50 years University of Mining and Geology “St. Ivan Rilski”
Annual, vol. 46, Part I, Geology and Geophysics, Sofia, 2003, pp.391-396

QUANTITATIVE INTERPRETATION OF THE HORIZONTAL GRAVITY GRADIENT FOR SEMI-INFINITE HORIZONTAL SLAB STRUCTURES ACCORDING TO DATA FROM STATISTICAL ANALYSIS

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ABSTRACT

The applied method for quantitative interpretation of the gravitational field caused by semi-infinite horizontal slab structures is based on the horizontal gradient U_{xz} distribution on different levels.

Representative characteristics are chosen, that can be obtained without ambiguity from the data after analysis of the measured gravitational field – the maximum value of the horizontal gradient U_{xz}^{max}, the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ and the widths of the two branches of the horizontal gradient distribution - $\Delta X_{1/2}^{-}$ and $\Delta X_{1/2}^{+}$.

Subject of the study is the relationship between these characteristics and the main parameters of a semi-infinite edged horizontal slab – the depth h_1 to the slab structure, the thickness of the slab Δh ($\Delta h=h_2-h_1$) and the angle of the edge α. On the base of statistical studies for a wide range of different model parameters are obtained correlation ties suitable for the aims of the quantitative interpretation.

INTRODUCTION

The semi-infinite edged horizontal slab is one of the most common cases of contact between rocks having different density (W. Telford *et al.*, 1990; Gravity surveying, 1990). In this geometrical model the anomaly forming mass is confined by two horizontal and one dipping planes. In fig.1 is illustrated the traditional 2-D model of a semi-infinite edged horizontal slab, as well as the notation of its basic parameters. The gravitational effect U_z of the edged slab is presented by the following analytical expression:

$$U_z = 2G\Delta\rho \left| h\left(\frac{\pi}{2} + \text{arctg}\frac{h.\text{ctg}\alpha + x}{h}\right) + x.\sin\alpha\left\{\frac{1}{2}\sin\alpha.\ln\frac{1}{\sin^2\alpha}\left[(h + x.\sin\alpha.\cos\alpha)^2 + x^2.\sin^4\alpha\right] - \cos\alpha.\text{arctg}\frac{h + x.\sin\alpha.\cos\alpha}{x.\sin^2\alpha}\right\}\right|_{h_1}^{h_2}$$

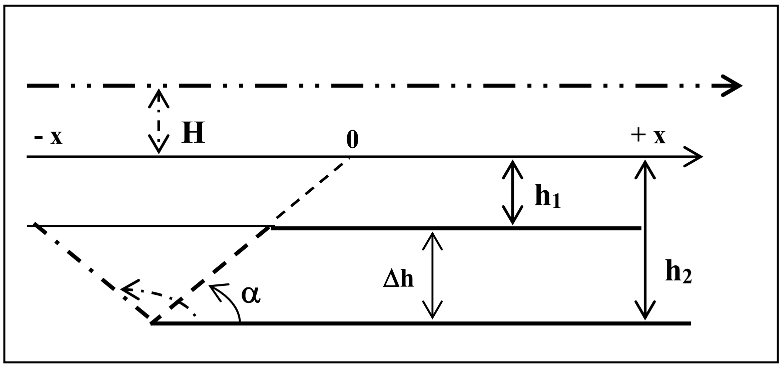


Figure 1. A 2-D model of a semi-infinite edged horizontal slab and notation of its basic parameters

Due to the quite complex expression there are no analytical methods for solving the reverse gravity problem. For that reason a detailed statistical analysis was performed over a wide range of different models solutions not only for the gravitational field, but also for its horizontal U_{xz} and vertical U_{zz} gradients. The aim of the studies is to obtain results for the geometrical parameters of the slab structure after excluding the influence of the density distribution. The analysis of a big volume of statistical data shows that it is most suitable to apply the distribution characteristics of the horizontal gradient U_{xz} on two levels – basic one and upward continuation on height H.

MAIN RESULTS FROM THE STATISTICAL STUDIES

After many statistical studies are selected representative characteristics that can be obtained without ambiguity from the data after analysis of the measured gravitational field – the maximum values of the horizontal gradient on the basic level $U_{xz}^{max,b}$ and on the upward continuation on level H - $U_{xz}^{max,H}$, as well as the difference ΔU_{xz}^{max} between these values; the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ and the widths of the two branches of the horizontal gradient distribution - $\Delta X_{1/2}^{-}$ and $\Delta X_{1/2}^{+}$ (fig.2).

