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MATHEMATICAL MODELS WITH IMPROVED CHARACTERISTICS IN THE CONTROL SYSTEMS OF WHEEL EXCAVATORS

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ABSTRACT
Determining the parameters of main working motions in the systems for programmed control of wheel excavators utilizes complex mathematical relationships requiring calculation of direct and inverse trigonometric functions. The minimization of computing operations may be achieved by using mathematical models. A basic problem in searching for such models consists in the circumstance that the initial information is acquired through constant-step scanning. This results in the occurrence of multicolinearity, which leads to finding coefficients of strongly differing orders. This impedes the program realization, decreases the reliability in verifying the model adequacy, and worsens the accuracy of forecasting.
In this article the results of applying three approaches to finding models with improved characteristics, namely those of factor normalization, regressor standardization and use of ridge-regression analysis, are presented.

FORMULATION OF THE PROBLEM

The wheel excavator work is connected with performing a cyclic sequence of motions. Their programmed control is performed on the basis of preset reference coordinate points within the face space. This allows determining the motion parameters with the aid of mathematical relationships.

The use of analytically derived expressions (Iliev, 2001; Iliev 2002) is connected with multifold calculation of radicals and multiple direct and inverse trigonometric functions, which does not allow meeting the requirements for fast response. For this reason it is necessary to seek other approaches to modelling in order to determine the parameters of main working motions with virtually satisfactory accuracy by carrying out a minimum number of computing operations.

A good solution proposed by Iliev (2002) consists in finding regression models estimated by the method of least squares (LSM). Using these methods not only permits building unified algorithms for programmed control that are invariant with respect to the excavator kinematic specificities, but also leads to a multifold increase in the required number of computing operations (see Table 1).

Table 1

Number of "multiplication" operations	Analytical relationships	Mathematical models	Analytical relationships	Mathematical models
Algorithm for	SRs 1200		SRs 2000	
Determining the parameters of cut sickles	4	7	4	6
Determining the number of layers	30	4	29	3
Determining the maximal depth of the block	131	4	109	7
Forming the cut-sickle thickness	203	6	-	-
Determining the angle of swinging of the wheel boom	112	16	-	-
Determining the parameters for transition to a new layer	424	29	124	29

A characteristic feature of the mathematical description obtained is the wide range of varying the coefficients in regression equations. It is a normal phenomenon to find coefficients lesser than 10^{-5} that cannot be assumed insignificant as their rejection will lead to considerable worsening of the models forecasting properties. Another problem is connected with the fact that the coefficient estimates are sought with the aid of computers and in a programming environment that differs considerably from those used in real industrial control systems. For instance, MATLAB operates with numbers represented by 16 significant digits and an order of $10^{\pm 308}$. Presetting the regression coefficients in such a form is virtually unrealizable for programmed control systems. However, the experience gained shows that the diminishment of the number of digits used for representing these coefficients leads to essential worsening of forecasting accuracy. This requires the application of other approaches to modelling the main working motions of wheel excavators, through which a smaller coefficient variation range will be provided and the sensitivity to changes in the number of digits representing the coefficients will be lessened in order to preserve the forecasting properties of models.

APPROACHES TO IMPROVING THE CHARACTERISTICS OF MATHEMATICAL DESCRIPTIONS OF THE MAIN WORKING MOTIONS OF WHEEL EXCAVATORS

The search for mathematical models describing the main working motions of wheel excavators is connected with the following specificities:

- An asymmetry of the factor space as a result of the essential differences in factor variation ranges;
- A presence of correlation between the columns of the augmented matrix from the experimental plan *F*, which is caused by the way of forming the input data.

This imposes the use of specific approaches to the information processing.

Working with normalized variables.

Input data for estimating the models that describe the main working motions of wheel excavators are obtained by scanning with a constant step. As a result there emerge significant correlations of the type $r(x_i, x_i^2)$, $r(x_i, x_i x_j)$, $r(x_i x_j, x_i^2)$, $r(x_i, x_i^3)$, although the factors x_i and x_j are independent. This is the cause for bad conditionality of the information matrix $G = F^T F$, and leads to problems in using the least-squares method, discussed by Vuchkov and Boyadjieva (1987).

In this case it is useful to perform factor centering and normalizing before proceeding with the search for regression coefficients. This approach was investigated and promoted by D. Marquardt (1980) :

$$\overset{o}{x}_{i,j} = \frac{x_{i,j} - \bar{x}_i}{\bar{x}_i - \min\limits_j x_{i,j}} \ , \ i = 1,...,m \ , \ j = 1,...,N \qquad (1)$$

where *m* is the number of factors, *N* the number of experiments, and $\bar{x}_i = \frac{1}{N}\sum\limits_{j=1}^{N} x_{i,j}$.

