Annual of the University of Mining and Geology "St. Ivan Rilski"
vol.44-45, part III, Mechanization, electrification and automation in mines, Sofia, 2002, pp. 29-32

A BOUNDARY ANALYSIS OF ROUND PLATES BY SHEAR FORCE CALCULATION

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ABSTRACT

A method is proposed which allows to solve Von Mise's nonlinear plasticity equation. An algorithm is suggested for solving a nonlinear differential equation on the basis of the balance equations which are suitable for defining some dimension less variables of Von Mise's plasticity conditions.

A software product has been developed with tabular and graphic appendices referring to accepted boundary conditions for propped and jammed plate.

The results can be used as a basis for certain studies on support elements in the mining industry.

INTRODUCTION

Boundary analysis of round ideally plastic plates is done by many authors (Brotchine, 1960; Guerlement, Lamblin, 1972; Mohaghegh, Coon, 1973; Sawczuk, Jaeder, 1963). One of the first published papers in this scientific area is (Sawczuk, Duszek, 1963). The criterium of maximal tangental stresses is used in it. In (Mohaghegh, Coon, 1973) are given the conditions for plasticity of a material, which follows Mise's criterium in intergral-parametric form, but the question about bending element and shearing stresses is not solved. In (Brotchine, 1960; Shapiro, 1961) a generalisation of previous results as a high non-linear equation of a surface and some particular cases are solved by using suitable linearisation of this equation.

In this article a method, systemizing all previous results and permitting to obtain solvation of non-linear Von Mise's equation is proposed.

EQUILIBRIUM EQUATION

On fig.1 cylindrical coordinates r, Q, z and positive directions of forces and moments are given.

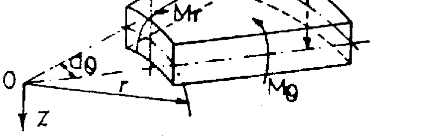

Figure 1.

The equilibrium equation could be written in the form:

$$\frac{d}{dr}\left(r.M_r\right) = M_\theta + r.T \qquad (1)$$

It is suitable non-dimension variables x; m; n; t and the dimensional q and p to be defined:

$$x = \frac{r}{R};\; m = \frac{M_r}{M_o};\; n = \frac{M_\theta}{M_o};\; q = \frac{T_e}{M_o};$$
$$t = \frac{r}{2R};\quad p = \frac{P}{2\pi M_o} \qquad (2)$$

where Mo, e, R are plastic moment, thicness and radius of the plate.

$$\frac{dm}{dx} = \frac{1}{x}\left(n - m\right) + \frac{TR}{M} \qquad (3)$$

CONDITIONS FOR PLASTICITY EXISTENSE

1. Independent action of bending moment and shearing force

On (Shapiro, 1961) is assumed that stresses, caused by shearing force could be discussed independently from those of bending moment. The condition for plasticity existence in this case is given by the equations

$$F_1\left(T\right) = 0; \qquad F_2\left(M_r, M_\theta\right) = 0 \qquad (4)$$

The arbitrary surface of the deformed plate could be assumed as small flat surfaces for which the equations (5) are valid:

$$F_1 = T \pm T_o = 0;$$
$$F_2 = \max\{|M_r|, |M_\theta|, |M_r - M_\theta|\} - M_o = 0 \tag{5}$$

where T_o is shearing force for one unit length and is solved using the relation

$$T_o = E.\tau_o, \tag{6}$$

where τ_o is tangential stress limit of plastic yield in pure shearing (fig.2).

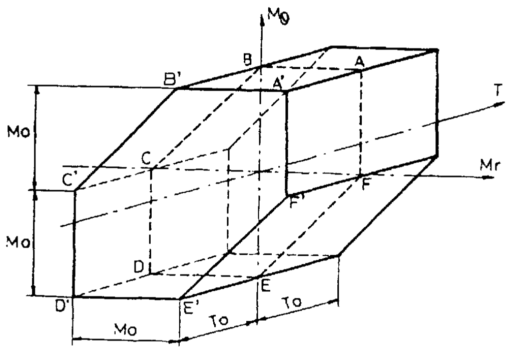

Figure 2.

2. Mutual action of shearing force and bending moment

2.1. Von Mise's plasticity conditions. Plasticity condition is written in the form (Brotchine, 1960; Sawczuk, Jaeder, 1963).

$$M_r^2 - M_r M_\theta + M_\theta^2 + \frac{3}{4}\left(\frac{TE}{2}\right)^2 - M_o^2 = 0 \tag{7}$$

and presents an ellipsoid in coordinate system Mr Ma T. Family of sections of ellipsoid with a plain perpendicular to the axis t (t could be determined from the equation t = T/To) are presented on fig.3.

