only the ability of the driving system to generate the necessary forces (torques with revolute joins) and velocities at each joint, but the strength of the structure as well.

The present paper is dedicated to trajectory investigation of five degree of freedom, hydraulically driven loading manipulator with revolute joins as shown on Figure 1. The first and the fifth link rotate about vertical axes, while the others arms – about three parallel horizontal axes.

The trajectory investigations are conducted by means of computer simulation and treat the capacity of the existing driving system to:

a/ generate the necessary driving forces (torques) in order to balance the set of external static and dynamic forces;

b/ generate the necessary velocities of arms in order to maintain desired trajectory;

c/ generate desired output power.

The acquired result from such a simulation could be easily utilized in wide area of additional research tasks, including stress analysis, which is out of the scope of this work.

Specifically we will consider transportation of a particular load with defined mass and dimensions from one starting location **S**, via some middle point **M** (without stopping there), to a particular final position **E**. For every one of these trajectory points, the position and orientation of the gravity center of the load in respect to the base coordinate system is known and given by transformation matrix:

$$ {}^{0}_{Q}T = \begin{pmatrix} cos(\varphi) & -sin(\varphi) & 0 & x_c \\ sin(\varphi) & cos(\varphi) & 0 & y_c \\ 0 & 0 & 1 & z_c \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Where x_c, y_c z_c and are the coordinates of the load's center of gravity, while φ represents the rotational angle about the vertical axis passing through that center.

The solution of the present task will be achieved utilizing the Newton – Euler interactive algorithm, which regards the state of static equilibrium of each manipulator's link under arrangement of external static and dynamic forces and reactions in joints. Numbering the links in ascending order staring from the immobile base of the manipulator and placing the local frames (connected rigidly to respective arms) in accordance to some rules, we can apply the Denavit-Hartenberg's transformation which gives the description between two neighboring frames: - Denavit and Hartenberg, 1955

$$ \cdot \; {}^{i}_{i+1}T = \begin{pmatrix} {}^{i}_{i+1}R & {}^{i}\vec{P}_{i+1O} \\ 0\ 0\ 0 & 1 \end{pmatrix}, $$

where ${}^{i}_{i=1}R$ is rotational 3x3 matrix describing the orientation between $i+1$-th and i- th frames, and ${}^{i}P_{i+1O}$ is the $i+1$ frame's origin vector described in respect to the i th coordinate system

The task of simulation will be performed in the following order:

- solving the inverse problem of manipulator kinematics. This will give a positional vector in joint space (five dimensional vector with particular values for each joint angle) corresponding to the starting middle and final trajectory points;

- generating time dependant functions for position, velocity and acceleration for each joint angle while passing between the trajectory points;

- applying the Newton – Euler interactive algorithm in order to determine the dual force-moment vectors acting in each joint.

Solving the inverse problem of manipulator kinematics is routine tasks and can be done considering the position and orientation of the fifth arm's coordinate system in respect to the base frame of the manipulator. The location of the load in this fifth frame has elementary description. One additional condition which somehow simplifies the solution is the fact that the load must be maintained in horizontal orientation during the motion. That condition could be ensured by different techniques (using the pantograph system for this particular example), and provides one simple dependency between joint variables.

