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MODELING THE TEMPERATURE FIELD AROUND HOT MAGMA BODY

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ABSTRACT
Modeling the temperature field around hot magma body implanted in the earth's crust is carried out in this article. The following model is treated: let in a layer of H thickness a magma body is implanted. The temperature T had to be found, it should satisfy this equation $a^2\Delta T = \frac{\partial T}{\partial t}$, with initial condition $T(x,y,z,t=0) = T_0(x,y,z)$ and boundary conditions $T(x,y,z=0,t) = 0$, $K\frac{\partial T(x,y,z=H,t)}{\partial z} = Q_H$. Solving this problem we find out after how much time and how the heat flow on earth's surface changes. In order to investigate the basic characteristics of the temperature field calculations for cubic and prismatic bodies were carried out. . We have investigated the behavior of temperature inside and around the body as a function of time. The behavior of cooling body on earth's crust is found. Numerical results of heat flows of the earth's surface are depending on time according to profiles passing through the center of the bodies.

INTRODUCTION

The question about the duration of cooling of magma body is important to geological science. It is easy to answer these questions if we know the heat characteristics of the investigated body and the rocks nearby.

But often these data are difficult to get because heat conductivity of rocks gets lower when temperature rises and heat capacity increases when the temperature reaches 400° C. Ignoring changes connected with the temperature we can obtain more or less valid average values.

FORMULATION AND SOLVING THE PROBLEM

Let's treat the following case: The anomalous change of temperature provoked by the influence of thermal body has to be found. (fig. 1).

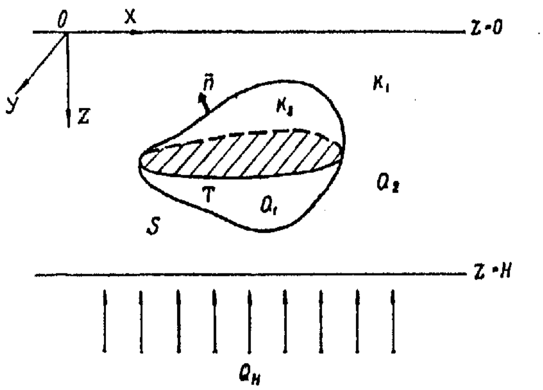

Figure 1.

Let in the layer with thickness H a body is implanted. We define the temperature satisfying the equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a^2}\frac{\partial T}{\partial t} \qquad (1)$$

where
$a^2 = a_1^2$ - the coefficient of heat conductivity of the body,

$a^2 = a_2^2$ -the coefficient of heat conductivity of the medium.

Initial and boundary conditions are:

$$T(M,t)\big|_{t=0} = T_0(M) = \begin{cases} \frac{Q_H}{K_1} z, & M \notin V, \\ T_1, & M \in V, \end{cases} \qquad (2)$$

$$T(M,t)\big|_{z=0} = 0; \qquad K_1 \frac{\partial T}{\partial z}\bigg|_{z=H} = Q_H = const \tag{3}$$

On the body surface we get the following boundary conditions:

$$[T]_s = 0; \qquad \left[K\frac{\partial T}{\partial n}\right]_s = 0 \tag{4}$$

Solving the boundary problem (1) - (4) we can know after how much time and how the heat flow will change on the earth's surface:

$$K_1 \frac{\partial T}{\partial z}\bigg|_{z=0} = P(x,y,t) \tag{5}$$

where

$$P(x,y,t)\big|_{t=0} = Q_H$$

In order to investigate the basic characteristics of the heat body - calculations of cooling the body are made (cube, prism).The cube is at 300m depth and has sides equal 300m, i.e. -150<x<150; -150<y<150; 300<z<600, And the prism is at the same depth but has dimensions 4 times bigger on the Ox, i.e. -600<x<600 The dimensions on the Oy and on the Oz are the same.

