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SOME PRELIMINARY RESULTS OF THE ANALYSIS OF THE LANZAROTE (Canary Islands) GRAVITY ANOMALY WITH ELEMENTARY SOURCES

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ABSTRACT. As is known, the method of D. Zidarov concerning the solution of the inverse problems of the potential fields with elementary sources through optimisation lets to obtain approximate information about the distribution of the underground density inhomogeneties on the basis of the corresponding observations. Here, this technique is used to study the deep underground structure of the Lanzarote (Canary Islands). For this purpose, a network of 296 gravity stations distributed over the whole island and a digital terrain model of about 45, 000 terrestrial and oceanic data (to determine the corresponding terrain corrections) , are used. The resulting Bouguer anomaly is analysed by the above mentioned method, which gave a mean level of observational noise of about 1.7 *mGal*. The final solution is presented by 14 point masses and a polynomial trend, obtaining the approximate location of the anomalies sources and their masses. The main quantity of masses (corresponding probably to the main intrusive body) is concentrated under the central area and can correspond to a dilated volcanic activity (of shield formation). The center of this body is located at a depth of about 8 - 10 *km*. Besides, the SW extreme area of the island shows a smaller positive body, approximately at the same depth, interpreted as a less developed magmatic intrusion. Similar results are obtained also in the previous works on the same subject. Here, however, no intrusion is established in the NE extreme area of the island. Probably the observed anomaly in this region is provoked by much deeper masses, located far away NE from this place. All these results seem to be more or less in agreement with the information known of orher sources .
Key words: Lanzarote island, Bouguer gravity anomaly, numerical modeling, inverse problems, optimization, point masses, dencity inhomogenities, volcanic structure.

НЯКОИ ПРЕДВАРИТЕЛНИ РЕЗУЛТАТИ ОТ АНАЛИЗА НА ГРАВИТАЦИОННАТА АНОМАЛИЯ ЛАНЗАРОТИ (Канарски Острови) С ЕЛЕМЕНТАРНИ ИЗТОЧНИЦИ

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РЕЗЮМЕ. Както е известно, методът на Д. Зидаров за решаване на обратните задачи на потенциалните полета с елементарни източници чрез оптимизация дава приблизителна информация за разпределението на плътностните нееднородности в горните слоеви на Земята въз основа на съответните наблюдения. В настоящата работа тази техника е използвана за изследване на дълбочинния строеж на остров Ланзароти (Канарски острови). За тази цел, са използвани 296 гравитационни станции, разпределени върху целия остров и цифров модел на терена от около 45,000 наземни и океански данни (за определяне на съответните корекции за релеф). Резултатната Буге аномалия се анализира с горе споменатия метод, при което се получава едно средно ниво на наблюдателния шум от порядъка на 1.7 *mlg*. Окончателното решение е представено от 14 точкови маси и полиноминален тренд, получавайки приблизителното месторазположение на аномалните източници и техните маси. Главното количество маси (вероятно съответстващи на основното интрусивно тяло) е съсредоточено под централната част и вероятно съответства на разширяваща се вулканска активност (формираща щтита). Центърът на това тяло се намира на дълбочина от около 8 - 10 км. Освен това, в югозападната крайна част на острова се забелязва наличието на сравнително по-малко положително тяло, приблизително на същата дълбочина, което може да се интерпретира като по-малко развита магмена интрузия. Подобни резултати се наблюдават и в предишните работи на тази тема. Тук обаче за разлика от тях, в североизточната крайна част на острова интрузия не се установява. По всяка вероятност, наблюдаваната там аномалия е предизвикана от значително по-дълбоко залягащи аномални маси, намиращи се далече на североизток от този район.

Introduction

The Canary Islands are one of the main volcanically active areas, with many Quaternary eruptions and several historic ones, among which the Lanzarote eruption between 1730 and 1736, one of the Earth's biggest, predominates. They are an old volcanic feature with origin still under debate, sited on top a Jurassic oceanic crust, located at the edge of the West African Continental Margin. The island of Lanzarote is located at the north-eastern extreme of the archipelago, in clear alignment with the island of Fuerteventura (see a map).