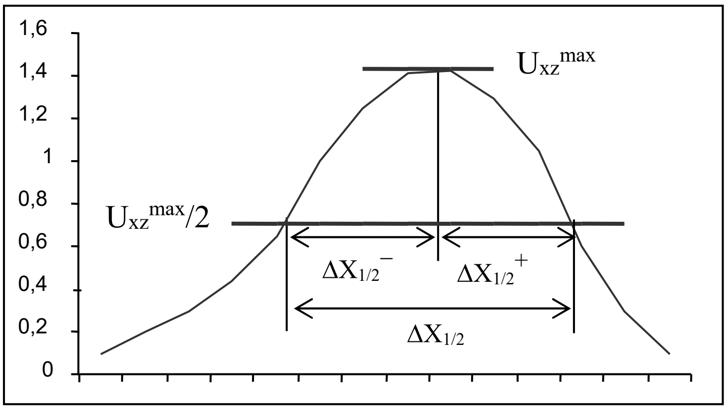
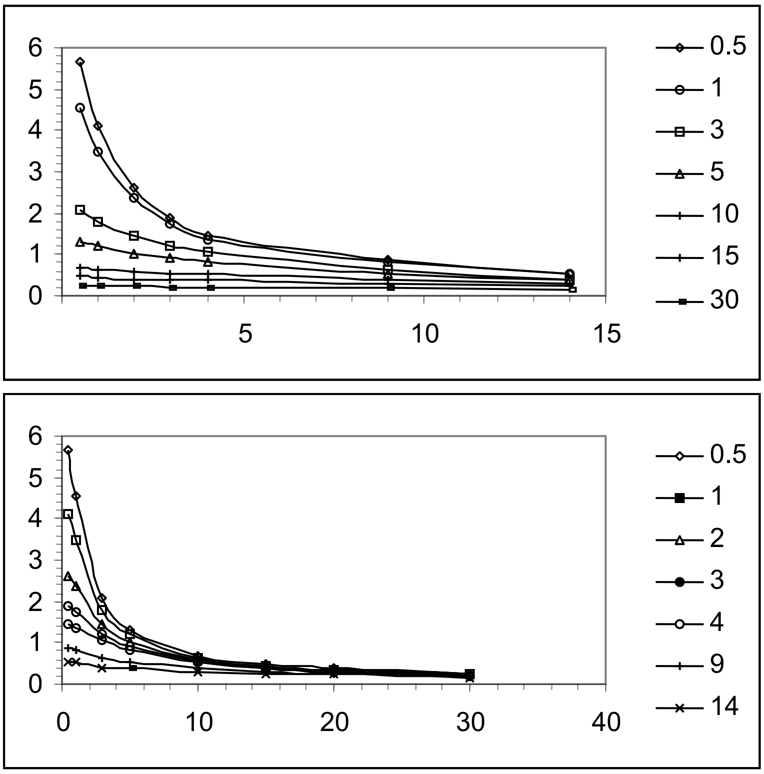


Figure 2. Notation of the parameters utilized in the quantitative interpretation of the horizontal gravity gradient for semi-infinite horizontal slab structures

Subject of the study is the relationship between these characteristics and the main parameters of a semi-infinite edged horizontal slab – the depth h_1 to the slab structure, the thickness of the slab Δh ($\Delta h=h_2-h_1$) and the angle of the edge α (fig.1). All parameters having dimension length are presented in the utilized scale (metres or kilometres).

The analysis of some of the obtained main statistical relations is of definite methodical and practical interest. In all presented cases is applied upward continuation on level H=1 toward the basic level.