At the beginning we will study how the temperature changes near the body and how the cooling the prism differs from that of the cube. In fig.2 we can see the changes of temperature in depth for different periods and point near the body.

It is easy to notice that in the direction of Ox passing through the center of the body the cooling of the prism is slower than that of the cube. The maximum temperature is concentrated around the body and for the first 100 years the temperature of the body goes down with 5%. After 1000 years the body temperature goes down with 25%. After 3000 years it remains 25% from the initial temperature, and after 10000 years 10% of the initial temperature is kept.

This can be seen in fig.3 and fig.4, where the changes of temperature in at around the body are given.

On fig.3 there is a change of temperature in the body:

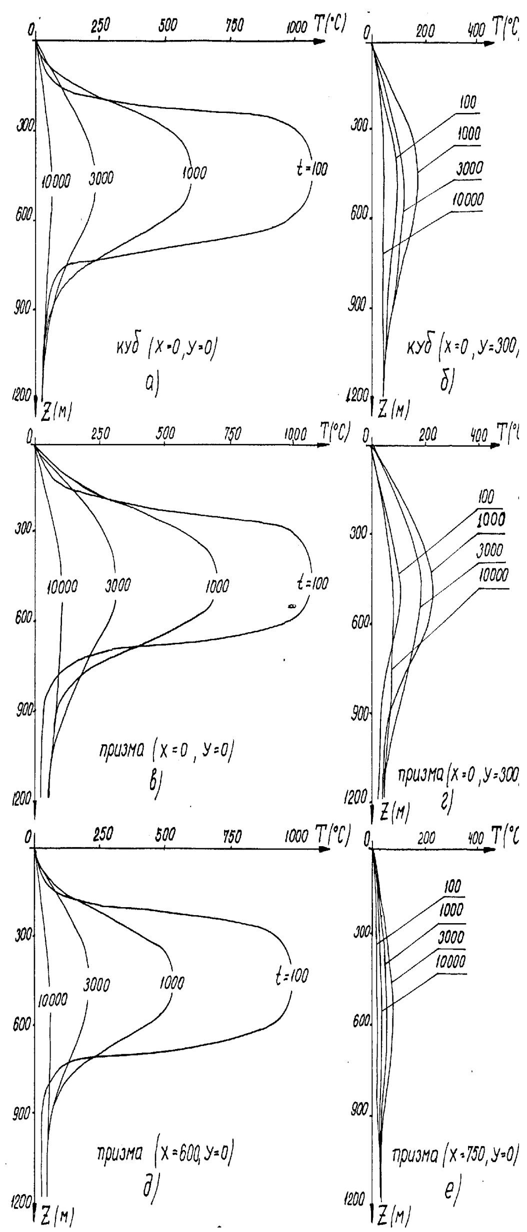

Figure 2.

———— center of the cube (x=0, y=0, z=450);

- - - - - - - center of the prism (x=0, y=0, z=450);

—·—·—·- prism (x=600, y=0, z=450);

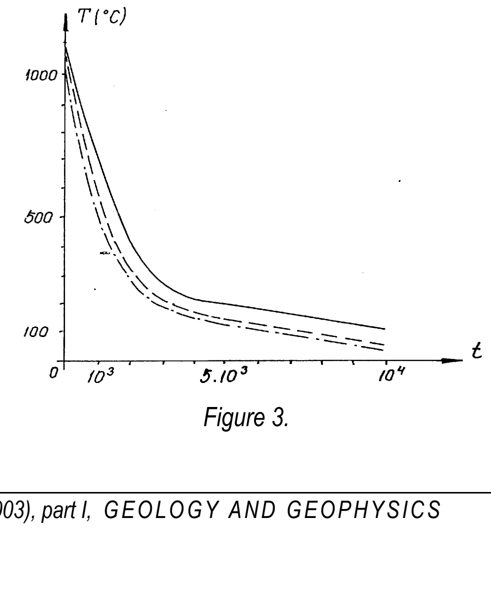

Figure 3.

