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MATHEMATICAL MODELS FOR INVESTIGATING MINE POWER SYSTEMS

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ABSTRACT
This report presents Markovian models of various complexity and capabilities intended for the investigation of mine power systems.
Keywords: Markovian models, power supply, reliability, safety.

INTRODUCTION

Creating absolutely reliable and safe technical systems is an unrealizable task. However, creating technical systems with a pre-set level of reliability and safety is a real and important task. Especially critical is the problem of developing mathematical models adequate to a corresponding technical system, based on which it would be possible to substantiate theoretically and select the most correct path of realizing a system with pre-set indices of reliability and safety.

This task is of special critical importance for mine power systems in connection with the necessity of quantitatively defining the risks in utilizing the electrical power in underground coal mines.

Markovian models of various complexity presented in this report are used for a mine power supply system. Their capabilities of analyzing and estimating the impact of individual parameters on general indices of the reliability and safety system are shown.

TWO-STATE MODEL OF THE MINE POWER SYSTEM

The mine power system (MPS) can be in one of two states: safe or dangerous [Makarov, 1990, 1996]. The term "safe state" shall be understood as such a state of MPS at which all requirements of the effective normative and technical documents are being met [Regulations-1967; Regulations-1992]. If the system is in a state at which just one of those requirements is not met, it should be considered "dangerous".

The first state (the safe one) will be denoted by 0 (zero), and the second (dangerous) state by 1 (one).

To formalize the MPS functioning process the following assumptions are made.

The probability of occurrence of one or another number of events during an arbitrary time interval Δt, in result of which MPS is transferred from one state into another, does not depend on what number of events have occurred for other time intervals not crossing that in question, i. e. it does not depend on the state of MPS in previous time instants, and this is a characteristic trait of Markovian processes.

The probability that for a given time interval Δt MPS will make two or more transitions is assumed to be an infinitesimal of order higher than that of the probability MPS will make one transition for the same time interval. The probability of occurrence of one or another number of events for a time segment of length τ, depends on the segment length only.

Based on the properties indicated above the flow of events that turns over the system from one state into another will be a Poisson's flow, and the process taking place in a system of discrete states and being continuous in time will be a Markovian process.

For a Markovian process of MPS functioning it is assumed that the occurring failures follow an exponential distribution law. The time for restoring the operable (safe) state of MPS are assumed to have an exponential law of distribution, which is confirmed by multiple investigations.

To determine the probabilities of safe and dangerous MPS states ($P_0(t)$ and $P_1(t)$, respectively) in an arbitrary instant of time t the following set of differential equations is composed. It describes the stochastic states of a discrete system of two states, in which a Markovian process, being continuous in time, takes place.

$$\left.\begin{aligned}\frac{aP_0(t)}{at} &= -\lambda_{01}P_0(t)+\lambda_{10}P_1(t)\\ \frac{aP_1(t)}{at} &= -\lambda_{10}P_1(t)+\lambda_{01}P_0(t)\end{aligned}\right| \quad (1)$$

For every instant of time t the condition

$$P_0(t)+P_1(t)=1 \quad (2)$$

is valid.

Set (1) under initial conditions $P_0(0) = 1$ and $P_1(0) = 0$, and if condition (2) is met, will have the following solution

$$P_0(t)=\frac{\lambda_{10}}{\lambda_{10}+\lambda_{01}}\left[1+\frac{\lambda_{01}}{\lambda_{10}}\,\ell^{-(\lambda_{01}+\lambda_{10})t}\right] \quad (3)$$

$$P_1(t)=\frac{\lambda_{01}}{\lambda_{01}+\lambda_{10}}\left[1-\ell^{-(\lambda_{01}+\lambda_{10})t}\right]$$

(4)

In such a way the assumption for a Markovian character of the MPS functioning process enables the relatively simple way of obtaining relationships between the probability for a safe state *P0(t)* of MPS and its reliability parameters (a flow parameter of dangerous failures λ_{01} and restoration intensity λ_{10}), which allows to perform, as a first approximation, an analysis of the most effective paths to system safety improvement.

The advisability of the assumption for Markovian processes taking places in MPS functioning is confirmed by the following two circumstances. In the first place, these are the results from multiple investigations confirming the exponential distribution law for the operational period between failures and the time for MPS restoration, which is a necessary and sufficient condition for the existence of homogeneous (Poisson's) flows, and hence of a Markovian process having discrete final number of states and being continuous in time [Makarov, 1990].