In fig.3 is presented the ratio $U_{xz}^{max}{}_{,b}/U_{z,b}$ as function of the depth h_1 towards the slab structure (a) and as function of the slab thickness Δh (b) respectively. The illustrated relations are showing that the ratio $U_{xz}^{max}{}_{,b}/U_{z,b}$ differentiates reasonably well the depth h_1 down to about 4 for slab structures having thickness Δh up to about 5.



a

b

Figure 3. a. The ratio $U_{xz}^{max}{}_{,b}/U_{z,b}$ as function of the depth h_1 towards the slab structure for various slab thicknesses Δh

b. The ratio $U_{xz}^{max}{}_{,b}/U_{z,b}$ as function of the slab thickness Δh for various depths h_1 towards the slab structure

In fig.4 is presented the ratio of the maximum gradient value $U_{xz}^{max}{}_{,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max}{}_{,b}$) and on level H=1 ($U_{xz}^{max}{}_{,H=1}$) as function of the depth h_1 towards the slab structure for various angles of the edge α and slab thickness Δh=10.

The compound analysis of the illustrated dependence as well as the similar relations for slab thicknesses Δh in the range

0,1-50 is showing that for depths $h_1>15$ the angle of the edge has practically no influence and that the complex parameter $\Delta U_{xz}^{max}/U_{xz}^{max},_b$ for Δh=const is connected linearly to the depth h_1 towards the slab structure.

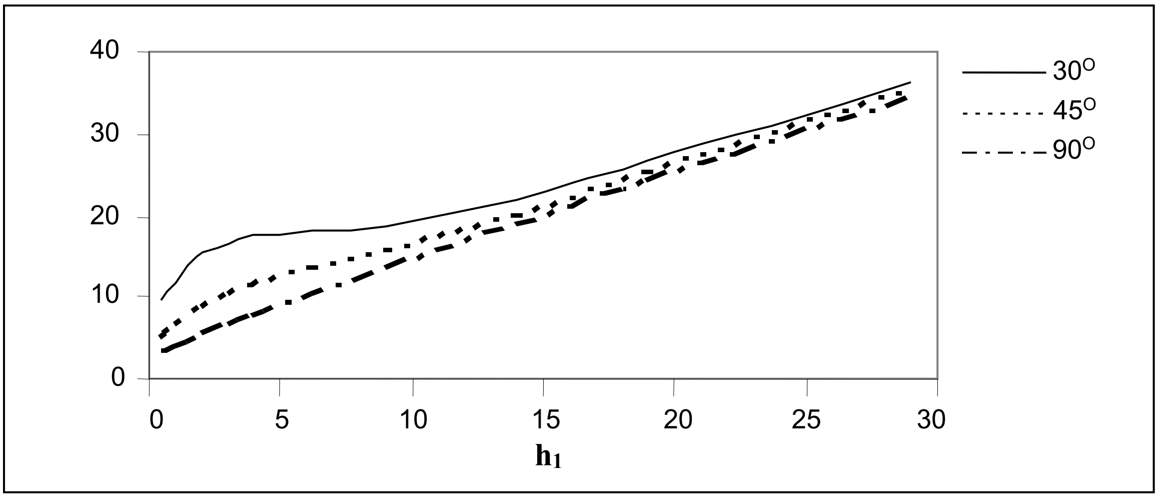


Figure 4. The ratio of the maximum gradient value $U_{xz}^{max},_b$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max},_b$) and on level H=1 ($U_{xz}^{max},_{H=1}$) as function of the depth h_1 towards the slab structure for various angles of the edge α and slab thickness Δh = 10

When $h_1<15$ the relationship is showing ambiguity. This is revealed clearly for $\alpha<30^o$ ($\alpha>150^o$ respectively). For example, for depth h_1=10 the values of the ratio $\Delta U_{xz}^{max}/U_{xz}^{max},_b$ for angles 60^o, 45^o and 30^o are 4%, 13% and 23% respectively if compared towards the value for 90^o. For depth h_1=20 these values decrease to 2%, 4% and 8% respectively.

In fig.5 is presented the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the slab thickness Δh for various depths h_1 towards the slab structure and angles of the edge $\alpha=90^o$ and $\alpha=30^o$ respectively. The analysis of the illustrated dependence as well as the similar relations for other angles of the edge is showing that for h_1=const and α=const the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ is connected almost linearly to the thickness Δh of the slab structure.

This dependence is detailed in fig.6. There is illustrated the relationship for two angles of the edge and two depths towards the slab structure – 0,5 and 14 respectively. For small values of Δh the angle of the edge has no influence and with the increase of the slab thickness the angle influence is increasing linearly.