In Fig.4 there is a change of temperature around the body:

——— around the cube (x=0, y=300, z=300);

- - - - - - - around the prism (x=0, y=300, z=450);

_._._._. around the prism (x=750, y=0, z=450).

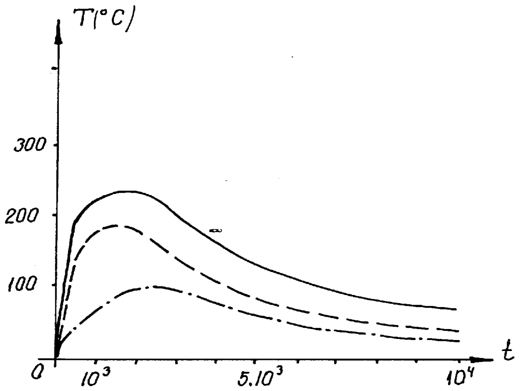

Figure 4.

We can see that in the body the first 2000 years the temperature goes down at the speed of 0,35 ÷ 0,4°C / y. With long periods the speed of cooling goes down quickly and at 4000 years the average speed is equal to 0,02 °C / y.

In the beginning near the body the temperature does up and then it begins to go down. The time for reaching the maximum depends on the distance to the body and the location of the point to the boundary of the cube maximal temperature 190 °C is reached for 1500 years from the initial moment of cooling the body. . We get different results for the prism depending on which side we investigate the temperature and how near it is. If we near the big side at a distance of 150 m. then the maximum temperature is bigger then that of the cube and it is 240°C. It is also reached for 1500 years.

If we are at the end of the prism then the maximum temperature is much lower 100° C, and it is reached much later – up to 2500 years from the beginning of the body cooling. After reaching the maximum near the body the temperature goes down very slowly – approximately 0.01 °C/y.

In tables 1, 2, 3 the calculations of non-stationary heat flow on the earth's surface are given. These results are presented in fig.5 for the cube and in fig.6 for the prism.

Table 1. Cube $q = \lambda u_z(t,\ x = 0,\ y,\ z = 0)$

y/t	0	150	300	450	600	750	900	1050	1200
100	0.796	0.431	0.086	0.062	0.06	0.06	0.06	0.06	0.06
1000	2.10	1.652	0.954	0.328	0.138	0.084	0.067	0.062	0.061
3000	1.182	1.008	0.672	0.356	0.192	0.118	0.086	0.072	0.069
10000	0.31	0.286	0.234	0.176	0.134	0.106	0.09	0.081	0.078

Table. 2. Prism $q(t,\ x = 0,\ y,\ z = 0)$

y/t	0	150	300	450	600	750	900	1050	1200
100	1.168	0.840	0.456	0.094	0.062	0.062	0.06	0.06	0.06
1000	2.636	2.114	1.264	0.282	0.198	0.104	0.074	0.064	0.062
3000	1.774	1.540	1.068	0.596	0.322	0.182	0.116	0.096	0.082
10000	0.542	0.504	0.414	0.310	0.226	0.168	0.132	0.114	0.108

Table 3. Prism $q(t,\ x,\ y = 0,\ z = 0)$

y/t	0	150	300	450	600	750	900	1050	1200
100	1.168	1.168	1.166	1.142	0.798	0.430	0.086	0.062	0.06
1000	2.636	2.624	2.570	2.376	1.743	0.978	0.338	0.148	0.108
3000	1.774	1.754	1.680	1.504	1.154	0.740	0.394	0.232	0.186
10000	0.542	0.534	0.506	0.456	0.382	0.298	0.226	0.184	0.170