Most of the geophysical works about the Canarian archipelago correspond to extend areas and marine data. To attain a better understanding of the inner structure and evolution of Lanzarote, several geophysical and geodynamical studies are carried out on the island by the Instituto de Astronomia y Geodesia (I.A.G.), Madrid (e.g., Camacho et al., 1991, 1992, 2001). Continious gravity tide observations have been conducted in two underground geodynamic laboratories (located in the island) for several years (e.g. Vieira et al., 1991; Arnoso et al., 1996), obtaining the respective local tidal gravity models, which help studying the anomalous crustal structure of the island in two zones (Arnoso et al., 1996).

The I.A.G. has also carried out several gravity studies on Lanzarote (e.g. Sevilla & Para, 1975). In 1988 a more detailed gravity survey is carried out in this island, and the analysis of the data and preliminary structural model are presented in successive works (Camacho et al., 1991; 1992). An improved interpretation of this gravity data by means of a 3-D inversion approach, to determine the geometry of the anomalous bodies (corresponding to the prescribed densities) is made by (Camacho et al., 2001).

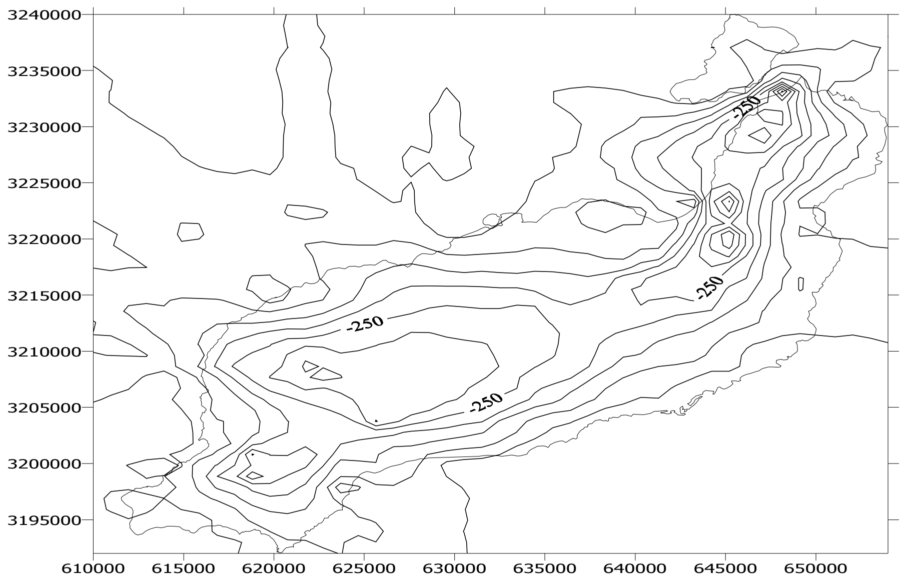

Fig. 1. Topographic model of the studied region. Contour interval 50 m, co-ordinates in meters. Besides the level lines of the terrain, the boundaries of the islands are also given