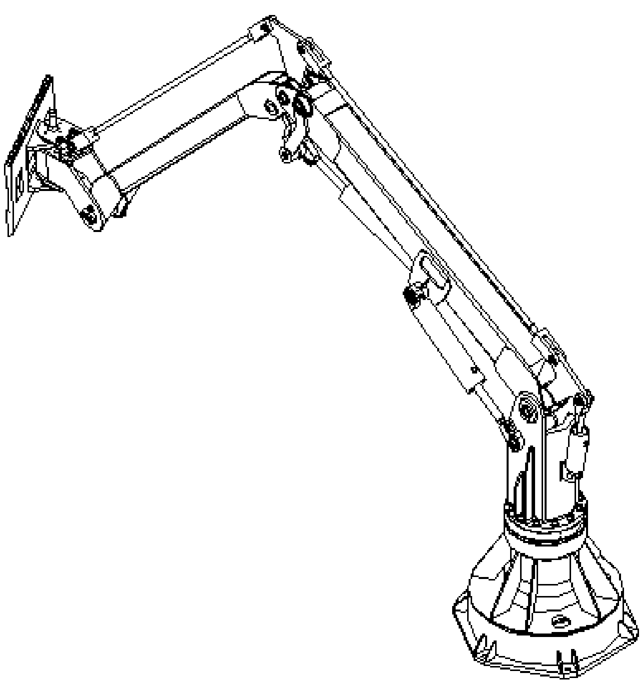

Figure 1. Five degree of freedom loading manipulator

The inverse problem solution produces as a result three values for each joint variable which values correspond to the given trajectory points. Hence the joint variable will vary $\Theta_{Si} \le \Theta_i \le \Theta_M$ for the first section of the motion (from the start to the middle position) and $\Theta_{Mi} \le \Theta_i \le \Theta_{Ei}$ for the second section.

The time elapsed between the trajectory points will be also considered as specified and we will denote $0 \le t \le t_1$ for the first section and for $0 \le t \le t_2$ the second section respectively.

To assure the smooth motion between the points, third order polynomials will be used to specify the time dependent position for each joint variable $\Theta_i(t)$.

$$ \begin{aligned} \Theta_i(t) &= a_{i0} + a_{i1}.t + a_{i2}.t^2 + a_{i3}.t^3 \\ \dot{\Theta}_i(t) &= a_{i1} + 2.a_{i2}.t + 3.a_{i3}.t^2 \end{aligned} \qquad (1) $$

The forth polynomial coefficients will be computed using the initial and final values of the function which are known:

- for the first section

$$ \Theta(t=0) = \Theta_S; \quad \dot{\Theta}(t=0) = 0 $$
$$ \Theta(t=t_1) = \Theta_M; \quad \dot{\Theta}(t=t_1) = \dot{\Theta}_M $$

whence:

$$a_{10} = \Theta_S; \qquad a_{11} = 0;$$
$$a_{12} = \frac{3.(\Theta_M - \Theta_S)}{t_1^2} - \frac{\dot{\Theta}_M}{t_1}; \qquad (2)$$
$$a_{13} = -\frac{2.(\Theta_M - \Theta_S)}{t_1^3} + \frac{\dot{\Theta}_M}{t_1^2};$$

- for the second section:

$$\Theta(t=0) = \Theta_M; \quad \dot{\Theta}(t=0) = \dot{\Theta}_M$$
$$\Theta(t=t_2) = \Theta_E; \quad \dot{\Theta}(t=t_2) = 0$$

whence

$$a_{20} = \Theta_M; \qquad a_{21} = \dot{\Theta}_M;$$
$$a_{22} = \frac{3.(\Theta_E - \Theta_M)}{t_2^2} - \frac{2.\dot{\Theta}_M}{t_2}; \qquad (3)$$
$$a_{23} = -\frac{2.(\Theta_E - \Theta_M)}{t_2^3} + \frac{\dot{\Theta}_M}{t_2^2};$$

We can compute the desired velocity at the "via point" assuming the equality of the acceleration at the end of the first section and the beginning of the second section.

$$\ddot{\Theta}(t=t_1) = \ddot{\Theta}(t=0)$$
$$2.a_{12} + 6a_{13}.t_1 = 2.a_{22}$$

Substituting the coefficients for the middle point we obtain:

$$\dot{\Theta}_M = \frac{3.[t_2^2.(\Theta_M - \Theta_S) + t_1^2.(\Theta_E - \Theta_M)]}{2.t_1.t_2.(t_1 + t_2)}$$

As it was mentioned above, each link of the manipulator could be regarded as in equilibrium when subjected to a set of external forces and joint reactions. When a particular load is being transported, the masses of the load itself and manipulator's arms represent the static external forces. Additional dynamic components are applied at mass centers due to the acceleration of the arms. Considering the balance of the last arm from the kinematical chain we can write:

$$\vec{f}_5 = -{}^5_0R.({}^0\vec{Q} + {}^0\vec{G}_5) + \vec{F}_Q + \vec{F}_5;$$
$$\vec{m}_5 = -\vec{P}_Q \times {}^5_0R.{}^0\vec{Q} - \vec{P}_{G5} \times {}^5_0R.{}^0G_5 + \vec{M}_Q + \qquad (4)$$
$$+ \vec{M}_5 + \vec{P}_Q \times \vec{F}_Q + \vec{P}_{C5} \times \vec{F}_{C5}$$

In the above equation all force and moment vectors are expressed in terms of the coordinate system of the fifth link. Here the following notations are made:

$\vec{f}_5$ - force vector applied at the fifth frame origin;

$\vec{m}_5$ - moment vector applied at the fifth frame origin;

${}^0\vec{Q}$, ${}^0\vec{G}_5$ - weights of the load and the link as vectors expressed in the base, motionless coordinate system;

$\vec{F}_Q$, $\vec{F}_5$ - dynamic forces applied at the mass centers of the load and the fifth link owing to the linear acceleration of the link (three dimensional vectors);

$\vec{M}_Q$, $\vec{M}_5$ - dynamic moments acting on the load and the link owing to the angular acceleration of the link (three dimensional vectors);

$\vec{P}_Q$, $\vec{P}_{C5}$ - positional vectors specifying the gravity centers locations of the load and the fifth link, expressed in respect to the same coordinate system.

It is natural that the weights of the load and each arm of the manipulator are best known in the base coordinate system, where the *Z* axis points vertically upwards. Then we can write a simple description: ${}^0\vec{Q} = [0 \quad 0 \quad -g.m_Q]^T$ In this case the rotational matrix ${}^5_0R$ gives the description of these vectors in respect to the fifth coordinate system, which is in accordance to the equation requirements.

We will use the Newton-Euler equations to compute the dynamic force and moment, Craig (1991):

$$\vec{F}_5 = m_5.\dot{v}_{c5}; \qquad \vec{F}_Q = m_Q.\dot{v}_Q;$$
$$\vec{M}_5 = I_5.\dot{\omega}_5 + \omega_5 \times I_5.\omega_5; \qquad (5)$$
$$\vec{M}_Q = I_Q.\dot{\omega}_5 + \omega_5 \times I_Q.\omega_5;$$

where:

$\dot{v}_{c5}$, $\dot{v}_Q$ - linear acceleration of the mass centers of the load and manipulator's link;

I_5, I_Q - inertia tensor of the load and the link in respect to the coordinate systems with origins at the mass centers, having the same orientation as the link's frame]

$\dot{\omega}_5$, ω_5 - angular acceleration and angular velocity of the link.

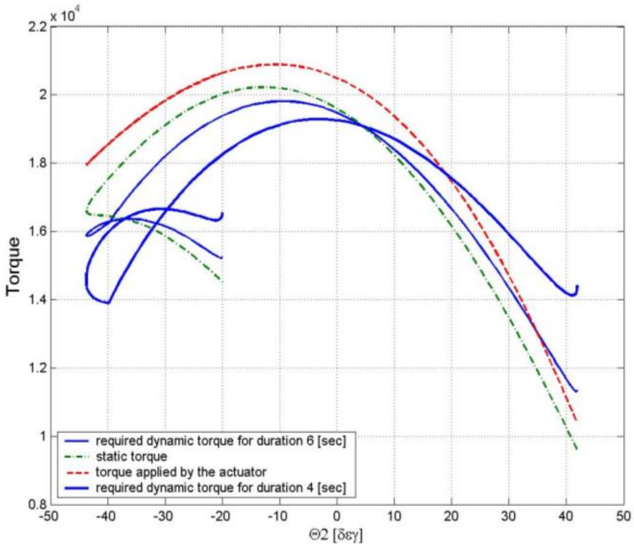

Figure 2 Static and dynamic torques at the second joint [N.m]

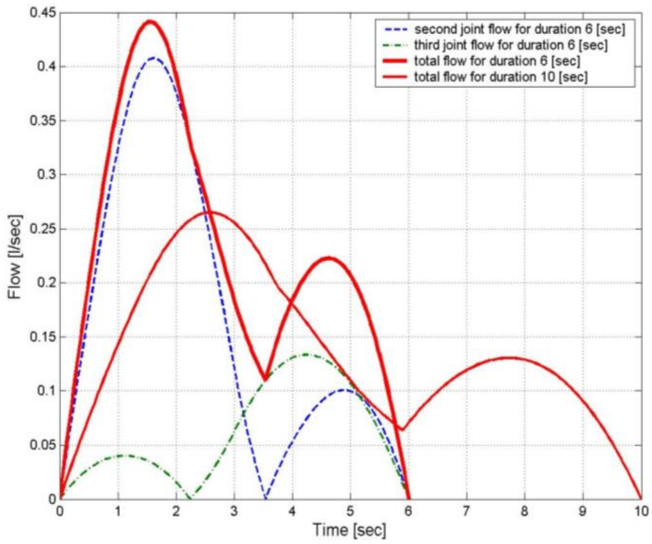

Figure 3 Desired overall flow [l/sec]