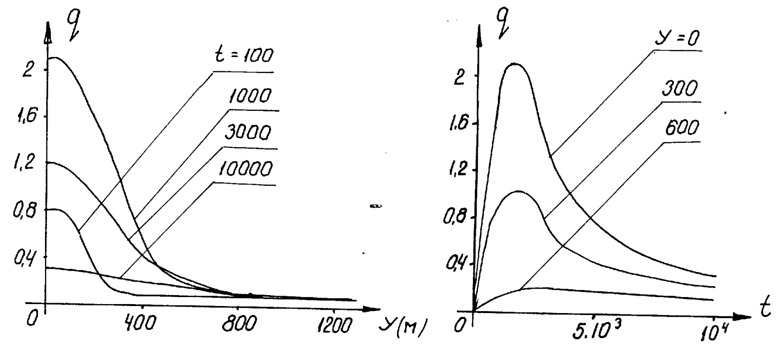

Figure 5.

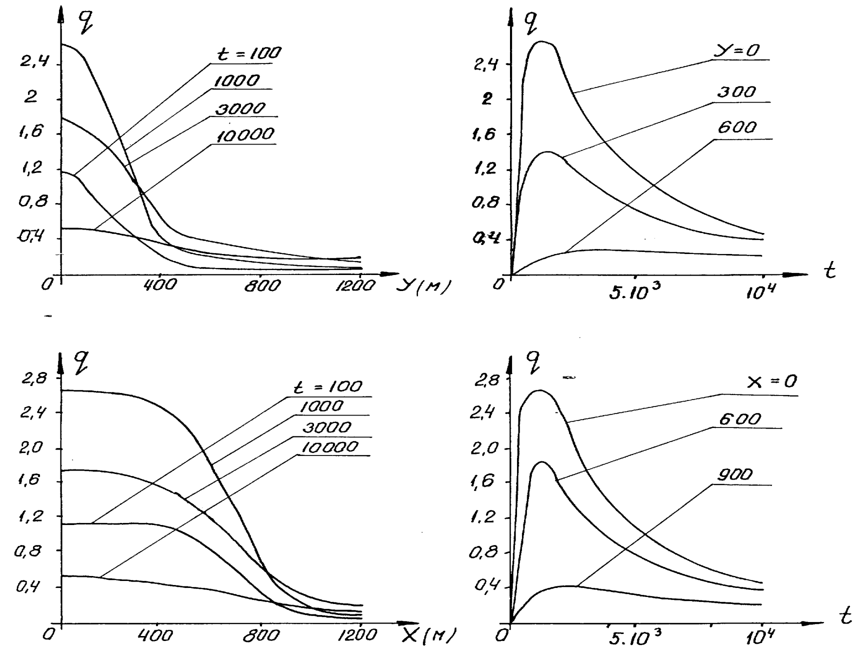

Figure 6.

CONCLUSION

The investigations carried out refer to a body that is not deep in the earth. If the body is at a great depth H, then all space characteristics increase (H/300) times, and those depending on time - $(H/300)^2$ times .

So for a body, which is at depth of 3000m. time characteristic increase 100 times. Then the maximum of heat flow on earth's surface for this body will be reached in 100-200 thousand years. The location of the heat flow will get worse 10 times. If the body is at depth (H=3000 m.) the flow be localized in an area with radius 4000-5000 m.

REFERENCES

Carslow G. and all. – Heat Conductivity, Moscow, 1964. (In Russian)

Numerical Mathematics and Teechnics in Geophysics. Geophysical Reference, Moscow, Nedra, 1982. (In Russian)

Vlasov, V., Lyubimova E. – Mathematical Models in Geothermics. Problems at Mathematical Physics and Numerical Mathematics. Moscow, 1977.(In Russian)

Kutas R. and all - Modelling of heat flow. Naukova Dumka, Kiev, 1989 (In Russian)

Kostyanev S. - Mathematical Models of Geothermal Fields. Moscow University, Moscow, 1979.(In Russian).

Kostyanev S. – Mathematical Modelling of Geophysical Fields in Gradient Media. MGU, Sofia, 1999,

(Monography- 200 pp.)

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