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ON THE VELOCITY OF AN OPTIONAL POINT OF CRANK MECHANISM CONNECTING ROD

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ABSTRACT
The paper deals with evaluation of the velocity of an optional point of crank mechanism connecting rod, by applying the so-called secondary model of the mechanism (Piperkov, 1987). Short formulae for fast evaluation of point velocity have been synthesized for the case, when the point is center of mass of the connecting rod. These formulae can be used in the dynamical study of assemblies comprising such mechanisms.

Evaluation of the velocity of optional point of crank mechanism connecting rod is an already solved problem, which as analytical solution leads to voluminous results. The velocity of the point is evaluated in the present paper by means of the so-called *secondary model* of the mechanism, proposed by D. L. Piperkov (1987). With the aid of this model short formulae could be synthesized for calculation of point velocity concerning the case that the point is center of mass of the connecting rod.

Fig. 1 shows arbitrary position of the axial crank mechanism *OAB*. For this position, the velocity $\mathbf{V}_S$ of the arbitrary point *S* of the symmetry axis of the connecting rod *2* is to be found, when applying the secondary model *OAB'* of the mechanism.

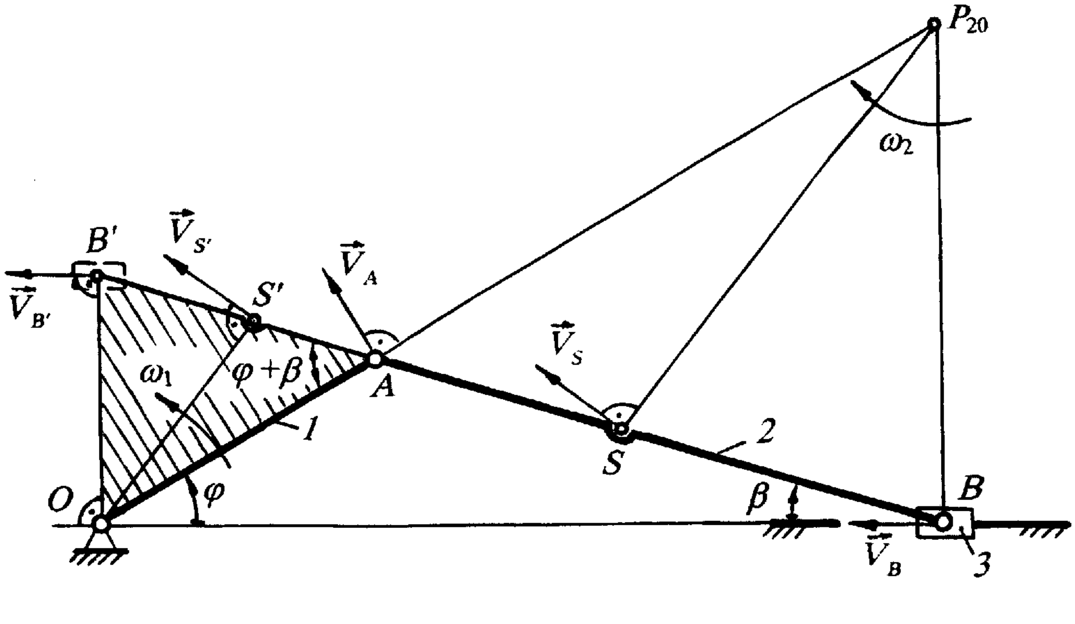

Figure 1

Point *B'* is intersection point of connecting rod direction *2* and the line passing through the axis *O* of the crank *1* and is perpendicular to the straight-line *OB*, which is the path of movement of slider *3*. Line segment *AB'* is called *image* of the line segment *AB*, representing the connecting rod *2*, and points *B'* and *S'* - *images* of the corresponding points *B* and S from the connecting rod. For each arbitrary position of the mechanism the relationship (1) is valid:

$$\frac{AS'}{AB'} = \frac{AS}{AB} = \xi \,, \tag{1}$$

where ξ is a known constant.

