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ABOUT THE GENERALISATION POSSIBILITIES IN A NEW STOCHSTICAL MODEL OF MINING SUBSIDENCE

Michael Vulkov

University of Mining and Geology "St. Ivan Rilsky"
Sofia, 1700, Bulgaria

ABSTRACT
This paper presents the mining subsidence trough from the viewpoint of the stochastic processes. Taking into account the laws of the chance processes and the elastic properties of the particles of the loose media a generalized nonlinear equation for calculating the mine subsidence trough is obtained. The new obtained equation generalizes the existing stochastic linear and nonlinear models in geomechanics and may be used as an universal basis for deeper understanding and analyzing the subsidence phenomenon.

When creating the stochastic geomechanical theory of mining subsidence J. Litwiniszyn (1956) considered the many time underworked rock mass as an accumulation of loose stone blocks (Fig.1). This system of bodies has such a degree of freedom that is seems logical to apply statistical methods in calculating its displacements.

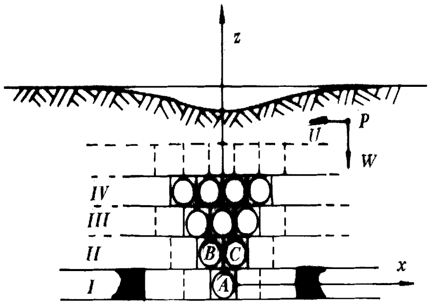

Figure 1

In this work we study the possibility to use a generalized nonlinear model of stochastic medium with application in the mining subsidence.

For calculating the displacements in rock mass and on the earth's surface, caused by underground mining works, let us assume that the following presumptions are fulfilled:

- the displacements of the rock mass are regarded from the view point of the stochastic processes;
- the geomaterial is stochastic medium built from elastic particles;
- the characteristics of the rock mass allow to use the model of the new stochastic medium, presented from M. Vulkov (1997).

In order to sketch this consideration let us assume a plane rhombic packing of particles in the Cartesian system of coordinates (x, z), as in Fig.2.

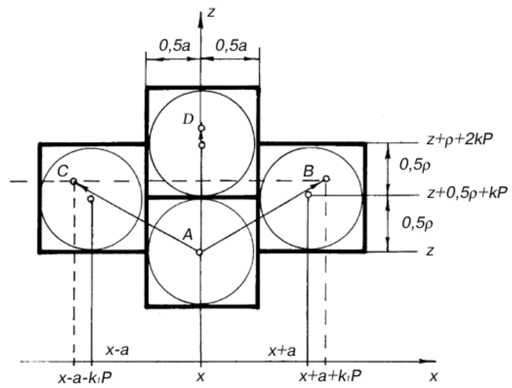

Figure 2

If the particle in field A is removed, the vacant space can only be filled by a particle from the neighboring higher fields B, C or D.

When an elastic particle migrates from higher to lower level of the loose media, this causes appearance of elastic displacement changes, as a result of the external load changes.

Let P(x, z) be the probability of a void appearance in a cage with a center-point coordinates (x, z). Following the assumption of J. Litwiniszyn (1974) for correspondence of that probability with the vertical displacements of the rock mass points for the stochastic medium presented above the following relationship could be written:

$$w(x,, z) = w_1 + P(x, z)w_2 \qquad (1),$$

where $\boldsymbol{w_1}$ is the classical stochastic displacement, by determining of which the rock mass particles are assumed for solid bodies after Litwiniszyn (1974);

w_2 is the increase of the elastic vertical displacement, caused by the change of places among particles in different levels i.e. by the change of the external load. Its value is proportional to the probability for removing at lower horizon.

Let p, q, and c be the chances of a void migration from cage A to B, C or D respectively. Then the following equality exists:

$$p + q + c = 1 \quad (2),$$

i.e. if a particle in field A is removed, the vacant space can only be filled by a particle from cages B, C or D.

In accordance with the mechanism of chance processes the following equation may be written:

$$P(x, z) = pP(x - a - k_1P, z + 0{,}5\rho + kP) + qP(x + a + k_1P, z + 0{,}5\rho + kP) + cP(x, z + \rho + 2kP). \quad (3),$$

where ***$k_1 = \Delta u(x,, z)$*** is the change of the horizontal elastic relative displacement of the particle's center-point (Fig.2); ***$k = \Delta w(x, z)$*** is the increase of the vertical elastic relative displacement of the particle's center-point.

The relationship (3) is the starting point for obtaining the generalized non linear equation for mining subsidence determining. After developing the right-side-terms of equation (3) in Tailor's series for x, z – point area, after substituting the obtained expressions in relationship (3), following M. Vulkov (2001) and after limiting process ***$a \to 0$, $\rho \to 0$*** by corresponding choice of the measurements, the following equation is constructed:

$$[A_{11}(P)P_x]_x + 2[A_{12}(P)P_x]_z + [A_{22}(P)P_z]_z + B_1(P)P_x + B_2(P)P_z = 0 \quad (4),$$

where $P_x = \partial P/\partial x$, $P_z = \partial P/\partial z$,; $A_{11}(P)$, $A_{12}(P)$, $A_{22}(P)$, $B_1(P)$ and $B_2(P)$ are rock mass characteristics.

By obtaining the equation (4) we take into consideration the fact that when passing to the limit expressions of higher order disappear and is assumed that appropriate limits such as by M. Vulkov (2001) exist.

The newly constructed equation (4) is a generalization of the classical linear models of J. Litwiniszyn (1956, 1974, 1974) and of the M. Vulkov's (1988, 1997)models.

So by $A_{12}(P) = A_{22}(P) = B_1(P) = 0$ from (4) follows the nonlinear parabolic equation

$$[A_{11}(P)P_x]_x + B_2(P)P_z = 0 \quad (5),$$

whose interesting solutions and their applications for mining subsidence engineering were presented from M. Vulkov (1989).

The newly obtained generalized nonlinear equation (4) may be used as a universal basis for the interpretation of the mining subsidence in different conditions and by different kinds of rock mass characteristics.

After A Tichonov (1972), it can be shown that the equation (4) can change its type from hyperbolic through parabolic into elliptic one.

In conclusion it can be said that the generalized nonlinear equation (4) obtained in this paper, offers interesting possibilities for formulating and solving strata movement problems caused by underground mining works. The last equation certainly enables us to complete the visions about the stochastic theory in mining subsidence and to enrich the models and methods for strata movement calculations.

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