

## MATHEMATICAL MODELING THE INFLUENCE OF SOME GEOLOGICAL STRUCTURES ON EARTH HEAT FLOW

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**ABSTRACT.** In this paper is composed and investigated a mathematical model concerning the influence of some geological structures on the non-stationary earth heat flow. The essence of the model consists in numerical solution of heat equation, describing distribution of non-stationary temperature in Earth crust with variable thermal characteristics and with corresponding initial and boundary conditions. The reached results allow investigation of the connection between measured heat flow and some thermal parameters in Earth crust.

## МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ НА ВЛИЯНИЕТО НА НЯКОИ ГЕОЛОЖКИ СТРУКТУРИ ВЪРХУ ЗЕМНИЯ ТОПЛИНЕН ПОТОК

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**РЕЗЮМЕ.** В тази статия е съставен и изследван математически модел, третиращ влиянието на някои геоложки структури върху нестационарния земен топлинен поток. Същността на модела се състои числено решаване на граничната задача за уравнението на топлопроводността, описващо разпределението на нестационарната температура в земна кора с променливи топлофизически характеристики и при съответните начални и гранични условия. Получените числени резултати позволяват да се изследва връзката между измерения топлинен поток на повърхността земната повърхност и някои структурни и топлофизични характеристики на земната кора.

### Introduction

Recently, the interest about determination of non-stationary temperature field in three-dimensional media is growing considerably. The main reason is the fact that generation of hydrogen carbon in such mediums depends on the time and the temperature. In some cases the heat is received from intrusive bodies, contributing formation of organic materials. The calculation of temperature field in these mediums is object of enormous interest for petroleum geology [1 – 4].

### Formulation and solution of the problem

Let investigate the following model (fig.1)

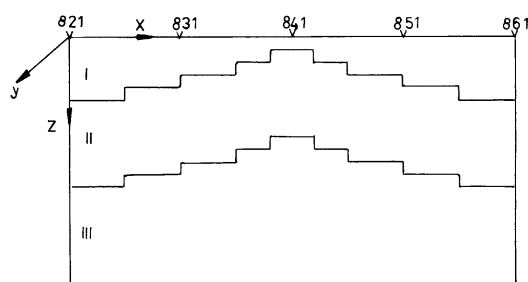


Fig. 1.

We have to find the function:

$$P(x, y, t) = K_1 \left. \frac{\partial T}{\partial z} \right|_{z=0} \quad (1)$$

where  $P(x, y, t)$  is heat flow on the Earth's surface. The temperature  $T(x, y, z, t)$  is defined from the boundary problem:

$$\text{div} \{ a^2(x, y, z) \text{grad} T(x, y, z, t) \} = \frac{\partial T}{\partial t}, \quad (2)$$

where:

$$a^2(x, y, z) = \frac{K(x, y, z)}{c\rho},$$

is coefficient of thermal diffusivity;  $K(x, y, z)$  - coefficient of thermal conductivity;  $c$  - specific heat capacity;  $\rho$  - density [5 - 6]. In our case  $K$  is:

$$K(x, y, z) = \begin{cases} K_1, & 0 < z < h_1(x, y), \\ K_2, & h_1(x, y) < z < h_1(x, y) + h_2(x, y), \\ K_3, & h_1(x, y) + h_2(x, y) < z < H. \end{cases}$$

On the surface  $z_1(x, y) = h_1(x, y)$  and  $z_2 = h_1(x, y) + h_2(x, y)$  have to be in force the following conditions:

$$[T]_{h_1, h_1 + h_2} = 0, \quad \left. \frac{\partial T}{\partial n} \right|_{h_1, h_1 + h_2} = 0. \quad (3)$$

If  $z=0$  and  $z=H$

$$T|_{z=0} = 0; \quad K_3 \frac{\partial T}{\partial z} \Big|_{z=H} = Q_H(t) \quad (4)$$

and the initial conditions are homogeneous:

$$\begin{aligned} T(x, y, z, t)|_{t=0} &= 0, \\ Q_H(t)|_{t=0} &= 0. \end{aligned} \quad (5)$$

Initially we make the following numerical study. We take  $Q_H(t)$  in the form:

$$Q_H(t) = \begin{cases} 0 & t < 0 \\ Q_H^0 = 50 \text{ mW/m}^2 & t > 0 \end{cases} \quad (6)$$

and determine the time ( $t_{\min}$ ) for appearance of deep heat flow on the Earth's surface and the time ( $t_0$ ) for establishment of stationary field.

From the fig. 2 are defined  $t_{\min}$  and  $t_0$ .

