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DYNAMICS EVALUATION OF RISK FACTORS

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ABSTRACT
IIn conventional studies of risk, the risk factors are assumed to be determined or random values. By the probabilistic method of study the statistical laws of distribution and the relevant numerical values are established. Nevertheless it takes into account the random character of risk factors, this approach is static and does not take into account their time dynamics.
In this work risk factors are analyzed as random processes on the basis of their continuous realizations.
It establishes: mathematical expectation and their dispersion; correlation function and its normalized value; spectral density and its normalized value.
The use of this approach allows evaluation of their time dynamics.
By applying the theory of random ejection two criteria are introduced – numbers of exceeding of the limit values per unit of time and duration of exceeding. On the basis of these criteria, as well as on the basis of the above-described characteristics, summarized evaluation of risk factors is made. The introduced criteria are established under specific confidence level, which is specified depending on the significance and degree of certainty of the examination. By this method the objective character of variation of risk factors is taken into account and evaluation of their dynamics is made.

Currently the methods of examination of risk factors are based on discrete measurements in time. Arithmetical mean values are determined and then they are compared to the limit values. On the basis of this comparison conclusions are made for the degree of compliance with the standard values.

In fact risk factors are dynamic processes that feature significant variations. The discrete measurements are not able to report that dynamics and so their evaluation is not objective.

In order to eliminate the mentioned disadvantages of discrete measurements, probabilistic-statistical method of analysis and evaluation is experimentally applied. It is based on the theory of random processes [1].

Continuous risk factors with normal distribution are examined. Function and density of distribution are used as its basic statistical characteristics [1].

Their probabilistic characteristics are derived:

- mathematical expectation $m_x(t)$;
- dispersion $\sigma_x^2(t)$;
- correlation function $R(\tau)$;
- spectral density $S(\omega)$.

Verification for stationariness and ergodicity is made:

Stationariness is examined in its wide sense, characterized with the equalities:

- $m_x(t) = m_x = \text{const.}$,
- $\sigma_x^2(t) = \sigma_x^2 = \text{const.}$,
  $R_x(t_1,t_2) = R_x(t_2 - t_1) = R_x(t)$.

Ergodicity is accepted on the basis of coincidence of the statistical characteristics, calculated according to the numbers of realizations, with those, calculated for continuous enough and averaged in time realization of immissions. As a result determination of probabilistic characteristics is simplified:

$$m[x] = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt;$$

$$\sigma_x^2[t] = \lim_{T\to\infty} \frac{1}{T}\int_0^T [\overset{o}{x}(t)]^2 dt;$$

$$R_x(t) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \overset{o}{x}(t)\,\overset{o}{x}(t+\tau)dt,$$

where $\overset{o}{x}(t) = x(t) - m(x)$ is the centred realization of random processes.

Correlation function and spectral density are applied to find out the internal structure of processes of risk factors. They are related to the transformations:

$$R_x(\tau) = \int_0^\infty S_x(\omega)\cos\omega\tau d\omega;$$

$$S_x(\omega) = \int_0^\infty R_x(\omega)\cos\omega\tau d\omega.$$

Practically the aim is that through the correlation function $R_x(\tau)$ to be established the process links and the character of progress as a function of time, and through the spectral density its $S_x(\omega)$ -frequency composition.

The normalized correlation functions $\rho(\tau)$ and the normalized spectral densities $\sigma(\omega)$ for the whole period of observation are established, as well as their characteristics: time τ_0 of correlation drop; the average half-period τ_ρ; attenuation frequency ω_ρ of correlation function; cutting frequency ω_c; frequency ω_0 of the maximum value of spectral density; spectral width $\Delta\omega$.

The modeling of continuous risk factors is made through the typical correlation function [1]

$$R(\tau) = \sigma_x{}^2 e^{-\mu|\tau|}(\cos\beta\tau + \frac{\mu}{\beta}\sin\beta\,|\,\tau\,|).$$

where μ, β are coefficient of the function.

According to the valid standards emissions and immissions of a part of the risk factors are limited one-sidedly by maximum value (sound pressure level, concentrations of harmful substances, etc.) or by minimum value (for example illuminance).

Another part of risk factors are standardized two-sidedly by maximum and by minimum value. Such are temperature, relative humidity, air speed, etc.

The carried out examinations, as well as examinations of other authors show that the continuous risk factors feature normal law of distribution.

By using these results for evaluation of emissions and immissions of risk factors, we apply the theory of random ejection [2,3].

We bring the task to determination of numbers of exceeding of standard values and duration of exceeding .

