

## MODELLING COMPLEX COMPRESSED AIR NETWORKS IN ORDER TO IMPROVE THE SPEED OF CALCULUS

**Ion Dosa**

*University of Petrosani, 332006 Petrosani, i\_dosa@hotmail.com*

**Abstract:** Solving the equations of fluid flow in ring shaped networks or in complex compressed air networks, requires a great amount of calculus. Therefore using computer program for this task is a must. Even so, the great amount of data and the complexity of fluid flow produce a great number of iterations, using most of the hardware resources of the computer. The paper presents possibilities of modeling the complex compressed air networks, in order to increase the speed of calculus and reduce the amount of hardware resources needed.

### МОДЕЛИРАНЕ НА СЛОЖНИ ПРЕНОСНИ СИСТЕМИ СЪС СГЪСТЕН ВЪЗДУХ С ОГЛЕД ПОДОБРЯВАНЕ СКОРОСТТА НА МЕТОДИТЕ ЗА ИЗЧИСЛЯВАНЕ

**Йон Доса**

*Петрошански университет, Петрошани, Румъния*

**РЕЗЮМЕ:** Решаването на уравненията за движение на флуидите в затворени транспортни системи или в системи със сгъстен въздух, изискват голям брой сложни изчисления. Ето защо е крайно необходимо използването на компютърна програма за решаването на тези задачи. Дори и при тези обстоятелства, голямо количество данни и сложността на движението на флуидите се получава много повторения, използвайки повече хардуерни способности на компютъра. Статията представя възможностите от моделиране на сложните преносни системи със сгъстен въздух, за да се повиши бързината на изчисляване и намали необходимото количество на хардуерните възможности.

### Introduction

Compressed air networks are complex structures due to their construction and the complexity of the compressed air flow in the network. The solutions for the equations of compressed air flow applied to networks in case of flow with friction, heat transfer and flow loss, can be obtained merely using numerical methods. These methods require a great volume of calculus. Therefore, the calculus of complex networks can be done only if the development of a computer program is considered.

The case of a mining compressed air network is studied, which by reason of his complexity, represents a special case of compressed air network.

Constructive peculiarities for such networks (Dosa, 1998):

- Developing as the mine site evolves. Therefore the length of the network, his structure and the gas flow is constantly changing. The network structure became more complicated with lots of rings, rings with common edges, and all these alternating with embranchments.

- Must follow the mine works, resulting elbows, deviations and multiple embranchments.

- The ducts can't be welded, so the pipelines must be joined through flanges, therefore the flow losses can't be eliminated.

- The maximum length of pipelines that can be entered in underground is 6 m, a great number of joints resulting; therefore the flow losses are high.

- There are hard exploitation conditions. The pipelines and fixtures can be damaged, so that the local pressure drops and the flow losses will grow. The corrosion of the pipelines is marked, which leads to the growth of the rugosity.

### Mathematical model of compressed air network

Developing the mathematical model of compressed air networks, must start from the definition of his functional role, settlement of his limits, identification of its components and the relations that exist between these.

The compressed air network must be able to transport the compressed air from the compressor to consumers assuring the optimum operation parameters for these.

The limits of the system are represented through the outlet section of the buffer reservoir (compressors with piston) or the outlet section of the last cooler (turbocompressors) representing the inlet section of the network, the inlet section of the consumers representing the outlet section of the network and the lateral area of the pipelines.

The compressed air flows from the compressor through the inlet section of the network toward the consumer (through the outlet section of the network) with friction, heat transfer and flow loss through lateral area of the pipelines.

The discrete components of the compressed air network from the point of view of this analysis are (Burducea and Leca, 1974; Leca et al. 1984):

- Pipelines and ducts which are the rectilinear elements of the network;
- Fittings used for modification of the section of flow, changing the direction of flow and the realization of the necessary embranchments.
- Fixtures that allow and direct the compressed air flow through pipelines, and also might adjust the parameters of the compressed air.
- Assembling parts that assure the connecting of components of the compressed air network.

Joining of these elements and their location in ground defines the structure of the compressed air network.

For modeling the structure of the compressed air network the representation of network as ordinary graph with the property that the maximum number of adjacent of a node is 4, is proposed.

According to the nodes of the graph different type of nodes of the network were defined:

- Compressor node – corresponding to the compressor (or an injection point in the network) with the property that has only one adjacent;
- Embranchment node – corresponding to the embranchments of network, can have 3 or 4 adjacent.
- Consumer node - corresponding to the pneumatic consumer, can have only one adjacent.

Transom of the compressed air network, was defined as the physical succession of the discrete components of the network lined up between two nodes and corresponds the edges defined by two nodes in the ordinary graph.

The compressed air pipeline was defined as the succession of ducts joined through one of the known methods (flanged, welded, etc.) having the same diameter, lined up between two discrete components (section lift, faucets, diaphragms etc.)