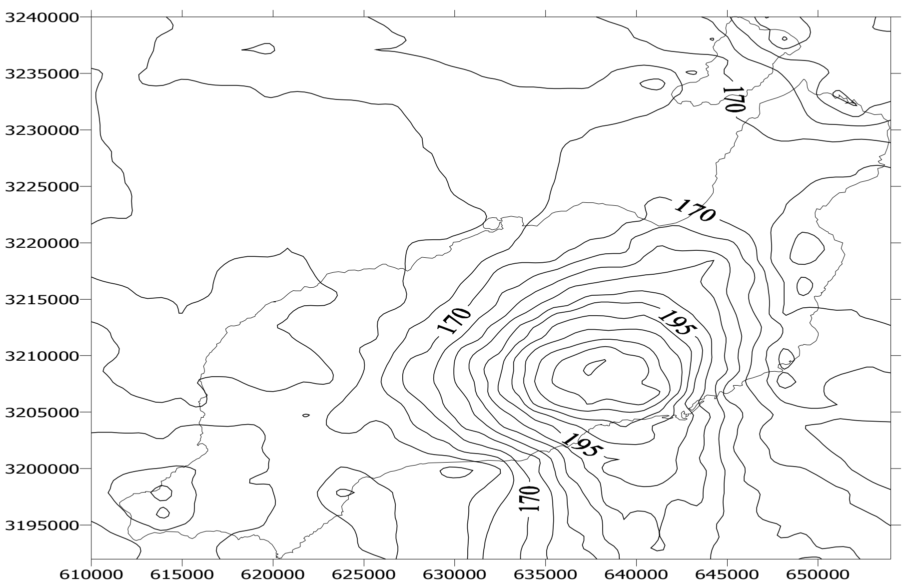

Fig. 2. Refined Bouger anomaly observed. Contour interval 5 m Gal, coordinates in meters. Besides the lines of the observations, the boundaries of the islands are also given

Different gravimetric methods have been also employed for studying the origin, structure or activity of some other volcanic areas and useful results have been obtained (Camacho et al., 1991; 1992; 1997; 2000 2001; 2002). In this sense, it may be said, that gravity modelling plays an important role in studying volcanic structures.

The aim of the present work is to contrast and to try to improve further the interpretation of the Lanzarote gravity data, using the above mentioned method of the solution of the inverse problems with a set of mobile elementary sources (ES) (Zidarov, 1965, 1968, 1990), which is known as very effective in similar situations (Zidarov, 1965, 1968, 1990; Bochev et al. 1974; Zidarov et al. 1970; Zhelev 1970, 1972, 1974, 1985, 1991, 1992, 1994; Zhelev et al. 1985, 1994). Besides, owing to the well known ambiguity and instability of the solution of these problems, the application of different methods for their solution is very encouraging and perspective, even necessary, for the achievement of more real results.

Gravimetric and topographic data

The islands of Lanzarote and La Graciosa are covered with 296 gravity stations with a nearly homogeneous distribution, separated mutually by a minimal distance of 1.5 km. The observation work is accomplished in April - 1988 and October - 1988, using a LaCoste-Romberg gravimeter with digital electronic reading. A total of 437 observations are registered (Camacho et al., 1991).

The station co-ordinates are determined by locating the points on detailed 1: 5,000 charts (Camacho et al., 1991).

To calculate the further terrain correction of the gravity data, a digital terrain model is obtained for Lanzarote, La Graciosa and surrounding oceanic areas by means of a dense regular digitalisation of 1:25,000 topographic charts (Camacho et al., 2001). The Fig. 1 shows the resulting terrain model (45,000 data points). The highest altitudes are about 600 *m*.

Using the 1980-normal gravity formula, and determining the terrain correction (extended to 45 km, according to the regularity of the effect beyond this distance), the refined Bouguer anomaly was determined (Fig. 2).

Mathematical formulation of the problem

The solution of the given problem by the above mentioned method (Zidarov, 1965, 1968, 1990) reduces mainly to the solution of the following non linear system of equations $f(x) = y$, where $x\ (x_1, x_2, \ldots, x_n)$ is the vector of the unknown parameters (co-ordinates - ξ_k, η_k, ζ_k and masses m_k, $k = 1, \ldots, n/4$ of the point sources (PS)), which must be determined on the basis of the vector of the observations $y^*\ (y_1, y_2, \ldots, y_N)$, while $f^*\ f_1, f_2, \ldots, f_N)$ is a vector of non linear functions (the symbol * means transposition). In this case the functions f_i, $i = 1, \ldots, N$, can be defined by the analytical expression about the gravity effect ($\gamma = 66.7 \cdot 10^{-12}$ $m^3\ kg^{-1}\ s^{-2}$ is the gravity constant) of a set of $n/4$ ES in N points of observation with co-ordinates - X_i, Y_i, Z_i, $i = 1, \ldots, N$, located over the Earth's surface.

$$f_i(x) = \sum_{k=1}^{n/4} \frac{\gamma\, m_k (z_i - \varsigma_k)}{R_{ik}^{3/2}}, \quad R_{ik}^2 = (X_i - \xi_k)^2 + (Y_i - \eta_k)^2 + (Z_i - \zeta_k)^2,$$