As $\overset{o}{X}$ has a zero mathematical expectation, the theoretical correlation coefficients also become equal to zero. Actually, they are small numbers and in the general case matrix *G* is well stipulated.

Standardization of regressors.

This approach finds application when the factors have diverse physical senses and dimensions, as it is also the case of modelling the working motions of wheel excavators..

The values of standardized regressors are determined in accordance with the relationship:

$$\overset{o}{f}_{ji-1} = \frac{f_{ji} - \bar{f}_i}{\sqrt{\sum\limits_{u=1}^{N}\left(f_{ui} - \bar{f}_i\right)^2}} \quad , i = 2,...,k, \quad j = 1,...,N \quad , \qquad (2)$$

where $\bar{f}_i$ is the arithmetic mean value of the *i*th column of *F*, and *k* the number of estimated coefficients.

The model is sought by applying the least-squares method, using the standardized information matrix.

Regularization

For the first time the regularization methods were proposed by Tikhonov (1979). They are an efficient means for estimating the models as regards the presence of multicolinearity (linear relationship between columns in the augmented matrix of the experimental plan). Obenchain (1997) demonstrated on the basis of the comparative analysis he had carried out that the most widespread and most frequently used method of this group is the ridge-regression analysis, in which the coefficient estimates are determined by the relation:

$$\vec{B}_p = \left(F^T F + P\right)^{-1} F^T Y \quad , \qquad (3)$$

where *Y* is a column-vector with the output data, and *P* is a positive definite matrix with dimension (*k* x *k*). Most frequently *P* is in the form $P = pE$, where E is a unit matrix, and *p* represents parameter regularization.

Hoerl and Kennard (2000) have shown that for the problems with multicolinearity it is possible to find a value p^*, for which the estimates $\vec{B}_p$, although displaced, are much closer to the theoretical coefficients in comparison to those obtained by using the classical least-squares method.

A basic problem of the application of ridge-regression analysis consists in finding the optimal value p^*. Selecting the parameter for regularization cannot be done uniquely because the degree of displacement of estimates $\vec{B}_p$ and the generalized root-mean-square error depend on the unknown theoretical regression coefficients $\vec{\beta}$. For this reason a large number of optimality criteria are proposed.

The determined criterion of Vinod is used for finding mathematical description of the main working motions of wheel excavators. For this criterion p^* is found from the condition for minimum of the function:

$$Q = \sum_{i=1}^{k-1}\left(\frac{(k-1)\delta_i^2}{\lambda_i.\theta} - 1\right)^2 , \qquad (4)$$

where $\delta_i = \dfrac{\lambda_i}{\lambda_i + p}$, and $\theta = \sum_{i=1}^{m} \dfrac{\lambda_i}{(\lambda_i + p)^2}$.

The eigen-values of standardized information matrix $\overset{o}{G}$ are designated by λ_i , $i = 1,...,k\text{-}1$.

According to the author the ridge-estimates found by using this criterion are the closest to those that would be obtained for an orthogonal matrix of the experimental plan.

RESULTS AND CONCLUSIONS

The considered approaches to finding models with improved characteristics have been realized by programming in MATLAB environment. They have been applied to estimating the coefficients in polynomials intended for determining the maximal depth of the block T_b in working with wheel excavators Rs 2000 and the maximal ratio between the thickness and width of the cut sickle i_{max} for excavators with fixed boom.

The programs developed have allowed to obtain and examine models of 1st, 2nd and 3 rd orders, the search having been accomplished through the classical least-squares method as well as by using the approaches proposed in Section II.

The coefficients found in regression equations that describe, respectively, T_b as a function of the inclination angle of the wheel boom and the position of the taking-out mechanism, and i_{max} as a function of the layer height and the preset productivity are shown in Tables 2a/ and 2b/.