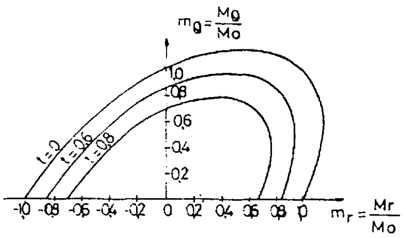

Figure 3.

Fig. 4 is a section of (7) with the plane o, t, n. Taking into account of non-dimensional variables (2), the equation (7) could be written in the form which could be taken as a square equation towards n.

$$f = m^2 - mn + n^2 + \frac{3}{16}q^2 - 1 = 0 \tag{8}$$

There fore:

$$n = \frac{1}{2}\left[m \pm \sqrt{4 - 3m^2 - \frac{3}{4}q^2}\right] \tag{9}$$

It is evident that

$$n = \frac{1}{2}\left[m + \sqrt{4 - 3m^2 - \frac{3}{4}q^2}\right] n \geq \frac{m}{2} \tag{9a}$$

$$n = \frac{1}{2}\left[m - \sqrt{4 - 3m^2 - \frac{3}{4}q^2}\right] n \leq \frac{m}{2} \tag{9b}$$

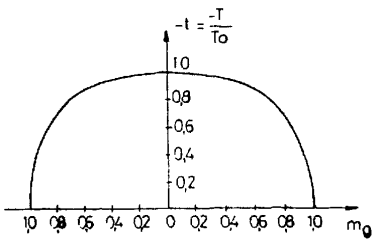

Figure 4.

2.2. Maximum of m. The m extremums are searched in plasticity conditions, static limits for the plate discussed to be defined.

For the purpose derivatives of (8) must be computed:

$$\frac{df}{dm} = 0, \quad \frac{df}{dn} = 0, \tag{8a}$$

from where the relations are defined

$$2n - m = 0, \quad 2m - n = 0 \tag{8b}$$

and the first one of them corresponds to m_{max}. From (8) could be found

$$m = \pm\sqrt{\frac{4}{3} - \frac{q^2}{4}} \tag{10}$$

METHOD PROPOSED

The elimination of n from (3) and (9) leads to a differential equation:

$$\frac{dm}{dn} = \frac{1}{2x}\left[-m \pm \sqrt{4 - 3m^2 - \frac{3}{4}q^2}\right] + \frac{TR}{M_o}, \tag{11}$$

Which is dependent of searched load parameters by q and T.

The algorithm proposed is as follows:

1. Equation (11) is integrated numerically into the interval (x_i, m_i), (x_r, m_{r1}). The result from the decision is saved in mr1, and x_r is between $x_i + x_f$.
2. Using the same method (11) is integrated for the second time, but in the interval (x_f, m_f) ÷ (x_r, m_{r2}) and m_{r2} is the solvation obtained.

3. Under the conditions $m_{r2} \geq \leq m_{r1}$ load parameters are corrected and then procedure 1° is repeated.
4. The procedure is stopped when the difference between m_{r1} and m_{r2} is negligably small, what could be put as an allowed punctuality of the solvation.

A software product is elaborated by following the algorithm above.

APPLICATIONS

1. Propped plate (fig.5a)
Boundary conditions

$$x_i = \frac{A}{R},\ m_i = \sqrt{\frac{4}{3} - \frac{P^2 t^2}{(A/R)^2}},\ x_f = 1,\ m_f = 0$$

Results obtained are given in tables 1, 2, 3, 4 and as graphs (fig. 5b).

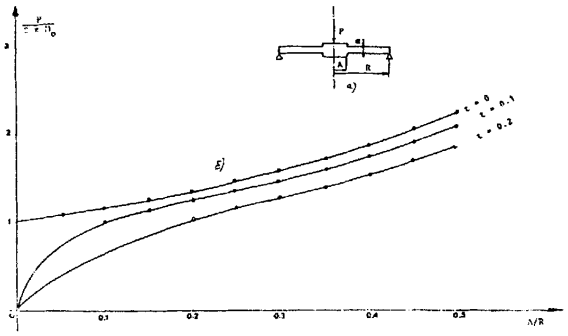

Figure 5

2. Fixed plate (fig.6a)
Boundary conditions

$$x_i = \frac{A}{R},\ m_i = \sqrt{\frac{4}{3} - \frac{P^2 t^2}{(A/R)^2}},\ x_f = 1,\ m_f = -\sqrt{\frac{4}{3} - P^2 t^2}$$