For the presentation of the field trend, when necessary, a part $a + bX + cY + \ldots$ (X and Y are the co-ordinates of the observational points) of a polynomial is used, whose coefficients - $a, b, c, \ldots$ are determined in the process of optimization, together with the rest of the unknowns (Zhelev, 1991, 1994). Alternatively, more PS can be included in the model for this purpose (Zhelev, 1991, 1994). Their parameters can be specified in the same way. Usually, in order to represent the trend, they must lie significantly deeper than the rest. Of course, the best thing to do here is to try to remove the trend before a further interpretation, but this is not always possible with the needed precision. Even so it must be done, because in all cases this can considerably ease the optimisation in the next step (Zhelev, 1991, 1994).

Numerical results

As was already mentioned, the above described gravity anomaly was treated with this method (Zidarov, 1965, 1968, 1990). An appropriate Computer Program on FORTRAN 77 worked out by Zh. Zhelev (1970, 1972, 1974, 1991, 1994), was applied for this purpose. The Bouguer gravity was used obtained on the basis of the surface registrations. The observed anomaly and the corresponding trend were presented with 14 PS and the linear part of a polynomial. The optimisation was carried out after the Marquardt method (Marquardt, 1963).

A part of the results obtained - the parameters of the elementary sources - x, y and z co-ordinates and the masses, respectively - ξ_k, η_k, ζ_k, m_k and some quantities connected with the corresponding errors in the solution, are presented in the Table. Besides, the following parameters are listed there for convenience:

- the functional

$$F(x) = \sum_{i=1}^{N} [y_i - f_i(x)]^2,$$

- the corresponding mean square deviation (MSD)

$$\sigma_v = [F(x)/(N - n - m)]^{1/2},$$

- the gradient of the functional

$$G(x) = \{\sum_{k=1}^{n} \left[\frac{\partial F(x)}{\partial x_k}\right]^2\}^{1/2},$$

- the coefficient of non-representativeness (Zhelev, 1970, 1972, 1974)

$$K_v = 100 \left(F/E\right)^{1/2} \ \%, \quad E = \sum y_i^2,$$

at the point of the minimum, etc.

Table 1.
Solution of the Inverse Gravity Problem with a system of elementary sources – fourteen point sources and a linear trend represent the local field of the "Lanzarote" gravity Anomaly (see Fig. 3 and Fig. 4)

F	*G*	*K*□*[%]*	*k*	□$_k$ *[km]*	□$_k$ *[km]*	□$_k$ *[km]*	m_k *[kg/1.5 . 10 ^{12}]*
.41 . 10 ^{4}	.26 . 10 ^{7}	2.14	1	635.00	3210.00	1.12	1.12
			2	637.00	213.00	.92	.151
			3	639.00	3207.00	13.05	280.300
Initial approximations:			4	643.00	3219.00	.69	1.232
			5	626.00	3205.00	11.89	27.010
			6	613.00	3198.00	13.18	92.467
			7	644.00	3198.00	6.60	6.486
			8	647.00	3224.00	11.86	9.829
			9	662.00	3206.00	.70	1994.900
			10	630.00	3229.00	15.66	214.550
			11	612.00	3220.00	19.45	756.630
			12	602.00	3166.00	91.15	81484.000
			13	659.00	3276.00	40.14	12901.000
			14	665.00	3238.00	45.80	8695.400
Parameters of the trend (a, b, c):				890.38	.0	.0	

.68 . 10 ^{3} .77 . 10 ^{2} .88	1	635.34	3209.54	1.09	.263
	2	637.18	3213.37	.88	150
	3	639.13	3206.63	13.05	13.05
Solution:	4	642.62	3219.33	64	1.233
	5	625.88	3204.75	12.03	27.492
N = 294 , n = 56 , m = 3 ;	6	613.58	3198.13	12.41	89.474
	7	643.72	3216.01	6.29	6.375
σ_ν = 1.70 mgal .	8	646.15	646.15	8.54	8.430
	9	661.72	.6585	59	2008.756
	10	629.68	3229.81	15.37	215.964
	11	611.55	3220.95	20.60	793.664
	12	601.37	3165.01	3165.01	81784.268
	13	659.29	3274.46	35.13	12785.100
	14	662.05	3232.87	44.57	8586.135
Parameters of the trend (a,b,s):		914.30	0	0	