Table 2a/

Linear model	LSM	Method II.1	Method II.2	Method II.3
b_0	0.03609986366880	5.56141309867896	5.561413098679	5.561405439806
b_1	0.00008969398502	-0.73771673156166	0.033977331725	-0.039940763667
b_2	0.92088553916836	5.52531323501016	106.301467997969	106.151626946256
b_{12}	-0.00616258842718	-0.73951061126207	-16.341320754085	-16.255017200151
Model of 2nd order	LSM	Method II.1	Method II.2	Method II.3
b_0	0.00507252211393	5.58698392508998	5.561413098679	5.561411631555
b_1	0.00008969398502	-0.73771673156166	0.033977331725	0.019696060482
b_2	0.93975159515699	5.52531323501016	108.479251622126	108.036482643396
b_{12}	-0.00616258842718	-0.73951061126207	-16.341320754085	–16.324665484406
b_{11}	-0.00003666142207	-0.01466456882859	-0.146843984164	-0.146804288370
b_{22}	-0.00157217133239	-0.05659816796589	-2.254689792396	-1.826529515609
Model of third order	LSM	Method II.1	Method II.2	Method II.3
b_0	0.00514132179850	5.58698392508998	5.561413098679	5.561412084968
b_1	0.00108172470731	-0.71787611711594	0.409772396745	0.398801996160
b_2	0.93967486402001	5.52544022889438	108.470394242832	106.880356392369
b_{12}	-0.00616258842718	-0.73951061126207	–16.341320754085	–16.329580483659
b_{11}	-0.00003666142207	-0.01466456882859	-0.146843984164	-0.146816554023
b_{12}	-0.00155585520166	-0.05659816796589	–2.231290425768	1.476569651841
b_{111}	-0.00000393975664	-0.03151805313061	-0.409863290312	-0.408954989081
b_{222}	-0.00000090645171	-0.00019579356881	-0.015057558788	–2.218628073790

Table 2b/

Linear model	LSM	Method II.1	Method II.2	Method II.3
b_0	-0.00000000000123	3.16399958250370	3.16399958250370	3.16397577899822
b_1	0.00000000000000	-1.22797028612256	0.00000000000004	-0.74220702702786
b_2	0.99485433987013	0.48676916653903	7.20840452332031	6.84410426289174
b_{12}	-0.00018774509869	-0.18891850555732	-8.50261808612360	-7.67371750385568
Model of 2nd order	LSM	Method II.1	Method II.2	Method II.3
b_0	3.26557591923311	2.98961806240783	3.16399958250370	3.16399918012368
b_1	-0.00254223182584	-1.22797028612256	-15.87272198086817	-15.82707525497650

b_2	0.99485433987700	0.48676916653903	7.20840452332028	7.16398037221861
b_{12}	-0.00018774509869	-0.18891850555732	-8.50261808612352	-8.50623610455675
b_{11}	0.00000046969641	0.47558596389783	15.95243667561529	15.91003156851794
b_{22}	-0.00000000000051	0	0.00000000000019	0.04594250197596
Model of third order	LSM	Method II.1	Method II.2	Method II.3
b_0	6.51871367108561	2.98961806240783	3.16399958250370	3.16399956929197
b_1	-0.00638907760531	-1.10692038572146	-39.89095388965000	-39.40273896827786
b_2	0.99485433947461	0.48676916653903	7.20840452340212	6.82934750530163
b_{12}	-0.00018774509869	-0.18891850555732	-8.50261808612335	-8.50371837352500
b_{11}	0.00000193561560	0.47558596389783	65.73987925325532	64.73479444587571
b_{12}	-0.00000000010559	0.00000000000000	0.00000000017701	0.76367381508999
b_{111}	-0.00000000018056	-0.18396641398343	-26.00208805321579	-25.47955791244682
b_{222}	0.00000000000634	0.00000000000000	0.00000000002746	-0.38491847157070

The impact of the maximal number of digits representing the model coefficients upon the accuracy of forecasting has been examined with the purpose of evaluating the applicability of models obtained to systems for programmed control of wheel excavators. That is why, using diverse degrees of rounding, the model-forecast values of the parameter examined $\hat{y}_i, i = 1,..., N$ have been determined, and:

- the residual sums of the squares $Q_o = \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$ and
- the maximal absolute error from forecasting $\Delta_{max} = \max_i |y_i - \hat{y}_i|$

have been calculated.

The results obtained are shown in Tables 3a/ $\div$ 3d/. Their first columns contain the type of the model and the boundary number of digits (3, 4 or 5) by which the coefficients are being represented.

Analyzing the characteristics of models found by using diverse approaches allows making the following conclusions:

1. The most significant difference in the orders of coefficients found is obtained when using the least-squares method; this effect becoming stronger with the increase in the order of the model.