In the considered position of the mechanism the secondary model *OAB′* represents a body (designated by shaded triangle), which rotates around the axis *O* with the known rotational speed ω_1 of the crank *1*. In this case there is no problem to evaluate the velocity $\boldsymbol{V}_{S'}$ of image *S′*. The directrix of this velocity is perpendicular to the line segment *OS′*, and its direction corresponds to the direction of ω_1. The magnitude of $\boldsymbol{V}_{S'}$ is

$$V_{S'} = \omega_1 . OS' \,. \tag{2}$$

With the evaluation o f $\boldsymbol{V}_{S'}$ the searched velocity $\boldsymbol{V}_S$ of the optional point *S* from the connecting rod *2* is also evaluated, because it comes out, that they are equal, i. e.

$$\boldsymbol{V}_S = \boldsymbol{V}_{S'} \,. \tag{3}$$

This statement can be proved, having in view, that the point P_{20} is the absolute instantaneous velocity center (IVC) for the connecting rod *2*, and that triangles ABP_{20} and *OAB′* are similar. For the magnitude of the velocity $\boldsymbol{V}_S$ can be declared

$$V_S = \omega_2 . P_{20}S \,, \tag{4}$$

where the rotational speed ω_2 of the connecting rod *2*, expressed by the velocity of point *A*, is

$$\omega_2 = \frac{V_A}{P_{20}A} = \frac{\omega_1 . OA}{P_{20}A} = \frac{\omega_1 . OS'}{P_{20}S} \,. \tag{5}$$

Then

$$V_S = \omega_1 . OS' \,. \tag{6}$$

After comparison of the expressions (6) and (2) is established, that velocities $\boldsymbol{V}_S$ and $\boldsymbol{V}_{S'}$ have equal magnitudes. The equivalence of directress and directions of these velocities follows from the parallelism of straight lines $P_{20}S$ and *OS′*, and from the direction of rotational speeds ω_2 and ω_1. By this means the statement (3) has been proved.

Creation of secondary model *OAB′* for each position of the crank mechanism *OAB* gives the possibility for graphic evaluation of the investigated velocity $\boldsymbol{V}_S$. At the same time, this model permits a relatively easy way for determining the analytical expression of the function $V_S = V_S(\varphi)$, where φ is geometrical parameter, defining the rotation of the crank *1* related to the horizontal line in the side of slider *3*. Using triangle *OAS′*, with the aid of the theorem of cosines, the line segment *OS′* is defined. This gives us the possibility to write down for the velocity $\boldsymbol{V}_S$ the following expression

$$V_S = V_{S'} = \omega_1 . OS' = \\ = \omega_1 \sqrt{(AS')^2 + r^2 - 2AS'.r\cos(\varphi + \beta)} \,. \tag{7}$$

Here for briefness is nominated *r = OA*, and β is the geometrical parameter, defining the rotation of connecting rod *2* against the horizontal line. Further, using the theorem of sines, from the triangle *OAB′* we determine

$$AB' = r\frac{\cos\varphi}{\cos\beta} \,. \tag{8}$$

Taking into consideration (1), after substituting (8) in (7), and using the equations $\sin\beta = \lambda\sin\varphi$ (triangle *OAB* - theorem of sines) and $\cos\beta = \sqrt{1-\lambda^2\sin^2\varphi}$ (for briefness here is nominated $\lambda = r/AB$), then for the function $V_S = V_S(\varphi)$ we obtain the following analytical expression

$$V_S = V_S(\varphi) = r\omega_1 \sqrt{\frac{\xi^2\cos^2\varphi}{1-\lambda^2\sin^2\varphi} + 1 - 2\xi\cos\varphi\left(\cos\varphi - \frac{\lambda\sin^2\varphi}{\sqrt{1-\lambda^2\sin^2\varphi}}\right)} \,. \tag{9}$$

It is important for the dynamical study of crank mechanism that we know how does proceed the variation of the gear ratio V_S/ω_1 as a function of φ at ω_1 = const , particularly when $\boldsymbol{V}_S$ is the velocity of the mass center *S* of the connecting rod *2*. Obviously, the answer is in the relationship (9), but for the requirements of practice it appears to be too complicated. The secondary model *OAB′* gives us the possibility to use the line segment *OS′* to notice easily the nature of change of this quantity, as well as to find short analytical relationships for its description.

