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ON THE NEUTRAL LOSSLESS TRANSMISSION LINE EQUATIONS WITH TUNNEL DIODE AND A LUMPED PARALLEL CAPACITANCE

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ABSTRACT
By means of a fixed point approach an existence theorem for neutral equations arising in transmission lines is proved.

The present paper continuous investigations on losless transmission lines equations with nonlinear resistive elements.

In a previous paper [1] we have studied neutral equations ([2]-[4]) with nonlinearities caused by a nonlinear $V-I$ characteristic which is of polynomial type. Here we consider an initial value problem for neutral equations with an exponential nonlinearity in the right-hand side. Such a problem is equivalent (cf. [5]-[7], [2]) to an initial-boundary value problem for linear hyperbolic system with a nonlinear boundary conditions. Indeed in some cases the approximation of the $V-I$ characteristic curve generated by the diffusion current in semiconductors [8]-[9] can be fitted by an exponential function. The relation between the diffusion current and the voltage is of the type $i(u)=(Ae^{\alpha u}-1)$, where A and α are constants. It is known that such type nonlinearities have also another applications (cf. [10], [11]).

Therefore formulating the initial value problem

$$\dot{u}(t)=F(u(\Delta_1,(t)),...,u(\Delta_m,(t)),\dot{u}(\gamma_1,(t)),...,\dot{u}(\gamma_n(t)), \quad t>0$$

$$u(t)=\varphi(t),\ t\le 0 \qquad (1)$$
$$\dot{u}(t)=\dot{\varphi}(t),\ t\le 0,$$

we have to impose such conditions on the right-hand side of (1), $F(u_1,...,u_m,v_1,...,v_n)$, in order to include exponential nonlinearities. For instance F can be chosen of the type

$$F(u_1,...,u_m,v_1,...,v_n)=A\sum_{k=1}^{n}(e^{\alpha_k u_k}-1)+B\sum_{s=1}^{n}v_s .$$

We reduce the problem (1) to the following one (putting $y(t)=\dot{u}(t)$ for $t>0$ and $\psi(t)=\dot{\varphi}(t)$ for $t\le 0$ assuming $\varphi(0)=0$):

$$y(t)=F\left(\int_0^{\Delta_1(t)}y(\tau)d\tau,...,\int_0^{\Delta_n(t)}y(\tau)d\tau, y(\gamma_1(t)),...,y(\gamma_n(t))\right), t\in[0,T_0]$$

$$y(t)=\psi(t),\ t\le 0 . \qquad (2)$$

We make the following assumptions (C):

(C1) functions $\Delta_i(t),\gamma_s(t):R_+^1\to R^1$

$(i=1,...,m; s=1,...,n)$ are continuous and $t-\Delta_i(t)\ge\Delta_0>0$, $t-\gamma_s(t)\ge\gamma_0>0$ for some constants Δ_0, γ_0.

(C2) the functions $F(u_1,...,u_m,v_1,...,v_n):R^{m+n}\to R^1$ and $\psi(.):R_-^1\to R^1$ are continuous and satisfies the condition

$$\psi(0)=F\left(\int_0^{\Delta_1(0)}\psi(s)ds,...,\int_0^{\Delta_m(0)}\psi(s)ds,\psi(\gamma_1(0)),...,\psi(\gamma_n(0))\right)=$$
$$=F(0,...,0,\psi(0),...,\psi(0)).$$

(C3) $|F(u_1,...,u_m,v_1,...,v_n)|\le A\sum_{k=1}^{n}|e^{\alpha_k u_k}-1|+B\sum_{s=1}^{n}|v_s|$

where A, B are positive constants.

(C4) $|F(u_1,...,u_m,v_1,...,v_n)|-|F(\bar{u}_1,\bar{u}_2...,\bar{u}_m,\bar{v}_1,...,\bar{v}_n)|\le$
$$\le A_1\sum_{k=1}^{m}|e^{\alpha_k u_k}-e^{\alpha_k\bar{u}_k}|+B_1\sum_{s=1}^{n}|v_s-\bar{v}_s| .$$

where A_1 and B_1 are positive constants.

(C5) $A\sum_{k=1}^{m}\frac{2|\alpha_k|T_0}{2-|\alpha_k|P_0T_0}+nB\le 1$, where $P_0>0$ is chosen such that $|\alpha_k|P_0T_0<2$.

Theorem 1. Under assumption (C) the initial value problem (2) has a unique continuous solution.

Proof. Consider the set X consisting of all continuous functions $f(t):[0,T_0]\to R^1$, $(T_0>0)$ whose restrictions on $(-\infty,0]$ coincide with $\psi(t)$. It becomes a uniform space endowed with the following saturated family of pseudometrics (metrics) (cf. [13]-[14]):

$$\mathscr{A}=\{\rho_\lambda(f,\tilde f):\lambda\in[0,\infty)\},$$

$$\rho(f,\tilde f)=\sup\{e^{-\lambda t}|f(t)-\tilde f(t)|:t\in[0,T_0]\}.$$

Introduce the following subset M of X:

$M=\{f(.)\in(X,\mathscr{A}):|f(t)|\le P_0,\ t\in[0,T_0]\}$,
where P_0 is a constant which does not depend on f. It is easy to see that M is bounded and closed set. Define the operator $T:(X,\mathscr{A})\to(X,\mathscr{A})$ by right-hand side of (2):

$$(Tf)(t)=\begin{cases}F\left(\int_0^{\Delta_1(t)}f(s)ds,\dots,\int_0^{\Delta_m(t)}f(s)ds,f(\gamma_1(t)),\dots,f(\gamma_n(t))\right), & t\in[0,T_0]\\ \psi(t), & t\le 0.\end{cases}$$

Obviously $(Tf)(t)$ is a continuous function.