Using the inward iterations we can compute the force and moment vectors as reactions at each successive joint in descending order

$4 \le i \le 1$:

$$\vec{f}_i = {}^{i}_{i+1}R.\vec{f}_{i+1} + \vec{F}_i - {}^{i}_{0}R.{}^{0}\vec{G}_i ;$$

$$\begin{aligned}\vec{m}_i = {}^{i}_{i+1}R.\vec{m}_{i+1} + \vec{M}_i - \vec{P}_{Ci} \times {}^{i}_{0}R.{}^{0}G_i + \\ + \vec{P}_{i+1} \times {}^{i}_{i+1}R.\vec{f}_{i+1} + \vec{P}_{C5} \times \vec{F}_i\end{aligned} \qquad (6)$$

$$\begin{aligned}\vec{F}_i &= m_i.\dot{v}_{ci} ; \\ \vec{M}_i &= I_i.\dot{\omega}_i + \omega_i \times I_i.\omega_i ;\end{aligned} \qquad (7)$$

Linear and angular velocities and accelerations of each link can be determined by outward iterations, having in mind that their values for the base frame are equal to zero, Craig (1991). The values for the joint variables, their velocities and accelerations at each moment of time are taken form generated trajectories (third order polynomials) for each degree of freedom (formulas 1-3).

Between them vectors $\vec{f}_i$ and $\vec{m}_i$ have six components altogether. All these components are resided by the structure of the mechanism itself except for the torque about the joint axis, which is to be counterbalanced by the driving system's actuator. As we place local *Z* axes along the axes of rotation between arms, the necessary torque, required to maintain the static equilibrium, will be determined by the dot product of the joint axis vector with the moment vector: $\tau_i = \vec{m}_i.\vec{\hat{z}}_i ; \quad \vec{\hat{z}}_i = [0 \quad 0 \quad 1]^T$.

Figure 2 shows the necessary torques to be applied by the driving system at the second joint (rotation about the first horizontal axis), while moving the load along predetermined trajectory in space. Two sets of values are computed applying the above described simulation method (formulas 4 – 7) – one for the overall duration of the motion of 6 seconds and one for the duration of 10 seconds. The necessary static torque as well as the actual torque, applied by the driving system's actuators are also shown there. It is clearly visible, that increasing the process by reducing the time elapsed leads to greater dynamic loads which can not be balanced by the existing driving system at some part of the trajectory and thus rendering such a trajectory unfeasible.

The ability of the driving system to provide desired link velocities (computed by formulas 1-3) depends mainly on the flow produced by the hydraulic pump, which usually generates flow needed to power several actuators at the same time. Determining the necessary flow requires transforming the angular velocities in joints to linear velocities of hydraulic actuators as soon as linear actuators are used to drive rotational joints. The schemes used to attach the linear actuators to the manipulator's arms (directly or using a kind of leverage) as well as the parameters of the attachments are paramount in such transformations. (Grigorov and Exsarov, 1981, Grigorov 1996).

Figure 3 gives graphical representation of the flow necessary to drive the second, third and fifth links of the manipulator while moving along the same trajectory. Analyzing the graphs, one can see that moving along the path in 6 seconds interval could not be achieved because the lack of the actual pump to supply desired flow.

The simulation presented is made by programming in Matlab mathematical package environment. The masses and center of gravity locations of the manipulator links as well as inertia tensors are determined through creating 3D models in Mechanical Desktop 6 CAD package

CONCLUSIONS

- A method to investigate the capability of hydraulically driven manipulator to transport a load along given trajectory in space is presented. This method is based on comparing the actual driving torques applied by the hydraulic actuators in each joint to the torques necessary to counterbalance the sum of static and dynamic forces acting on links and the load during the motion. One additional comparison is made regarding the flow supplied by the pump and the flow needed to provide desired velocities of the manipulator's links.

- This method can be used to investigate a real machine as well as in the design stage in order to choose suitable driving system parameters, provided a concrete application for the manipulator is given.

- Some additional results such as computing the reaction forces in each joint can be utilized for other purposes such as stress analysis or mechanical design of manipulator's links.

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