	1	.1934	.2137	6784	1341
Corresponding confidence inter-	2	2343	.4473	.8978	.0867
vals : $\delta x_k = \pm\sigma_k\, t_{(N-n-m),\alpha}$,	3	1197	1146	1146	6.1083
$\sigma_k = \sigma_\nu w_\kappa$, $w_\kappa = [\alpha_{\kappa\kappa}]^{1/2}$;	4	2158	.2037	.8946	6249
$\sigma_\nu = 1.70$ *mgal* ,	5	7329	.6136	7849	5.1847
($t_{235,0.05} = 1.971$	6	.2883	2526	3712	7.3728
$t_{235,0.01} = 2.599$) ,	7	4478	6502	.7941	2.0781
	8	9249	.8954	1.2438	3.3977
a_{kk} , *k=1,...,n+m are the diagonal*	9	.6585	.6585	.0835	249.2237
elements of the inverse	10	6903	6903	.6361	15.8731
of the matrix of the correspond-	11	4952	5388	.4264	25.5180
ing normal equations at the point	12	3481	2402	1871	226.0883
of the minimum, arranged conse-	13	7361	3030	9477	148.6145
quently by rows :	14	3194	4186	1780	57.2575
Parameters of the trend *(a,b,s)* :		1 7276	0027	0005	

Obviously, to obtain the masses in kilograms, the given quantities in the table must be multiplied by 1.5 10^{12}. For example: spheres with 0.360 gr/sm^3 density (i.e. - one approximately real density contrast) and 5 km radius, have a mass about 1.8 10^{14} kg, t.e. more or less of the same order, as the masses of the central intrusive body of the studied anomaly.