Table 3a/

Maximal absolute errors for a model of finding the maximal depth of the block for a wheel excavator SRs 1200				
Linear model	LSM	Method II.1	Method II.2	Method II.3
3	0.95469138888250	0.09274441086981	0.07869054142926	0.07499479568782
4	0.13074441086981	0.09074441086981	0.09187473548691	0.09001067201056
5	0.08364441086981	0.09224441086980	0.09218422742794	0.08826960345126
Model of 2nd order				
3	1.16469138888250	0.05274441086981	0.04159556525676	0.05115751837770
4	0.08830861111750	0.04474441086981	0.04730690651268	0.05074082135723
5	0.04613441086981	0.04654441086980	0.04660969825073	0.05279765663714
Model of third order				
3	1.16469138888250	0.04274441086981	0.03708861367869	0.03618193969953
4	0.10830861111750	0.03274441086981	0.03606557766521	0.02936596718147
5	0.06784441086981	0.03474441086981	0.03479828338201	0.02873311174606

Table 3b/

Maximal absolute errors for a model of finding i_{max} for a wheel excavator SRs 2000.				
Linear model	LSM	Method II.1	Method II.2	Method II.3
3	4.90456363636364	0.43418823529412	0.44058991490449	0.46110764298111
4	4.94206363636364	0.43618823529412	0.43654528567511	0.45580572074388
5	0.62743636363636	0.43648823529412	0.43653283449078	0.45579175205944
Model of 2nd order				
3	8.17456363636364	0.12418823529412	0.13464448464654	0.14236019941366
4	4.24732000000000	0.13418823529412	0.13571774888725	0.13538822073170
5	6.64308636363637	0.13528823529412	0.13527839506169	0.13586465479281

Model of third order				
3	27.0083200000000	0.06418823529412	0.06962886333570	0.07336275142821
4	12.1318200000000	0.07118823529412	0.07285499375310	0.07419283840457
5	17.8687363636364	0.07238823529412	0.07243248658507	0.07369985095839

Table 3c/

Maximal absolute errors for a model of finding the maximal depth of the block for a wheel excavator SRs 1200				
Linear model	LSM	Method II.1	Method II.2	Method II.3
3	104.568184126320	0.42995879717320	0.51225155028232	0.51274988623360
4	0.58032784002766	0.41815138703630	0.41805920814596	0.43012376376252
5	0.42708363599184	0.41788069583451	0.41788278686114	0.44279995114598
Model of 2nd order				
3	117.467975043315	0.08086797254300	0.28429311321500	0.07526384405400
4	0.79513754222256	0.05561947979853	0.05602753337193	0.07238815930581
5	0.05581125225909	0.05543295352681	0.05543347721596	0.06845674125306
Model of third order				
3	117.467975043315	0.05278344276400	0.25679269149700	0.07167215451700
4	1.19287264013240	0.02889645867371	0.02973880317474	0.05829324927401
5	0.18277354255024	0.02866642911099	0.02866600400534	0.05852915786512

Table 3d/

Maximal absolute errors for a model of finding i_{max} for a wheel excavator SRs 2000.				
Linear model	LSM	Method II.1	Method II.2	Method II.3
3	1198.15646527233	2.68050471573000	2.67978859755000	2.68646370927000
4	1220.85745805931	2.67809889041000	2.67809637444000	2.68465299462000
5	7.85199363013467	2.67809602165858	2.67809593143896	2.68465526220022
Model of 2nd order				
3	4567.10695196704	0.14253097003000	0.14359871766000	0.14307270693000
4	392.409447372075	0.14119791902900	0.14116723562000	0.14119198275700
5	1711.51674750314	0.14116323378000	0.14116321259000	0.14118564599000
Model of third order				
3	34343.5368087833	0.03700908270000	0.03940754570000	0.03833278750000
4	5228.47454898258	0.03577750966000	0.03575057723000	0.03579117847000
5	14382.7828136771	0.03574242620000	0.03574237840000	0.03578639630000

2. The approach used for estimating regression models exerts virtually no effect on the maximal absolute error and the value of the residual sum of the squares.

3. For the linear models the best results are obtained when working with centered factors, but for cubic regression the ridge-regression analysis should be preferred because of the pronounced multicolinearity.

4. In all cases decreasing the number of digits used for the presentation of coefficients in the models found by the least-squares method leads to an inadmissible increase not only in Q_o, but also in Δ_{max}. This tendency becomes stronger with increasing the order of the model.

5. Modifying the number of digits in the presentation of coefficients found by using the approaches described in Section II does not lead virtually to any change in the forecasting properties of models.

All the investigations performed have shown that the use of ridge-regression analysis or standardization of factors or regressors leads to finding such models describing the basic working parameters of wheel excavators, which have improved characteristics in comparison with those obtained through the least-squares method. They can be successfully used in the systems for programmed control without any need of imposing special requirements regarding the control system hardware.

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