Table 1 t=0.05

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.193510	1.156638	3.19
0.15	1.276320	1.249567	2.14
0.20	1.365865	1.342972	1.70
0.25	1.465136	1.443725	1.48
0.30	1.577100	1.555844	1.37
0.35	1.705235	1.683139	1.31
0.40	1.853964	1.830071	1.31
0.45	2.029181	2.002376	1.33
0.50	2.239053	2.207852	1.41

Table 2 t=0.1

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.193510	1.037124	15.07
0.15	1.276320	1.171673	8.93
0.20	1.365865	1.277697	6.90
0.25	1.465136	1.383078	5.93
0.30	1.577100	1.495829	5.43
0.35	1.705235	1.620916	5.20
0.40	1.853964	1.762994	5.15
0.45	2.029181	1.927436	5.27
0.50	2.239053	2.121118	5.56

Table 3 t=0.15

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.193510	-	-
0.15	1.276320	1.046800	21.92
0.20	1.365865	1.178598	15.89
0.25	1.465136	1.292342	13.37
0.30	1.577100	1.406607	12.12
0.35	1.705235	1.528858	11.53
0.40	1.853964	1.664303	11.39
0.45	2.029181	1.817979	11.61
0.50	2.239053	1.995693	12.19

Table 4 t=0.20

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.193510	-	-
0.15	1.276320	-	-
0.20	1.365865	1.055572	29.39
0.25	1.465136	1.182634	23.88
0.30	1.577100	1.299774	21.33
0.35	1.705235	1.419349	20.14
0.40	1.853964	1.547722	19.78
0.45	2.029181	1.689841	20.08
0.50	2.239053	1.85063	20.99

Tables 5, 6, 7 contain results from numerical integration and fig.6b illustrates the calculations.

Table 5 t=0.05

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.897169	1.749513	8.44
0.15	2.097663	1.994682	5.16
0.20	2.304407	2.216200	3.98
0.25	2.526285	2.442476	3.43
0.30	2.770641	2.685430	3.17
0.35	3.045226	2.954035	3.09
0.40	3.359381	3.257489	3.13
0.45	3.725231	3.606797	3.28
0.50	4.159343	4.016234	3.56

Table 6 t=0.1

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0.1	1.897169	-	-
0.15	2.0976668	1.672251	25.44
0.20	2.304407	1.964258	17.32
0.25	2.526285	2.210647	14.28
0.30	2.770641	2.453915	12.91
0.35	3.045226	2.709750	12.38
0.40	3.359381	2.988364	12.42
0.45	3.725231	3.298988	12.92
0.50	4.159343	3.651618	13.90

Table 7 t=0.15

A/R	$\frac{P_1}{2\pi M_o}$ without shearing force	$\frac{P_2}{2\pi M_o}$ with shearing force	$\Delta = \frac{P_1 - P_2}{P_2}$ %
0	1.230570	-	-
0.1	1.897169	-	-
0.15	1.976668	-	-
0.20	2.304407	-	-
0.25	2.526285	1.865976	35.39
0.30	2.770641	2.125957	30.32
0.35	3.045226	2.374025	28.27
0.40	3.359381	2.628064	27.83
0.45	3.725231	2.897946	28.55
0.50	4.159343	3.191302	30.33

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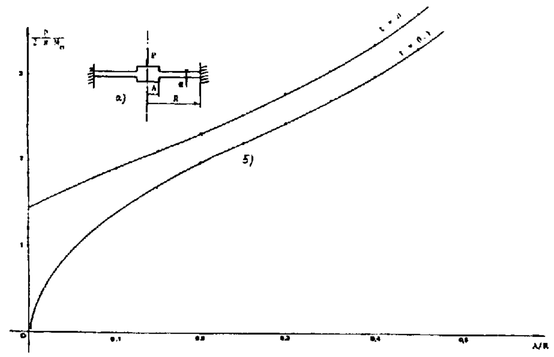

Figure 6.

CONCLUSION

Taking into account the computations made the following conclusions could be made:

a) For a propped plate the classic theory gives enough good results if

$$\frac{3R}{20} \le e \le \frac{R}{5}$$

b) For a fixed plate classic theory is suitable if

$$\frac{3R}{20} \le e \le \frac{R}{10}$$

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