In what follows we show that T maps the set M into itself. Indeed let $f\in M$. Then in view of $(C3)$ we have

$$|(Tf)(t)|\le A\sum_{k=1}^{m}\left|e^{\alpha_k\int_0^{\Delta_k(t)}f(s)ds}-1\right|+B\sum_{s=1}^{n}|f(\gamma_s(t)|\le$$

$$\le\left[A\sum_{k=1}^{m}(e^{|\alpha_k|P_0|\Delta_k(t)|}-1)+nBP_0\right].$$

It is know that for $0<w<2$ we have:

$$e^w-1=w+\frac{w^2}{2!}+\frac{w^3}{3!}+\dots+<w\left(1+\frac{w}{2}+\left(\frac{w}{2}\right)^2+\dots\right)=\frac{2w}{2-w}.$$

Therefore

$$\left|\alpha_k\int_0^{\Delta_k(t)}f(s)ds\right|\le|\alpha_k|P_0|\Delta_k(t)|\le|\alpha_k|P_0T_0$$

and then

$$\left[A\sum_{k=1}^{m}(e^{|\alpha_k|P_0|\Delta_k(t)|}-1)+nBP_0\right]\le A\sum_{k=1}^{m}\frac{2|\alpha_k|P_0T_0}{2-|\alpha_k|P_0T_0}+nBP_0\le P_0,$$

provided $A\sum_{k=1}^{m}\frac{2|\alpha_k|T_0}{2-|\alpha_k|T_0}+nB\le 1$ which is $(C5)$.

It remains to show that T is contractive operator. Indeed for every f and $\bar f\in M$ and in view of the inequalities $t-\Delta_0\ge\Delta_k(t)\ge\frac{|\alpha_k|P_0\Delta_k(t)}{\lambda}$ (for sufficiently large $\lambda>0$) $\Rightarrow -\lambda t+|\alpha_k|P_0\Delta_k(t)\le-\lambda\Delta_0$

we have for $t\in[0,T_0]$ for which $\Delta_k(t)>0$:

$$|(Tf)(t)-(T\bar f(t)|\le A_1\sum_{k=1}^{m}\left|e^{\alpha_k\int_0^{\Delta_k(t)}f(s)ds}-e^{\alpha_k\int_0^{\Delta_k(t)}\bar f(s)ds}\right|+$$

$$+B_1\sum_{s=1}^{n}|f(\gamma_s(t))-\bar f(\gamma_s(t))|\le$$

$$\le A_1\sum_{k=1}^{m}|\alpha_k|e^{\alpha_k\int_0^{\Delta_k(t)}\hat f(s)ds}\left|\int_0^{\Delta_k(t)}f(s)ds-\int_0^{\Delta_k(t)}\bar f(s)ds\right|+$$

$$+B_1\sum_{s=1}^{n}e^{-\lambda\gamma_s(t)}\left|f(\gamma_s(t))-\bar f(\gamma_s(t))\right|e^{\lambda\gamma_s(t)}\le$$

$$\le A_1\sum_{k=1}^{m}|\alpha_k|e^{\alpha_kP_0\Delta_k(t)}\left|\int_0^{\Delta_k(t)}e^{\lambda s}ds\right|\rho_\lambda(f,\bar f)+$$

$$+B_1\sum_{s=1}^{n}e^{\lambda\gamma_s(t)}\rho_\lambda(f,\bar f)\le$$

$$\le e^{\lambda t}\left[A_1\sum_{k=1}^{m}|\alpha_k|e^{-\lambda t+\alpha_kP_0\Delta_k(t)}\left|\frac{e^{\lambda\Delta_k(t)}-1}{\lambda}\right|+\right.$$

$$\left.+B_1\sum_{s=1}^{n}e^{-\lambda t+\lambda\gamma_s(t)}\right]\rho_\lambda(f,\bar f)\le$$

$$\le e^{\lambda t}\rho_\lambda(f,\bar f)\left[A_1\sum_{k=1}^{m}|\alpha_k|\frac{e^{-\lambda\Delta_0}e^{\lambda t}}{\lambda}+B_1ne^{-\lambda\gamma_0}\right]\le$$

$$\le e^{\lambda t}\rho_\lambda(f,\bar f)\left[\frac{A_1 e^{-\lambda\Delta_0}e^{\lambda T_0}\sum_{k=1}^{m}|\alpha_k|}{\lambda}+B_1 n e^{-\lambda\gamma_0}\right].$$

Consequently

$$\rho_\lambda(Tf,T\bar f)\le\left[A_1\sum_{k=1}^{m}|\alpha_k|\frac{e^{\lambda T_0-\lambda\Delta_0}}{\lambda}+nB_1e^{-\lambda\gamma_0}\right]\rho_\lambda(f,\bar f).$$

But for sufficiently large λ the expression in the bracket is smaller than 1 (if $T_0-\Delta_0\le 0$) and therefore T is contractive operator.

Finally we have to choose an initial approximation. Indeed let
$$x_0(t)=\begin{cases}\psi(t), t\le 0\\ \psi(0), t>0\end{cases}.$$

Then
$$|(Tx_0)(t)-x_0(t)|=\begin{cases}0, t\le 0\\ |F(0,...,0,\psi(0),...,\psi(0))|, t>0\end{cases}$$

The map $j:A\to A$ in this case is $j(\lambda)=\lambda\Rightarrow$
$\Rightarrow j^k(\lambda)=\lambda$, i.e.

$$\rho_{j^k(\lambda)}(x_0,Tx_0)\le|F(0,...,0,\psi(0),...,\psi(0))|<\infty\ (k=1,2,...)\ .$$

Therefore T has a unique fixed point which a solution of (2). Theorem is thus proved.

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