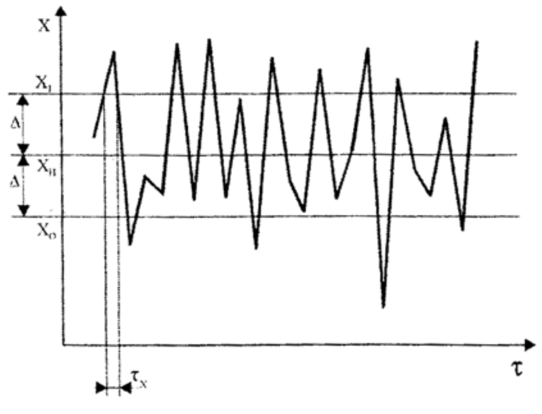

Figure1. One-sided and two-sided limitation of hazardous values

We designate with X the characteristics of risk factors, and with X_o – the limit values that shall not be exceeded one-sidedly.

The condition not to exceed X_o, expressed through the probability P the value X to be less than X_o may be written as

$P[X < Xo] < \alpha$

where α is the level of confidence.

The level of confidence is assumed depending on the nature of the risk factor and the degree of its influence on the particular objects (α = 0,95; 0,99; 0,999; 0,9999).

We designate the mathematical expectation with $m_x = < x >$ and the dispersion with $G_x^2 = D(x)$.

Based on the normal law of distribution the level of confidence α will be:

$$\alpha = 0{,}5\left[1 + \Phi\left(\frac{x_0 - m_x}{\sigma_x}\right)\right], \qquad (1)$$

where $\Phi(x)$ is the Laplas's function, defined as:

$$\Phi(x) = \sqrt{\frac{2}{\pi}}\int_0^x e^{-\frac{t^2}{2}}dt \qquad (2)$$

From (2) follows that

$$\Phi\left(\frac{x_0 - m_x}{\sigma_x}\right) = 2\alpha - 1 \qquad (3)$$

We accept the designation

$$t_{2\alpha-1} = \Phi^{-1}(2\alpha - 1) \Rightarrow \frac{x_0 - m_x}{\sigma_x} = t_{2\alpha-1} \qquad (4)$$

where $t_{2\alpha-1}$ is the quantity of the inverse function of Laplas.

From here we derive the basic condition for the limitation

$$x_0 - m_x \geq t_{2\alpha-1}\sigma_x \qquad (5)$$

The values 2α-1 and $t_{2\alpha-1}$ are given in table 1.

Table 1 The quantity of values $2\alpha - 1$ и $t_{2\alpha-1}$ as a function of α for emissions and immissions of risk factors with one-sided limitation:

α	0,95	0,99	0,999	0,9999
2α-1	0,886	0,98	0,998	0,9998
$t_{2\alpha-1}$	1,96	2,33	3,09	3,72

The condition for one-sided limitation of mathematical expectation m_x is expressed as:

$$x_0 - m_x \geq t_{2\alpha-1}\sigma_x,$$

or

$$m_x \leq x_0 - t_{2\alpha-1}.$$

For two-sided limitation the following condition is to be kept:

$$x_0 \leq x < x_1$$

where x_0 and x_1 are respectively lower and upper limit.
The condition for keeping these limits is:

$$P[x_0 \leq x < x_1] = \alpha. \quad (6)$$

From the condition for normal distribution follows:

$$0{,}5\left[\Phi\left(\frac{x_1 - m_x}{\sigma_x}\right) - \Phi\left(\frac{x_0 - m_x}{\sigma_x}\right) = \alpha\right] \quad (7)$$

We assume $m_x = (x_0 + x_1)/2$, from where

$$\Phi\left(\frac{x_1 - x_o}{2\sigma_x}\right) = \alpha \quad or \quad \frac{x_1 - x_o}{2\sigma_x} = t_\alpha \Rightarrow \sigma_x = \frac{x_1 - x_o}{2t_\alpha}$$

where $t_\alpha = \Phi(\alpha)$ is the argument of Laplas (tabl.2).

Table 2 The values of t_α for emissions and immissions of risk factors with two-sided limitation

α	0,95	0,99	-0,999	0,9999
t_α	1,42	2,58	3,29	3,9

Therefore, for risk factors with two-sided limitation the following condition shall be kept:

$$\sigma_x \leq \frac{x_1 - x_0}{2t_\alpha} \quad (8)$$

From the above two basic conclusions follow:

- The one-sided limitation of risk factors leads to limitation of their average value – the mathematical expectation;
- The two-sided limitation of hazardous values leads to limitation of the their dispersion characteristics – the root-meansquare deviation.

The mentioned conclusions provide grounds to define the tolerance Δ, which is to be introduced for specific examinations of emissions and immissions:

$$\Delta_{HX} = |x_H - m_x| \text{ и } \sigma_x = \sigma_H + \Delta_\sigma,$$

where x_H и σ_H are the standard values of the mathematical expectation and the root-meansquare deviation.

We examine the limitations x_H and σ_H with reference to the limitations of the values of characteristics of emissions and immissions of risk factors.