In fig. 2 is presented graphically the variation of the gear ratio V_S/ω_1 for the axial crank mechanism *AOB,* when φ varies from 0° to 180°. In the interval of change of φ from 180° to 360°, the variation is symmetrical to the straight line *OB*. As it can be seen, at $\varphi = 0$ for the gear ratio could be obtained

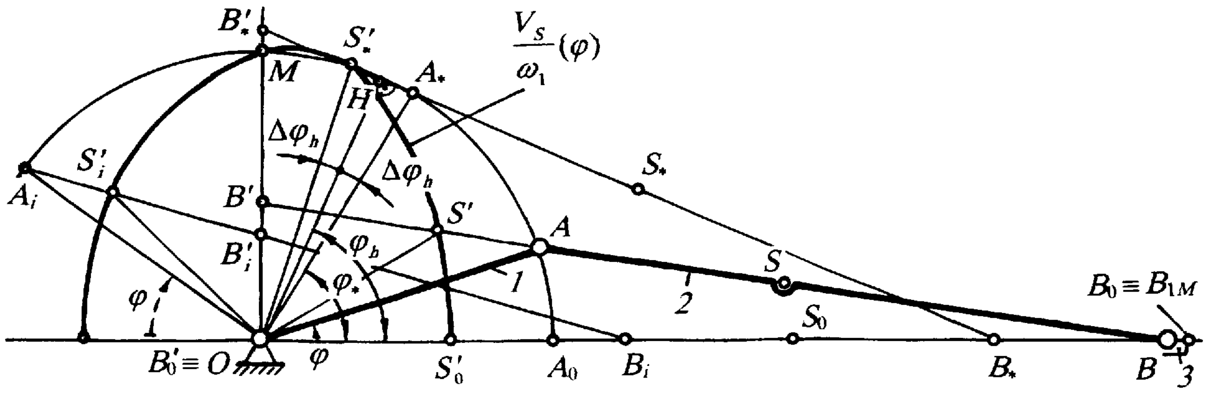

Figure 2

$$OS' = \frac{V_S}{\omega_1} = r - r\frac{A_0 S'_0}{A_0 B'_0} = r(1-\xi). \qquad (10)$$

The increase of φ leads to increment of the gear ratio V_S/ω_1 too. For one characteristic angle $\varphi = \varphi_*$

$$OS' = \frac{V_S}{\omega_1} = r\ , \qquad (11)$$

and after that

$$OS' = \frac{V_S}{\omega_1} > r\ . \qquad (12)$$

At $\varphi = 90^o$ the ratio V_S/ω_1 arrives at the value (11) again, after that it begins continuously to decrease, until it reaches at $\varphi = 180^o$ its initial magnitude (10).

The characteristic angle φ_* is determined by the position OA_*B_* of the crank mechanism, where

$$OA_* = OS'_* = r\ . \qquad (13)$$

The triangle $OA_*S'_*$ is isosceles and its height OH with angular coordinate φ_h divides the central angle $A_*OS'_*$ to two equal angles $\Delta\varphi_h$. Then for the characteristic angle φ_* can be written

$$\varphi_* = \varphi_h - \Delta\varphi_h\ . \qquad (14)$$

Further the point S is regarded as mass center of the connecting rode *2*. For the crank mechanism *OAB* very often $\lambda = OA/AB < 1/3$, and for the location of the mass center S is valid the condition $\xi = AS/AB < 1/2$. At these limitations we can assume with enough accuracy that

$$\varphi_h \approx \arctan\frac{A_*B_*}{OA_*} = \arctan\frac{1}{\lambda} \qquad (15)$$

and

$$\frac{2\Delta\varphi_h}{90^0 - \varphi_*} = \frac{\overset{\frown}{A_*S'_*}}{\overset{\frown}{A_*M}} \approx \frac{A_*S'_*}{A_*B'_*} = \xi\ , \qquad (16)$$

from there

$$\Delta\varphi_h \approx \frac{\xi}{2}\left(90^0 - \varphi_h\right). \qquad (17)$$