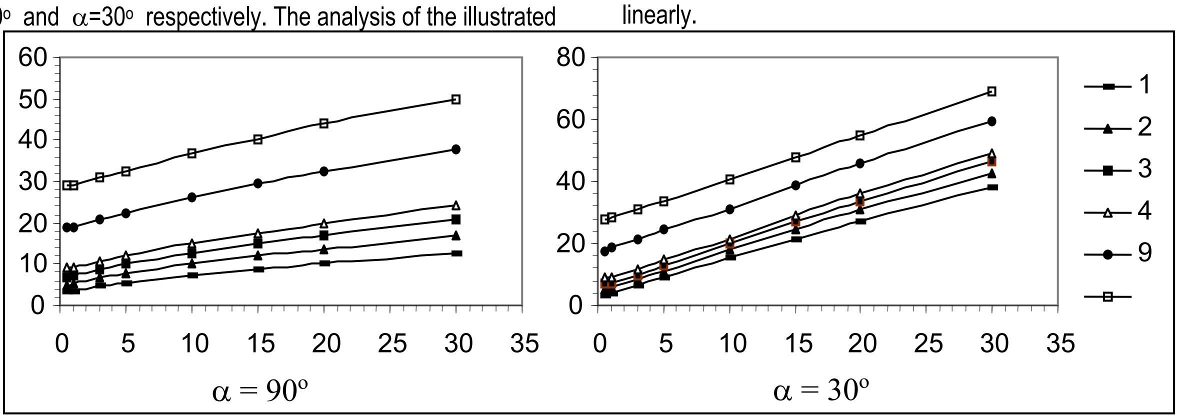


Figure 5. The anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the slab thickness Δh for various depths h_1 towards the slab structure and angles of the edge $\alpha = 90^o$ and $\alpha = 30^o$ respectively

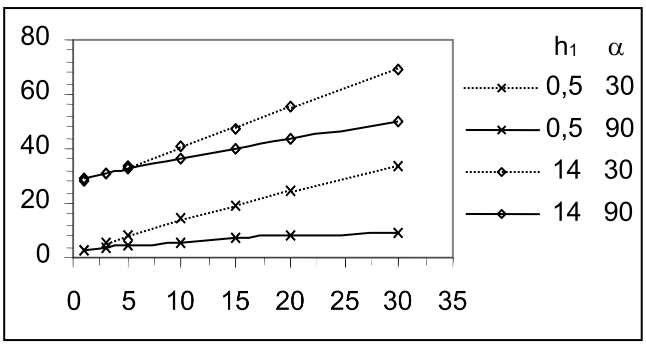


Figure 6. The anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the slab thickness Δh for depths towards the slab structure h_1 = 0,5 and h_1 = 14, and angles of the edge $\alpha = 90^o$ and $\alpha = 30^o$ respectively

In fig.7 is presented the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the depth h_1 towards the slab structure for various slab thicknesses Δh and angles of the edge α=90° and α=30° respectively. The dependence is showing that the connection is exponential and for depths greater than 5 the tie becomes linear.

The function between the depth h_1 towards the slab structure and the anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ is detailed in fig.8. There is illustrated the relationship for two angles of the edge – 30° and 90°, and two thicknesses of the slab structure – 5 and 30 respectively. On this figure the influence of the angle of the edge and the slab thickness is becoming clearer. For values of Δh smaller than 5 the angle of the edge has no influence. With the increase of the slab thickness the anomaly width $\Delta X_{1/2}$ for angle 30° increases compared to the width for angle 90°. This tendency is decreasing with the increase of the depth towards the slab structure.