In the second place, for most of the problems of applied nature the replacement of non-Poisson's flows of events by Poisson's flows of the same intensity leads to obtaining solutions which differ relatively little from the actual ones. Moreover, the error obtained is in principle within the accuracy limits of the initial data which also are most often known rather approximately. [Ovcharov,1969]

It is proposed to analyze expression (3) with the purpose of finding relationships between the probability for a safe state of MPS and the coefficient of danger K_0 as well as of clarifying the duration of time interval t_c after which a stationary value of $P_0(t)$, not depending on time, is established. In essence, this stationary value of $[P_0(\infty) = K_S]$ is that constant stationary probability with which MPS will be in a safe state at every instant of time t after t_c.

The condition of stationarity of $P_0(t)$ allows to simplify considerably the solution of differential equations which describe the stochastic states by replacing them with algebraic equations. This is important for the further analytical investigations regarding the safety of MRS in cases where the number of states considered is higher than two.

Equation (3) can be written in the following form:

$$P_0(t)=\frac{1}{1+K_0}\left[1+K_0\ell^{-(\lambda_{01}+\lambda_{10})t}\right] \quad (5)$$

where K_0 is called coefficient of danger, T_0 average operational period to a dangerous failure, T_y average period for which the system is in a dangerous state (average duration of the dangerous state), λ_{01} parameter of dangerous failures, λ_{10} intensity of restoring the safe state (intensity of eliminating the dangerous failures).

The coefficient of danger K_0 is determined by the expression

$$K_0=\frac{T_y}{T_0}=\frac{\lambda_{01}}{\lambda_{10}}$$

(6)

The following important conclusions can be derived from the relationships obtained:

1. The duration of time interval t_c within which a stationary probability value for system's safe operation is established, i. e. $P_0(t) = P_0(\infty)$ for MPS, is not great and in fact it is of the order of 12 hours. Taking into account the actual resource, this allows to eliminate the transition process in analyzing the state of MPS, i. e. to consider the system state after the time $t > t_c$.

In a stationary process

$$P_0(t)=P_0(\infty)=K_S=\frac{1}{1+K_0} \quad (7)$$

2. With the increase of the value of coefficient K_0 the probability of a safe state is abruptly diminished. That is why it is possible to attain a high level of safety not only at the expense of increasing the safety by realizing a longer operational period to a dangerous failure T_a and by decreasing the time for eliminating a dangerous failure T_y, but also at the expense of fulfilling the conditions

$$K_0=\frac{T_y}{T_0}=\frac{\lambda_{01}}{\lambda_{10}}<<1$$

(8)

Even for a low initial reliability.

3. To obtain a small value of the coefficient of danger K_0 in accordance with condition (6) it is possible to apply two approaches: by increasing T_0 or by diminishing T_y. The world experience gained in this field shows that it is connected with

considerably lower costs, and is also relatively simpler and more effective to use for a second time, by considerably increasing the maintainability of the electric equipment in MPS as well as by automatic turning on of the section (ATS), when a new element is virtually immediately put into operation instead of the failed system element, by the telemechanical control of high-voltage explosion-proof switchgears, by the use of built-in or mobile diagnostic devices, by applying search algorithms for dangerous failures, etc.

To the presentation made so far it should be also added the requirement for a certain correspondence between the values of T_0 and T_y: the smaller T_0, the greater T_y should be, or in other words: the worse the failure-free operation, the better the maintainability should be.

4. For a pre-set level of safety it is possible to define normative requirements for the reliability of MPS. For this purpose, by using equation (5) with taking into account the dependence of the coefficient of danger in accordance with (6), and pre-setting the necessary level of safety, the requirements for both indices T_0 and T_y being in strict correspondence to each other shall be simultaneously determined. In such a way multiple couples of the values of T_0 and T_y are obtained, which allows to find the most effective solution for attaining the pre-set level of safety: either at the expense of improving the safety, or at the expense of improving the maintainability.

THREE-STATE MODEL APPLICABLE TO A MINE CABLE NETWORK

To ensure a safe and failure-free electric power supply to the underground mines very severe requirements are defined for the mine cable network. Such cable parameters as the insulation and shield resistances as well as the value of transitional resistances in places of connecting the cable to electric power consumers and electric control apparatuses. The value of insulation resistance not only effects an essential influence on the risks of human-injuring accidents caused by electric current, but also on the possibility of fire occurrence.

In principle the values of parameters indicated are subject to normalization. If these values do meet the determinate norms the cable will be in a safe state. If not, it will be in a dangerous state.

Thus, from the viewpoint of the requirements for a safe and reliable functionality the following three states are characteristic for the cables:

I). A safe operable state, where the cable meets all requirements of the normative and technical documentation (NTD) regarding the values of parameters given above and is capable to provide electric power to consumers in the underground mine (state 1).