For one-sided limitation we get:

$$x_H \leq x_0 - t_{2\alpha-1}(\sigma_H + \Delta\sigma) - \Delta_{HX} \quad (9)$$

Through normalization with the values x_H and σ_H the relative tolerances are obtained:

$$\overline{\Delta}_\sigma = \frac{\Delta_\sigma}{\sigma_H}; \quad \overline{\Delta}_x = \frac{\Delta_{HX}}{x_H}. \quad (10)$$

Then we obtain

$$\frac{x_0}{x_H} \geq 1 + t_{2\alpha-1}(1 + \overline{\Delta}_\sigma)\tilde{\sigma}_H + \overline{\Delta}_x \quad (11)$$

where: $\tilde{\sigma}_H = \frac{\sigma_H}{x_H}$ is the relative root-meansquare deviation by nominal values. It is related to the variance coefficient through the relation

$$V_x = \frac{\sigma_x}{m_x},$$

Finally the limitation for the normalized parameter is obtained:

$$x_H \leq \frac{x_0}{1 + t_{2\alpha-1}(1 + \overline{\Delta}_\sigma)\tilde{\sigma} + \overline{\Delta}_x} \quad (12)$$

The regularities of variation of ratio x_H/x_0 as a function from $\overline{\Delta}_x$ when $\overline{\Delta}_\sigma = 0$ and different $\tilde{\sigma}_H$ for hazardous values with one-sided limitation are shown on fig. 2.

In case of two-sided limitations after introduction of tolerances we may express α as follows:

$$\alpha \leq \frac{1}{2}\left\{\Phi\left[\frac{x_1 - (x_H + \Delta_{HX})}{\sigma_H \pm \overline{\Delta}_\sigma}\right] - \Phi\left[\frac{x_0 - (x_H \pm \Delta_{HX}}{\sigma_H \pm \Delta_\sigma}\right]\right\} < 1$$

When choosing the sign we put the condition not to be violated in the more dangerous case, i.e.:

$$\alpha \leq \frac{1}{2}\left\{\Phi\left[\frac{x_1 - (x_H + \Delta_{HX}}{\sigma_H + \overline{\Delta}_\sigma}\right] - \Phi\left[\frac{x_0 - (x_H - \Delta_{HX}}{\sigma_H - \Delta_\sigma}\right]\right\} < 1.$$

Since α tends to one, the addends will also tend to one by absolute value. The determinant value will be the one with smaller argument. On that basis we obtain

$$-\Phi\left[\frac{x_0 - (x_H - \Delta_{HX})}{\sigma_H - \Delta_\sigma}\right] = \Phi\left(\frac{x_H - (x_0 - \Delta_{HX})}{\sigma_H - \Delta_\sigma}\right) \cong 1$$

So the expression gets the form

$$2\alpha \le 1+\Phi\left(\frac{x_1-x_H-\Delta_{HX}}{\sigma_H+\Delta_\sigma}\right)<2 \qquad (13)$$

Taking into account the lower and upper limitation

$$x_H=\frac{x_1+x_0}{2} \qquad (14)$$

we get

$$\Phi\left(\frac{\frac{x_1-x_0}{2}-\Delta_{HX}}{\sigma_H+\Delta_\sigma}\right)\ge 2\alpha-1 \qquad (15)$$

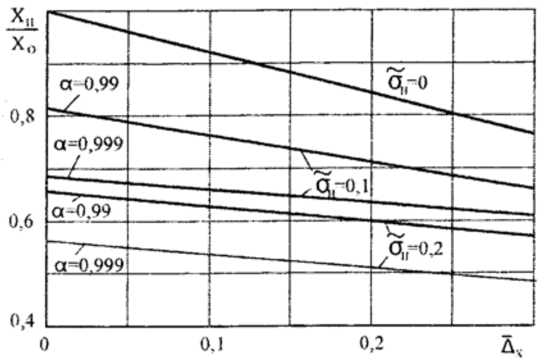

Figure 2. Regularities of variation of x_H/x_0 as a function of $\overline{\Delta}_x$ for risk factors with one-sided limitation

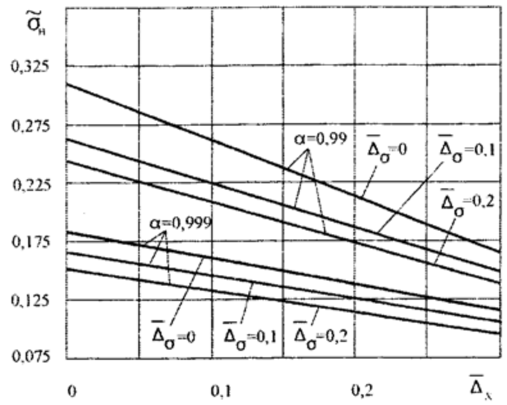

Figure 3. Regularities of variation of $\overline{\sigma}_H$ as a function of $\overline{\Delta}_x$ for risk factors with two-sided limitation.