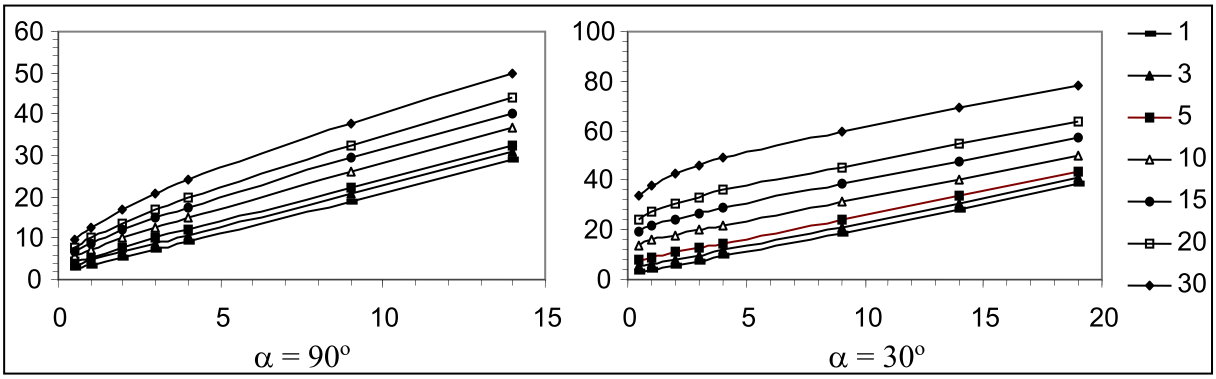


Figure 7. The anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the depth h_1 towards the slab structure for various slab thicknesses Δh and angles of the edge α = 90° and α = 30° respectively

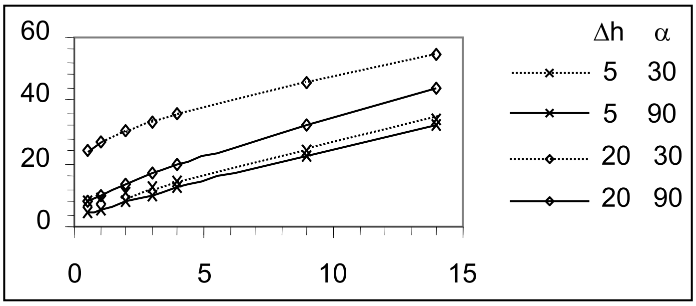


Figure 8. The anomaly width $\Delta X_{1/2}$ for gradient value $U_{xz}^{max}/2$ as function of the depth h_1 towards the slab structure for slab thicknesses Δh = 5 and Δh = 20, and angles of the edge α = 90° and α = 30° respectively

In fig.9 is presented the ratio of the maximum gradient value $U_{xz}^{max}{}_{,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max}{}_{,b}$) and on level H=1 ($U_{xz}^{max}{}_{,H=1}$) as function of the depth h_1 towards the slab structure for various slab thicknesses Δh and angle of the edge α=60°. The dependence is showing that the connection is exponential and for depths greater than 5 the tie becomes linear.

In fig.10 is illustrated the ratio of the maximum gradient value $U_{xz}^{max}{}_{,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max}{}_{,b}$) and on level H=1 ($U_{xz}^{max}{}_{,H=1}$) as function of the slab thickness Δh for various depths h_1 towards the slab structure and angle of the edge α=60°. The dependence is showing that the connection is exponential as the above-mentioned one.

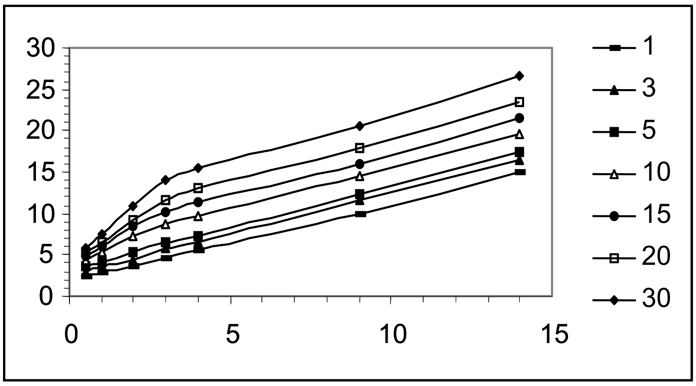


Figure 9. The ratio of the maximum gradient value $U_{xz}^{max}{}_{,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max}{}_{,b}$) and on level H=1 ($U_{xz}^{max}{}_{,H=1}$) as function of the depth h_1 towards the slab structure for various thicknesses Δh of the slab and angle of the edge α = 60°