Conclusively, the mathematical model of the compressed air network shall have two components:

- The algorithm of determination the structure of network, that describes the relations between different elements, the way of go through and the succession of calculus for the parameters of flow through the discrete elements of the network.
- Mathematical models for the calculus of parameters of compressed air flow through the elements of the network like: pipelines, faucets, elbows, flaps, valves, diaphragms, embranchments etc.

The analytic determination of parameters of the compressed air flow through pipelines is possible through solving of the fundamental equations of gas dynamics singularized for compressed air networks.

Starting from the equations of gas dynamics applied to compressed air networks (Irimie and Matei, 1994):

- The continuity equation:

$$\frac{d(\rho \cdot w)}{dx} = \frac{a \cdot d \cdot p^{1.3}}{\frac{\pi}{4} \cdot d^2} \quad (1)$$

- The momentum equation:

$$w \cdot \frac{dw}{dx} = g_x - \frac{1}{\rho} \cdot \frac{dp}{dx} - \lambda \cdot \frac{w^2}{2 \cdot d} \quad (2)$$

- The energy equation:

$$\frac{dT}{dx} = \frac{4 \cdot K}{\rho \cdot w \cdot d \cdot c_p} \cdot (T - T_m) \quad (3)$$

- Equation of state:

$$p = \rho \cdot R \cdot T \quad (4)$$

relations in which:  $w$  is the average speed in section [ $m \cdot s^{-1}$ ],  $T$  - fluid temperature [K],  $T_m$  - surrounding temperature [K],  $p$  - fluid pressure [ $N \cdot m^{-2}$ ],  $d$  - hydraulic diameter of the duct [m],  $\rho$  - density of the fluid [ $kg \cdot m^{-3}$ ],  $\lambda$  - friction coefficient,  $K$  - global heat transfer coefficient [ $W \cdot m^{-2} \cdot K^{-1}$ ],  $a$  - flow loss coefficient through leakiness.

From the relations (1), (2), (3), (4) above (Irimie and Matei, 1994):

$$w' = \frac{1}{w^2 - R \cdot T} \cdot (A \cdot w^3 + g_x \cdot w - R \cdot w \cdot T' - B \cdot R \cdot \frac{T}{\rho}) \quad (5)$$

$$\rho' = \frac{1}{w^2 - R \cdot T} \cdot (B \cdot w - A \cdot \rho \cdot w^2 - g_x \cdot \rho + R \cdot \rho \cdot T')$$

in which A and B:

$$A = \frac{\lambda}{2 \cdot d}; \quad B = \frac{4 \cdot a \cdot p^{1.3}}{\pi \cdot d}$$

The coefficients having the sign “-“ for the calculus of transoms in the sense of fluid flow and the sign “+“ for the calculus in opposite direction of fluid flow.

For numerical solution of the system (5), different methods can be applied (Dodescu and Toma, 1976; Larionescu 1989; Roşculeţ 1984; Salvadori and Baron 1972; Simionescu 1995). In the work (Irimie and Matei 1994) is recommended the use of the cubical spline functions for the approximation of the solution of differential equations, due to the convergence of the method and steps of iteration that have relatively big values.

The use of a specific method for solving the differential equations of fluid flow in pipelines is important when thinking in

terms of speed of the calculus, but also other aspects of network structure must be considered when thinking of overall calculus speed.

Because of the great complexity of compressed air networks, a great amount of data must be processed to make additional calculus related to other structural elements, modeling the structure and creating a friendly user interface. These aspects are presented as follows.

### Algorithm for determination of network structure

For simple network configurations, exists a big variety of calculation procedures, their complexity grow up as possibility of automate calculus improved, and especially along with the appearance of the electronic computers.

Among first calculation procedures counted the one applied of acad. M.M. Fedorov (Ilicev, 1951) in which is considered the variation of the state of the compressed air flow in pipelines.

The state of parameters of the air in any portion of the pipeline is determined in function of initial state parameters from the beginning of the pipeline.

The calculus of the network started from the compressor station toward to consumers.

The compressed air network can be projected choosing percentage of loss of pressure depending on value of admissible loss, so is obtained a minimum for the costs of the pipeline and the energy.

Acad. A.P. Gherman proposes the calculus of treelike networks from the consumer towards compressor (Ilicev, 1951).

In this case the pressure to consumers is the same, and is necessary the equalization of loss of pressure along of the branches that are not part of their nodes.

In behalf of a computer program realization an algorithm (Irimie and Matei, 1994) was developed, that require the division of some transoms with invariable geometric parameters in supplementary sectors, in order to obtain iterations with identical number of steps. Known parameters are: the configuration of the network, the length of the transoms, the demand of air input for the consumers, the temperature and the pressure of the compressed air at the outlet of the compressor station, the polytropic exponent of the flow on the transom.

Parameters resulting from calculus: loss of pressure on transoms, loss of flow on transoms, losses at the consumers, temperature on transom.