II). A dangerous operable state, where the values of one or several of the parameters given above do not meet the requirements defined by NTD, but the cable is in position to provide electric power to consumers (state 2).

III). A safe inoperable state, where the cable is turned off (state 3).

In such a way, in contrast to MPS, the mine cables can be in one of the three states (for them the dangerous inoperable state is excluded).

The flow of events that transfers the cables from the 1st state into the 3rd one is a flow of dangerous partial failures at which the cable's technological function is not disturbed, but the parameters influencing the safety do not meet the admissible norms. The intensity of this flow of events is equal to $\lambda_{12} = T_1^{-1}$, where T_1 is the average operational period of a partial dangerous failure.

Mine cables can pass from the 2nd state into the 1st one primarily through the 3rd state (upon switching off the cable and eliminating the causes for a partial failure). However, sometimes they can go into state 1 with increasing the insulation resistance as a result of cable drying by the heat generated by the working current (the intensity of this transition is denoted by λ_{21}).

The flow of events in resulting of which the cable performs a transition between the 2nd and 3rd states with intensity λ_{23} is the flow for switching off the cable by the automatic protection or operational personnel when partial dangerous failures occur. The intensity of this transition is equal to $\lambda_{23} = T_2^{-1}$, where T_2 is the average period from the occurrence of a partial dangerous failure to the instant of switching off the cable. It is assumed that the partial dangerous failure will be eliminated in state 3. For this reason the transition from the 3rd state into the 2nd one is impossible. From the 3rd state only a transition to the 1st state is possible with intensity $\lambda_{31} = T_3^{-1}$, where T_3 is the average period of eliminating complete and partial cable failures.

The transition between the 1st and 3rd states results from the occurrence of complete failures connected with disturbing the technological function of the mine cable (supplying power to underground consumers), which leads to actuating the corresponding protection devices and switching off the cable. The intensity of this transition is equal to $\lambda_{13} = T_4^{-1}$, where T_4 is the average operational period at a complete failure.

Such a system of three states is described by the following set of differential equations

$$\left|\begin{aligned} \frac{aP_1}{at} &= -(\lambda_{13}+\lambda_{12})P_1(t)+\lambda_{21}P_2(t)+\lambda_{31}P_3(t) \\ \frac{aP_2}{at} &= -(\lambda_{23}+\lambda_{21})P_2(t)+\lambda_{12}P_1(t) \\ \frac{aP_3}{at} &= \lambda_{13}P_1(t)+\lambda_{23}P_2(t)-\lambda_{31}P_3(t) \end{aligned}\right. \tag{9}$$

The following normalized condition corresponds to equation set (9):

$$P_1(t)+P_2(t)+P_3(t)=1 \tag{10}$$

The initial conditions have the following appearance:

$$P_1(0) = 1, \quad P_2(0) = P_3(0) = 0 \tag{11}$$

In a stationary process the set of equations (9) will be in the following form:

$$\begin{cases} -(\lambda_{13} + \lambda_{12})P_1 + \lambda_{21}P_2 + \lambda_{31}P_3 = 0 \\ -(\lambda_{23} + \lambda_{21})P_2 + \lambda_{12}P_1 = 0 \\ \lambda_{13}P_1 + \lambda_{23}P_2 - \lambda_{31}P_3 = 0 \\ P_1 + P_2 + P_3 = 1 \end{cases} \tag{12}$$

The probability for the cable to be in state 2 (dangerous operable state) is determined according to the following expression:

$$P_2 = \frac{\lambda_{31}\lambda_{12}}{(\lambda_{13} + \lambda_{31})(\lambda_{23} + \lambda_{21}) + (\lambda_{23} + \lambda_{31})\lambda_{12}} \tag{13}$$

Analyzing expression (13) in accordance with the numerical values of quantities shows that the probability of dangerous state most essentially depends on ratio $\lambda_{12}/\lambda_{23}$.

In such a way, to improve the degree of safe operation of mine cables requires, on one hand, an improvement in the failure-free work of cables at the expense of decreasing the parameter of partial dangerous failure flow, and on the other hand, a decrease in the period from the time of partial dangerous failure occurrence to the time of switching of the cable, e. g. by the automatic protection not allowing the operation of a cable of low insulation resistance.

The relationship (13) allows to establish a normative safety level for mine cables in respect to partial dangerous failures depending on the effectiveness of technical means for detecting those failures at a pre-set safety level.

CONCLUSION

Using Markovian processes of MPS functioning for two, three, or more states, it is possible to obtain, in a relatively simple manner, relationships between the safe state probability and the reliability parameters as well as to perform an analysis of the most effective ways of safety improvement. Eliminating the transition process of MPS functioning makes the analysis much simpler.

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