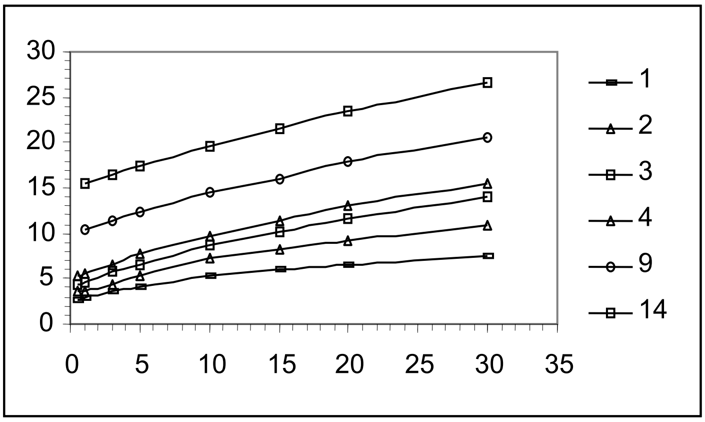


Figure 10. The ratio of the maximum gradient value $U_{xz}^{max,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max,b}$) and on level H=1 ($U_{xz}^{max,H=1}$) as function of the slab thickness Δh for various depths h_1 towards the slab structure and angle of the edge $\alpha = 60^o$

In fig.11 is illustrated the ratio $\Delta X_{1/2}^- / \Delta X_{1/2}^+$ as function of the depth h_1 towards the slab structure for various angles of the edge α and slab thickness Δh=10. For values of the depth greater than 7 the ratio inclines towards 1. For smaller depths the ratio is increasing with the decrease in the depth towards the slab structure and for angle $\alpha=30^o$ and h_1=0,5 it has a value of about 6.

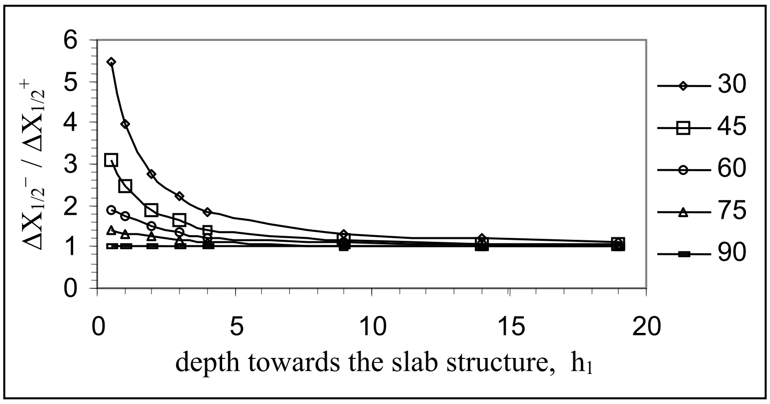


Figure 11. The proportion $\Delta X_{1/2}^- / \Delta X_{1/2}^+$ as function of the depth h_1 towards the slab structure for various angles of the edge α and slab thickness Δh = 10

In fig.12 is presented the ratio $\Delta X_{1/2}^- / \Delta X_{1/2}^+$ as function of the angle of the edge α for various depths h_1 towards the slab structure and slab thickness Δh=10. The illustrated relation is showing that the angles of the edge $\alpha < 75^o$ ($\alpha > 105^o$) are forming a well-pronounced asymmetry in the case of relatively small values for the depth ($h_1 < 7$).

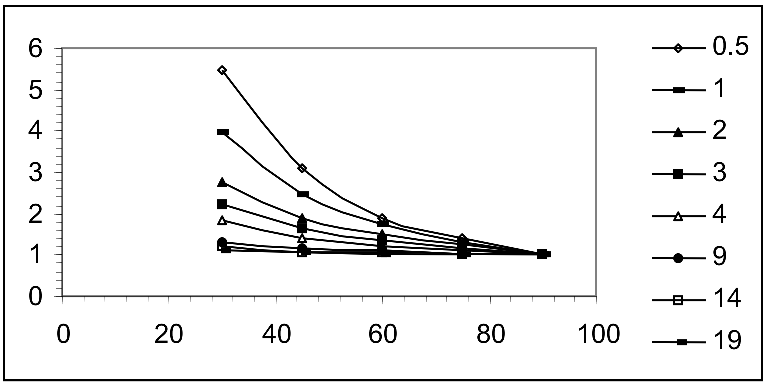


Figure 12. The proportion $\Delta X_{1/2}^- / \Delta X_{1/2}^+$ as function of the angle of the edge α for various depths h_1 towards the slab structure and slab thickness Δh = 10

Some more detailed information is presented in fig.13. It illustrates the ratio $\Delta X_{1/2}^- / \Delta X_{1/2}^+$ as function of the depth h_1 towards the slab structure for various slab thicknesses Δh and angle of the edge $\alpha=30^o$. This dependence reveals the quick increase of the asymmetry related to the increase of the slab thickness and the decrease of the depth towards the slab structure.