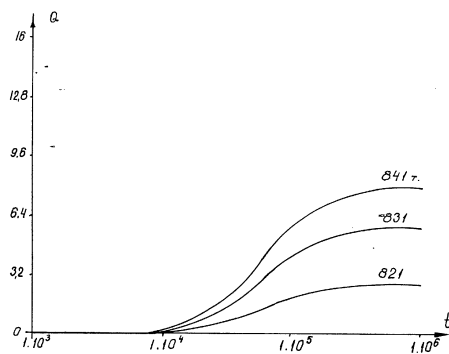


Fig. 2.

We can see, that the change of heat flow from depth of 2000 m lead to appearance of heat flow on the Earth's surface when  $t > 20000$  years and appearance of stationary case when  $t > 500000$  years.

On the fig. 3 is shown, that the anomaly in Earth's heat flow repeat the anomaly layer (if the layer is heat conducting).

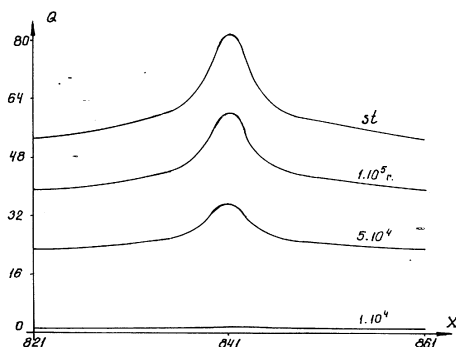


Fig. 3.

On the fig. 4 is shown  $Q(x)$  in case of different values of  $t$ , when the layer is bad conductor. Then the anomaly of heat flow is turned.

We calculate when  $Q_H(t)$  is altering in the following way:

$$Q_H(t) = \begin{cases} Q_H^0 & t < 0 \\ Q_H^i & t > 0 \end{cases} \quad (7)$$

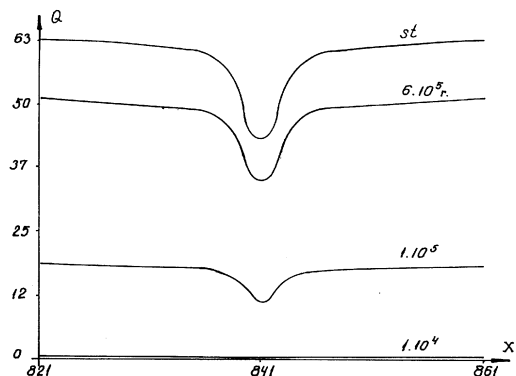


Fig. 4.

where  $Q_H^i \approx (1.5 \div 2)Q_H^0$ . The initial condition  $T(M, t=0) = T_0(M)$  is taken from the established field in problem (2). Furthermore  $P(x, y, t=0) = P_0(x, y)$  – this is the result of problem (2). Now we begin calculation about the time  $0 < t < t_{\min}$ , i.e. when on the Earth's surface the flow  $P(x, y)$  practically does not distinguish from  $P_0(x, y)$ . Moreover, it is found that for 10000 years the temperature in depth considerably has changed and the heat flow on the Earth's surface practically is the same (fig. 5).

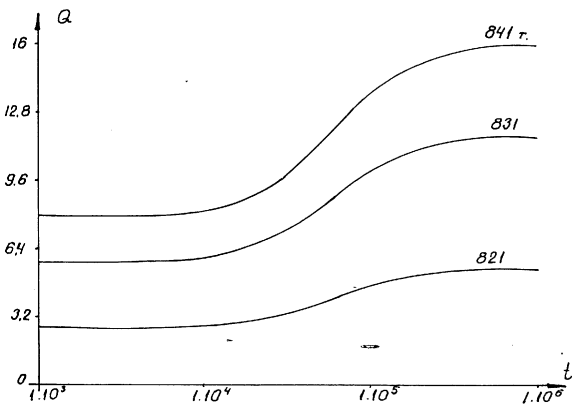


Fig. 5.

That shows the following: if we measure today the heat flow on the Earth's surface and continue the temperature field in depth, we can receive the stationary distribution, which had been 10000 years ago. In fact the current temperature distribution in depth is other.

## References

Carslow G. and all. – *Heat Conductivity*, Moscow, 1964. (In Russian).

*Numerical Mathematics and Technics in Geophysics.* Geophysical Reference, Moscow, Nedra, 1982. (In Russian).

Vlasov, V., Lyubimova E. – *Mathematical Models in Geothermics.* Problems at Mathematical Physics and Numerical Mathematics. Moscow, 1977. (In Russian).

Kutas R. and all - *Modelling of heat flow.* Naukova Dumka, Kiev, 1989 (In Russian).

Kostyanov S. - *Mathematical Models of Geothermal Fields.* Moscow University, Moscow, 1979.(In Russian).

Kostyanov S. – *Mathematical Modelling of Geophysical Fields in Gradient Media.* MGU, Sofia, 1999, (Monograph - 200 pp.)

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