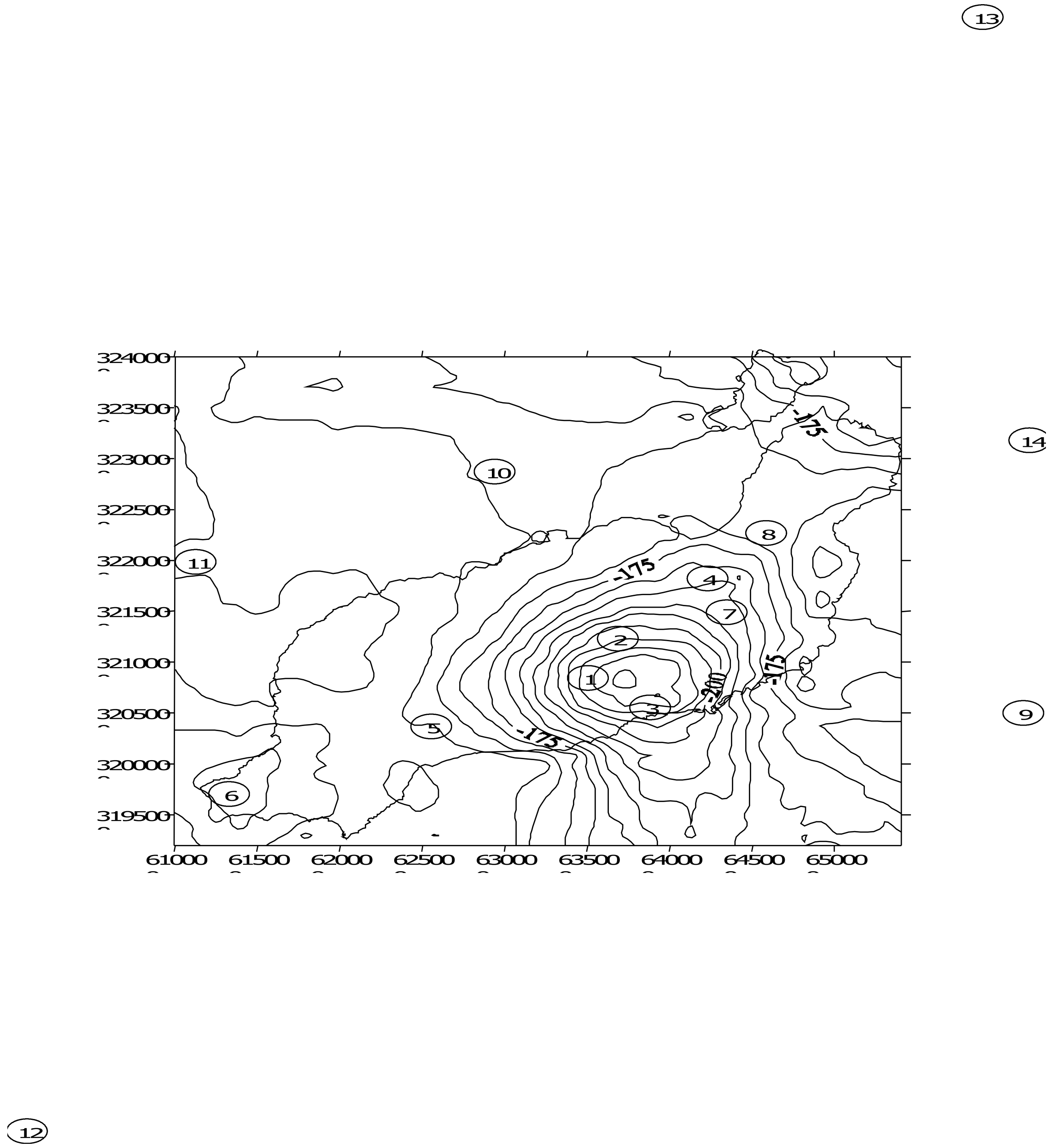

Fig. 3. The model's .field and a plan view of the obtained solution – the location of the corresponding 14 point sources are marked with small curcles (the numbers in them corresponds to those shown in the "k" column of the table). Contour interval 5 m Gal, coordinates in meters

Moreover, some of the results are also presented on suitable illustrations - (Figs. 3, 4 and 5).

On Fig. 3, beside the model's field, a plan view of the obtained solution - a set of 14 PS, which can easily be transferred into spherical bodies with different radii - corresponding to their masses, are shown. The number of these PS (given in them) corresponds to the one presented in the "k-th" column of the table.

On Fig. 4 and 5, the residuals (the difference between the observations and the models field) and the refined trend are represented, respectively.

On the figures, some other details are also given for convenience.

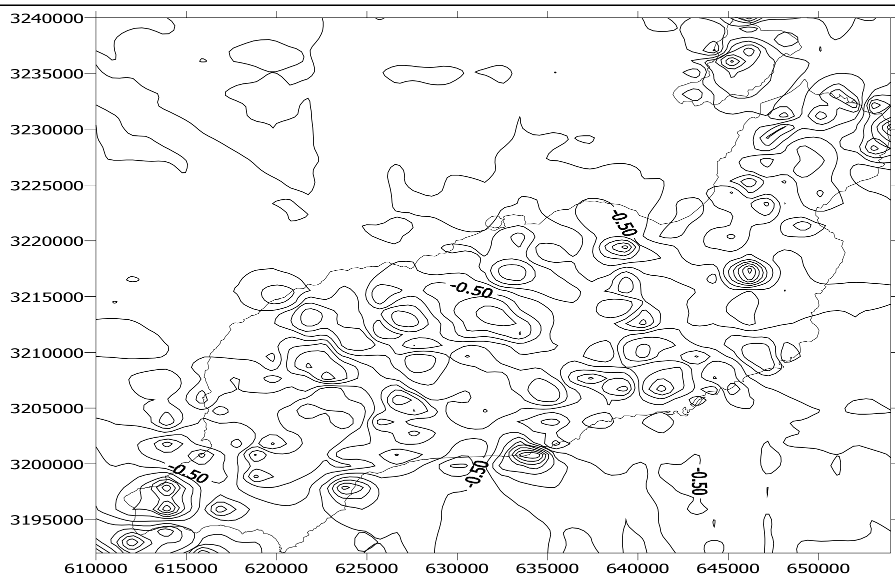

Fig. 4. The residuals. Contour interval 0.4 mGal , coordinates in. meters. Boundaries of the islands are also given