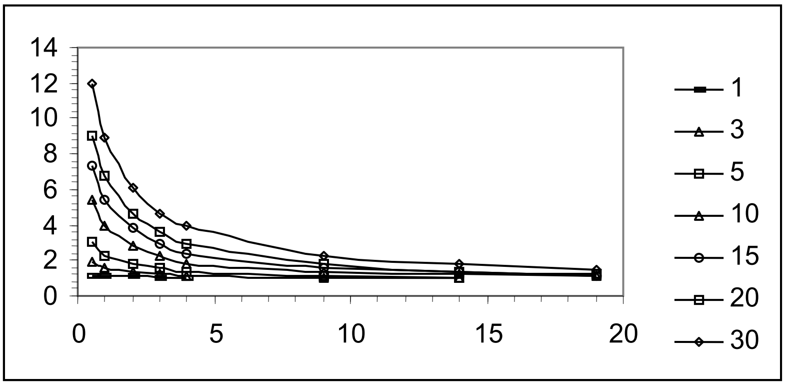


Figure 13. The proportion $\Delta X_{1/2}^{-}$ / $\Delta X_{1/2}^{+}$ as function of the depth h_1 towards the slab structure for various slab thicknesses Δh and angle of the edge α = 30°

CONCLUSIONS

On the base of analytical calculations of the gravitational field horizontal gradient U_{xz} for multiple models of semi-infinite edged horizontal slab are performed statistical studies that make possible the proposal of a method for estimating the slab structure parameters. This method includes three successive stages:

- For estimating the depth h_1 towards the slab structure is used a multiple regression with independent variables the ratio $\frac{U_{xz,b}^{max}}{\Delta U_{xz}^{max}}$ of the maximum gradient value $U_{xz}^{max}{}_{,b}$ on the basic level towards the difference ΔU_{xz}^{max} between the maximum gradient values on the basic level ($U_{xz}^{max}{}_{,b}$) and on level H=1 ($U_{xz}^{max}{}_{,H=1}$), the anomaly width $\Delta X_{1/2}$ for gradient value $\frac{U_{xz}^{max}}{2}$ and the natural logarithm of the ratio $\frac{\Delta X_{1/2}^{-}}{\Delta X_{1/2}^{+}}$ between the widths of the two branches of the horizontal gradient distribution on the basic level:

$$h_1 = f\left(\frac{U_{xz,b}^{max}}{\Delta U_{xz}^{max}}, \Delta X_{1/2}, \ln\frac{\Delta X_{1/2}^{-}}{\Delta X_{1/2}^{+}}\right)$$

The R-squared value of the multiple regression is very high. After limiting the range of the independent variables $\frac{U_{xz,b}^{max}}{\Delta U_{xz}^{max}}$, $\Delta X_{1/2}$ and $\frac{\Delta X_{1/2}^{-}}{\Delta X_{1/2}^{+}}$, the R-squared value is above 0,99. The established close relationship provides a high level of confidence in the estimation of the depth h_1 towards the slab structure.

- Once the depth h_1 towards the slab structure is found, the thickness Δh of the slab can be estimated. The following multiple regression is used:

$$\Delta h = f\left(h_1, \frac{U_{xz,b}^{max}}{\Delta U_{xz}^{max}}, \Delta X_{1/2}, \ln\frac{\Delta X_{1/2}^{-}}{\Delta X_{1/2}^{+}}\right).$$

- When the values for h_1 and Δh are known the angle of the edge α can be estimated using the following multiple regression:

$$\alpha = f\left(h_1, \Delta h, \frac{U_{xz,b}^{max}}{\Delta U_{xz}^{max}}, \Delta X_{1/2}, \ln\frac{\Delta X_{1/2}^{-}}{\Delta X_{1/2}^{+}}\right).$$

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