After substitution of (15) and (17) in (14), for the characteristic angle φ_* is obtained

$$\varphi_* \approx \frac{2\arctan\frac{1}{\lambda} - 90^0\xi}{2-\xi}\ . \qquad (18)$$

The variation of the gear ratio V_S/ω_1 in the interval of φ from 0^o to 180^o, which is graphically shown in fig. 2 , can be described approximately in the sections with the aid of short trigonometrical relationships. Obviously these relationships should obtain the mentioned values of V_S/ω_1 for $\varphi = 0^o$, φ_*, 90^o, 180^o, as well as for the limit case $\lambda = \xi = 0$, at which, going out from (9) it appears, that $V_S/\omega_1 = r$. On the base of these requirements and the observation of the dimensional correspondence, as well as based on a lot of numerical experiments, aiming the achievement of a specific relative error, have been synthesized reasonably short trigonometrical relationships, given in table 1.

When the geometrical parameter φ of the crank *1* has been read from the horizontal line, which related to the axis O is situated on the opposite side of the slider *3* (in fig. 2 the parameter φ for that case is given with dash-line), the graphical form of the gear ratio

Table 1

Limitations: $\lambda \le \frac{1}{3}$ и $\xi \le \frac{1}{2}$		
$0 \le \varphi \le \varphi_*$	$\rightarrow$	$\frac{V_S}{\omega_1} \approx r\left[1-\xi\cos^2\left(90^0\frac{\varphi}{\varphi_*}\right)-0{,}16\xi(2\lambda-\xi)\sin\left(180^0\frac{\varphi}{\varphi_*}\right)\right]$
$\varphi_* \le \varphi \le 90^o$	$\rightarrow$	$\frac{V_S}{\omega_1} \approx r$
$90^o \le \varphi \le 180^o$	$\rightarrow$	$\frac{V_S}{\omega_1} \approx r\left[1-\xi\cos^2\varphi+0{,}56\frac{180^0-\varphi}{90^0}\xi^2(3\lambda-\xi)\sin 2\varphi\right]$

V_S / ω_1 as a function of φ (Fig. 2) is kept the same, but according to the new situation the characteristic angle φ_* is given with the expression

$$\varphi_* \approx 180^0 - \frac{2\arctan\frac{1}{\lambda} - 90^0\xi}{2-\xi} . \qquad (19)$$

The short relationships from table 1 for this case are also subjected to changes. The formula (19) and the modified relationships are obtained correspondingly from the expression (18) and the relationships in table 1, when we substitute formally φ_* by $180^o - \varphi_*$, and φ by $180^o - \varphi$.

The relative error of the short relationships at values of λ in the limits of $1/10 \le \lambda \le 1/3$ and the above said values of ξ, attains the magnitude 2%. These limits of variation of λ are mostly used in practice. At $\lambda = 0$ and $\xi = 1/2$ this error increases to 2,5%, but its sense is generally theoretical. Without the last summand in the first and the third formula of table 1 the relative error is higher, but it is not higher than 6% for the specified limitations of the parameters λ и ξ.

CONCLUSION

Through the application of the secondary model of crank mechanism has been regarded the possibility of determination of the velocity of optional point from the axis of symmetry of connecting rod. Short formulae have been synthesized for the evaluation of velocity (gear ratio) of the mass center of connecting rod. The relative error of the obtained by these formulae results (in the limits of the parameters λ and ξ, where the mechanism is being used) is not higher than 2%.

REFERENCES

Piperkov, D. L. 1987. Secondary modeling in theoretical mechanics. *Author's summary of dissertation for doctor degree "candidate of technical sciences"*. Sofia, Technical University.

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