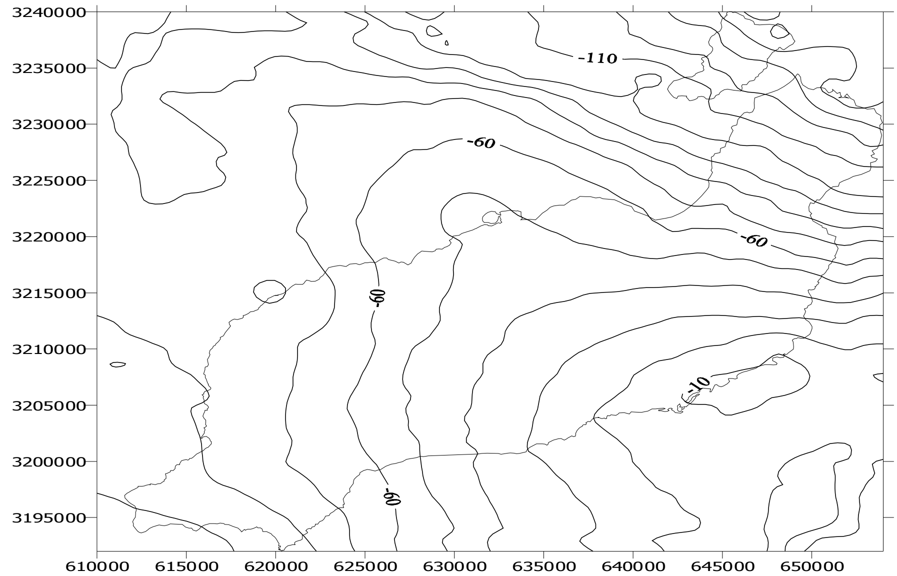

Fig. 5. The refined trend. Contour interval 10 mGal, coordinates in meters. Boundaries of the islands are also given

Taking into acount that the last four PS (from numbers 10 to14), together with the polynomial of first degree, represent mainly the trend of the field in this region, the left ten PS confirm more or less the structural model, obtained by the other authors (Camacho et al., 1991, 2001). As can be easily seen from the table and Fig. 3, the main anomalous masses (about 90 % of them), obviously related with the main intrusive body of this region, are concentrated in the middle part of the island, approximately at the depth - 10 - 12 kms. A comparatively much smaller part of them (the anomalous masses), probably connected with smaller intrusive bodies, is focused in the south region of the island approximately to the same depth. However the present investigation gives also something new - here (on the basis of the obtained results - see the Table and Fig. 3) it is suggested, that in the north-east region of the island there is not any concentration of anomalous masses, which are larger than the normal. The corresponding anomalous observations in this domain of the island, however, are probably connected with some anomalous masses, away of this place in north-eastern direction of it, located much deeper - at a depth of about 35 km (in the region of PS 13 and 14 - see Fig. 3).

At a greater depth, one elongation of the structures in a perpendicular (to the island), approximately east-west, direction is observed (see Fig. 5), which may be related to the existence of some fracture system, corresponding to the structural stress in this region.

The PS number 9, located to the right of the central part of the anomaly, which is lying at a very shallow depth, may hardly be connected with some real anomalous masses. It only indirectly shows that the number of the PS used in this investigation is too great, and there are not additional details for presentation by a more complicated model. Thus, the observational field is exhausted with this model and there is no reason for further complications in the present study - even the model used is too complicated for this anomaly.

The values of the functional, the corresponding MSD and the coefficient of non-representativeness (see the Table) show that a satisfactory solution is found, and the model fits the observations comparatively well. The corresponding gradient points out that the optimum of the functional has been approximately achieved. Besides, the coefficient of non-representativeness shows that part (in percentage) of the observations, which is not presented by the model used.

The corresponding MSD is about 1.7 *mgal*, i.e. it is approximately of the order of the observational error, which is more or less in agreement with the theory (Zhelev, 1991, 1994; Draper et al., 1986; Tihonov, 1965; Wiggins, 1972).

