**50 years University of Mining and Geology "St. Ivan Rilski"**
**Annual, vol. 46, part III, Mechanization, Electrification and Automation in Mines, Sofia, 2003, pp.225-226**

# STATIONARY STREAM OF VISCOSE FLUID IN A PIPE WITH FINITE LENGTH

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ABSTRACT
A finite length pipe, filled with viscose fluid, with bases rotating in opposite directions with constant angle velocities, dragging the fluid, is looked over. The velocity field is defined.

Finite cylinder with radius R and length l, filled with viscose fluid which has up and down bases rotating in opposite directions with angle velocities of ω and $-\omega_1$ respectively (see Fig.1), is considered. Defining the velocity field is the target of the present paper.

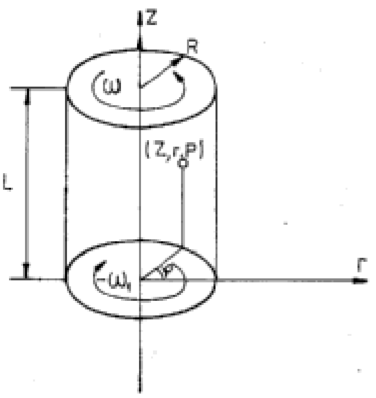


*Figure 1*

Basic equations of hydromechanics of viscose fluids in this case are set to [1]

$$\rho\frac{v_\varphi^2}{r}=\frac{\partial}{\partial r}(p+\Omega)=0 \qquad (1a)$$

$$\mu\left(\frac{\partial^2 v_\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial v_\varphi}{\partial r}-\frac{v_\varphi}{r^2}+\frac{\partial^2 v_\varphi}{\partial z^2}\right)=0 \qquad (1b)$$

$$\frac{\partial}{\partial z}(p+\Omega)=0 \qquad (1c)$$

where p is the pressure (scalar), Ω is the potential of volume powers, ρ is the coefficient of viscosity of the fluid and $v_\varphi$ is the tangent velocity.

The basic problem is brought to finding the solution of the equation (1b) with the following limit conditions:

$$\begin{aligned} &v_\varphi(0,z)=v_\varphi(R,z)=0 && ,0\le z\le l \\ &v_\varphi(r,0)=-\omega_1 r && ,0\le r\le R \qquad (2) \\ &v_\varphi(r,l)=\omega r && ,0\le r\le R \end{aligned}$$

where $v_\varphi$ is the tangent velocity.

Applying the method of Fourier, i.e putting

$$v_\varphi(r,z)=f(R).g(z) \qquad (3)$$

from (3) and (1b) is obtained

$$\frac{r^2\dfrac{d^2 f}{dr^2}+r\dfrac{df}{dr}-f}{fr^2}=\frac{\dfrac{d^2 g}{dz^2}}{g}=\lambda\ ,\ (-\lambda=a^2) \qquad (4)$$

or

$$\begin{aligned} &\frac{d^2 g}{dz^2}+\lambda g=0 \\ &\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{df}{dr}+(-\lambda-\frac{1}{r^2})f=0 \qquad (5) \end{aligned}$$

where

$$\begin{aligned} &g(z)=C_1 shaz+C_2 chaz \\ &f(r)=AJ_1(ar)+BY_1(ar) \qquad (6) \end{aligned}$$

where $J_1$ is a cylindric function (Bessel функция) of first type and first row, $Y_1$- cylindric function (Weber function) of second type and first row.

From (6) and from the first condition (2) it comes that the function $v_\varphi(r,z)$ can be sought in the following appearance

$$v_\varphi(r,z) = \sum_{i=1}^{\infty}(A_i sha_i z + B_i cha_i z) J_1(a_i z) \qquad (7)$$

where $a_i = \frac{\xi_i}{R}$, $\xi_i$ – zeros of $J_1(r)$, i=1,2,3,….., and from the last two conditions (2) is obtained the following system

$$\sum_{i=1}^{\infty} B_i J_1(a_i r) = -\omega_1 r \qquad (8a)$$

$$\sum_{i=1}^{\infty}(A_i sha_i l + B_i cha_i l) J_1(a_i r) = \omega r \qquad (8b)$$

Putting $r = Rx\,(0 \le x \le l,\ 0 \le r \le R)$ (8a) takes the appearance

$$-\omega_1 Rx = \sum_{i=1}^{\infty} B_i J_1(\xi_i x)\ ,$$

then multiplying this equation to $xJ_1(\xi_k x)$ and integrating it in the limits from 0 to 1 is obtained

$$\omega_1 R\int_0^1 x^2 J_1(\xi_k x)dx = \sum_{i=1}^{\infty} B_i \int_0^1 xJ_1(\xi_k x)dx$$

and in finite

$$B_k = -\frac{2\omega_1}{a_k J_2(\xi_k)}, \qquad k = 1,2,3,........$$

where $J_2$ is a cylindric function of first type and second row.

In absolutely the same way, designating

$$\beta_i = A_i sha_i l + B_i cha_i l$$

is obtained

$$\beta_k = \frac{2\omega}{a_k J_2(\xi_k)}, \qquad k = 1,2,3,....$$

so getting to the system

$$A_k sha_k l + B_k cha_k l = \frac{2\omega}{a_k J_2(\xi_k)}$$
$$B_k = -\frac{2\omega_1}{a_k J_2(\xi_k)} \qquad (9)$$

which has the solution leading to

$$A_k = \frac{2(\omega_1 cha_k l + \omega}{a_k J_2(\xi_k) sha_k l}$$
$$B_k = -\frac{2\omega_1}{a_k J_2(\xi_k)}$$

and finally is written

$$v_\varphi(r,z) = \sum_{k=1}^{\infty}\frac{2\omega_1 sha_k(z-l) + 2\omega sha_k z}{a_k J_2(\xi_k) sha_k l} J_1(a_k r) \quad (10)$$

Equation (10) is a solution of the basic problem just defined. It becomes evident that if $|\omega| = |\omega_1|$ area z=l/2 remains unmovable. The numeric experiments show that if $|\omega| \ne |\omega_1|$ the field of the tangent velocities $v_\varphi = v_\varphi(r,z)$ has the appearance turbulence.

It should be stressed that the problem just solved arises in global Earth sciences too.

## REFERENCES

Litwiniszyn J-A certain Problem of stationary laminary Flow of a Viscous Liquid , Bull. Polon.Sci. Letters , v.1 № 4, 1951

*Recommended for publication by Department of Mine Automation, Faculty of Mining Electromechanics*