Thereof follows that

$$\Phi\left(\frac{\frac{x_1-x_0}{2}-\Delta_{HX}}{\sigma_H+\Delta_\sigma}\right)\ge 2\alpha-1 \qquad (15)$$

The relative tolerance will be

$$\tilde{\sigma}_H \le \frac{\overline{\delta}_x-\overline{\Delta}_x}{(1+\overline{\Delta}_\sigma)t_{2\alpha-1}} \qquad (16)$$

where $\overline{\delta}_x=(x_1-x_0)/(x_1+x_0)$ is the relative value of the field of limitation of the risk factor.

The regularity of variation of $\overline{\delta}_x$ when $\overline{\delta}_x$ = 0,7 and different $\overline{\Delta}_\sigma$ and α for risk factors with two-sided limitation is shown on fig. 3.

Exceeding of risk factors out of the area of limitations well be characterized as follows:

On the basis of the assumption that the risk factor x is a random, stationary process with mathematical expectation m_x and with the above-mentioned correlation function:

- The average numbers of exceeding n_{x_0} above $x = x_0$ in case of one-sided tolerance is

$$n_{x_0}=\frac{\sqrt{\mu^2+\beta^2}}{2\pi}e^{-\frac{(x_0-m_x)^2}{2\sigma_x^2}}$$

- The average duration τ_{x_0} of one ejection above the limit x_0 is

$$\tau_{x_0}=\frac{\pi}{\sqrt{\mu^2+\beta^2}}e^{\frac{(x_0-m_x)^2}{2\sigma_x^2}}\left[1-\Phi\left(\frac{x_0-m_x}{\sigma_x}\right)\right].$$

The characteristics x_H and $\overline{\sigma}_H$ of emissions and immissions of risk factors are chosen in such a way that the probability to violate the conditions of limitation to be lower than the level of confidence α.

For that reason when calculating the characteristics of exceeding the more hazardous case will be taken into account i.e. for one-sided tolerance, the following equality will be valid:

$$x_0-m_x=t_{2\alpha-1}\sigma_x;$$

and for two-sided tolerance

$$m_x-x_0=x_1-m_x=t_\alpha\sigma_x.$$

Then the average numbers of exceeding n_{x0} and their average duration τ_{x0} will be:

- For risk factors with one-sided limitation

$$n_{x_0}\le\frac{\sqrt{\mu^2+\beta^2}}{2\pi}e^{-\frac{t^2_{2\alpha-1}}{2}};$$

$$\tau_{x_0}\le\frac{2\pi}{\sqrt{\mu^2+\beta^2}}e^{-\frac{t^2_{2\alpha-1}}{2}}(1-\alpha);$$

- for risk factors with two-sided limitation

$$n_{x_0} \le \frac{\sqrt{\mu^2+\beta^2}}{\pi} e^{-\frac{t^2_\alpha}{2}} ;$$

$$n_{x_0} \le \frac{\pi}{\sqrt{\mu^2+\beta^2}} e^{\frac{t^2_\alpha}{2}} (1-\alpha) .$$

The analysis of the derived dependencies shows the influence of the selection of a level of confidence α. The values of the expressions $n_{x_0} / \sqrt{\mu^2+\beta^2}$ and $\tau_{x_0} \sqrt{\mu^2+\beta^2}$, which are given in Table 3 depend on it.

Table 3 The values of $n_{x_0} / \sqrt{\mu^2+\beta^2}$ and $\tau_{x_0} \sqrt{\mu^2+\beta^2}$ depending on the level of confidence α

Level of confidence α	One-sided limitation	
	$n_{x_0} / \sqrt{\mu^2+\beta^2}$	$\tau_{x_0} \sqrt{\mu^2+\beta^2}$
0,99	0,01054	0,94853
0,999	0,001344	0,7439
0,9999	0,0001573	0,6356
Level of confidence α	Two-sided limitation	
	$n_{x_0} / \sqrt{\mu^2+\beta^2}$	$\tau_{x_0} \sqrt{\mu^2+\beta^2}$
0,99	0,01141	0,8761
0,999	0,001421	0,704
0,9999	0,0001585	0,6309

The presented data provide opportunity to determine the numbers of exceeding and its average duration. By other side their analysis shows that upon increase of the level of confidence by one order, the time during which one limit exceeding occurs also increases approximately with an order.

The presented approach allows:

- To establish the probabilistic characteristics of risk factors, as well as the links of the process of progress;
- To determine their frequency composition;
- To evaluate the numbers and duration of exceeding of limit values in case of one-sided and two-sided limitation;
- To evaluate completely the dynamics of risk factors, which complies with their objective progress in time.

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