As a matter of fact, the obtained MSD for the whole observational region is a little larger than the corresponding mean square observational error. This is mainly connected with the comparatively large residuals (systematic part) in the whole observational area (see Fig. 4), related mainly with the terrain variations due to the different volcanic eruptions and corresponding volcanic cones, created during their activity, as they could not be presented exactly by the model used in this investigation. And, indeed, all of them almost coincide with the ones seen on the corresponding map of the terrain (compare the maps of the terrain and the residuals, Figs. 1 and 4, respectively). This is probably connected mainly with the lack of sufficiently detailed gravity information for stable determination of the respective real sources with comparatively small dimensions.

Stability and errors in the solution

As is known (Draper et al., 1986), the problem concerning the exact evaluation of the errors in the solution in the non linear case is not satisfactorily solved yet. But as in the close vicinity of the minimum, a linear representation is usually acceptable, the well-known formalism concerning statistical estimations of linear systems (Draper et al., 1986) can be used in this and similar cases to study the stability of the solution. An approximation of the corresponding confidence intervals δx_k of the unknowns can be obtained by the following expression (Draper et.al., 1986)

$$\delta x_k = \pm \sigma_k t_{(N-n-m),\alpha}, \ \sigma_k = \sigma_v \omega_k, \omega_k = [a_{kk}]^{1/2},$$

where a_{kk}, $k = 1, \dots, n$ are the diagonal elements of the inverse matrix of the respective normal system of equations, and $t_{(N-n-m),\alpha}$ is the corresponding *t* score for the respective degrees of freedom *(N-n-m)* (*m* - the number of the trend parameters) and level of certainty α . Thus, we can have an approximate idea about the confidence intervals, suggesting an almost linear connection between the unknowns and the observations at the point of the minimum and its surroundings.

As can be easily seen from the table, almost all the PS and the trend are comparatively well determined - the correspond-ding errors in the solution are within acceptable limits.

Naturally, when this method is used, the question how to determine the optimal number of model parameters is essential. Although the optimisation method used automatically eliminates the extra parameters of the model, in order to ease the optimization process however, the following additional method (Zhelev, 1991, 1994) can be employed for this purpose.

The problem can be solved for different numbers of elementary sources. The number n at which the corresponding MSD has a minimum, had to be chosen as an optimal one. It is not difficult to show, that if the number of the observations is large enough, there is a number of the ES at which this criterion has a minimum and this optimum coincides with the real number of the parameters of the source - the respective proof can be seen in (Zhelev, 1991, 1994). Obviously, the minimum value of the MSD thus obtained, must be approximately equal (or a little less) to (than) the corresponding mean square error in the observations, as it is its unbiased estimate (if a good representation is achieved) (see Zhelev, 1991, 1994) . Thus, instead of searching for the minimum, we can look for that *n* for which the corresponding MSD coincides with (or is a little less than) the respective mean square error in the observations, when it is known of course (Zhelev, 1991, 1994).

Conclusion

It may be said in conclusion, that some new results are obtained here, by a different (mathematically well-grounded) method, which confirm the structural model obtained in previous works and gives with greater certainty and precision a more detailed idea about the distribution of the anomalous masses in depth in this region. More specially, some details in the central, south-western and north-eastern regions of the island are specified. The systematic part of the corresponding residuals almost coinside with the volcanic cones, which can be clearly noticed on the terrain map of the island, as they could not be represented exactly by the model used in this investigation. An idea about the direction of the fracture system, corresponding to the structural stress in depth, is given here. On the basis of all this and the proven in the practice possibilities of the method used here (Zidarov, 1965, 1968, 1990; Bochev et al., 1974; Zidarov et al., 1970; Zhelev, 1970, 1972, 1974, 1985, 1991, 1992, 1994, 1996; Zhelev et al., 1985; 1996), we can hope that now we already have one more precise and real idea about the underground structure in this region.

It must be added at the end, that all these results are obtained only on the basis of gravity observations and topographic information - without including other geophysical dada. Indeed, better results may be expected on the basis of some new, more detailed and precise observations on a larger region (including also some parts of the surrounding ocean and even of the neighbour island of Fuerteventura, by an updated (improved) method. Of course, additional improvements can be expected also, including some magnetic observations in this study.

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