There are calculation procedures for treelike networks fed from one source, from two sources, for simple ring networks, and ring networks with common edges, for which the algorithms are depicted in the works (Burducea and Leca, 1974; Leca et al. 1986; Sârbu, 1997).

For ring shaped networks the method of simple iteration or Lobacev method and for ring shaped networks with common edges the Hardy-Cross method (Sârbu, 1997) are widely used.

All these methods have a great disadvantage; they can be applied to networks with known configuration and in a differentiated way. The compressed air networks from underground are the result of the development in time of the mining works, have a complicated configuration, many branches and rings in different zones of the network.

Conclusively, an algorithm for compressed air networks must assure the way of go through the network in the sight of calculus and the identification of different sort of configurations.

Developing the algorithm for the calculus of the compressed air networks due to open with choosing the point from which the calculus began. According as, there are two possibilities: to go from the compressor to consumer, which presupposes a great number of iterations, and going from the consumer to compressor.

The second variant was chosen due to the fact that mathematical model permits the determination of transom parameters calculating from opposite direction of flow, and the number of iterations will decrease.

Is reminded as, the compressed air network is represented as an ordinary graph (Cristea et al., 1993; Ionescu Texe and Zsako, 1990), and in this case the calculus presupposes going in depth of the structure of graph, until to reach the consumer node.

Having in sight that a direct method of calculus doesn't exists, due to the complexity of the problem, a method that shall solve the problem through partial solutions must be found.

Such method is the Backtracking (Cristea et al., 1993), in which the solutions are built progressively.

Application of the method assumes the definition of stacks (static or dynamic) which in shall kept the visited nodes and which will be erased only after the nodes are solved.

An embranchment node can be solved if known at least  $n-1$  flows where  $n$  represent the number of adjacent.

Applying the principle of mass conservation, the value of the missing flow can be found, and on the strength of the flows and geometric sizes of the embranchment the resistance (Idlecik, 1984) and the missing pressure can be found, in assumption that the temperature is the same in all branches.

The ordinary graph defined through the nodes of the network and the proper transoms, is represented in the shape of a list of adjacent in a database. Although exist another solution of representation (Cristea et al., 1993), the choice was made since there are no limitations regarding the number of nodes, from the computer memory size point of view. In the computer memory stood at one time solely the nodes visited and unsolved, reducing the size of used memory.

For the description of the algorithm of determination what nodes belong to a ring, have to start up from the definition (Ionescu Texe and Zsako, 1990; Vrânceanu and Mititelu, 1984) of strongly connected graph, bi-connected graph and chain.

A limited chain which leaves from one point and comes back in the same point defines a ring.

The way of go through applied, assures that all the nodes in the graph will be visited.

Is noticed as, if we have rings with common transoms (edges), we have a lot of rings in the same structure, therefore many different roads from point  $v$  to point  $w$ , and although at one moment the return is happening in a node with visited neighbors, not all the nodes visited an unsolved shall belong to the same ring.

From the definition of the bi-connected component of the graph results as, any ring represents a bi-connected component, and any bi-connected component is due to have at least one ring, where through once eliminated one node, exists a chain between any among the remnant nodes.

Therefore, the first step in the determination of the structure of the network is the determination of the bi-connected components of the subgraph defined by the visited nodes. The algorithm for the determination of the bi-connected components of the graph is depicted in the work (Cristea et al., 1993).

The determination of the rings from the bi-connected subgraph can be achieved using the algorithm of minimum distance in graphs (Cristea et al., 1993), modified for the concrete established conditions through the definition of the subgraph, and applied repetitively until the subgraph has no more nodes that are not included in rings.

The results obtained using the algorithms presented therein before can followed using the computer program named "REȚEA" (NETWORK), which has an option that permits in depth visiting of the graph nodes and the identification of ring components of a graph (compressed air network).

## Results obtained, conclusions

The first analysis of the problem revealed, that in case of the compressed air network, the description of the nodes and transoms, the geometrical characteristics of different elements like ducts, elbows, faucets, fixtures etc., a great amount of data is used. A program was developed (Dosa, 1998) for compressed air network calculus, and includes the algorithms presented above, and also has many other features. Once nodes of a network defined properly, transoms will be generated automatically. After that different elements can be added and deleted easily from the transom, names of nodes and transoms can be changed. In fig. 1, on the right side all available elements for building a transom are given. On the left side the composing elements of a transom were listed. Adding an element can be done by selecting the element from the right side, and than pressing the "Add" button on the middle of the screen.

Nodes or transoms can be deleted or added quickly or even exergetic balance of network can be calculated, using the program.

The program has, for all windows of data input, functions that validate the correctness of data entered. Also, for avoiding the start of calculus of a network with wrong or absent data, a menu for data validation was provided.

These functions verify the network structure, initial data of nodes and transoms for consistency. Each validation function creates an error log file that can be consulted for the correction of errors appeared, in the file is stipulated clearly the character of error.

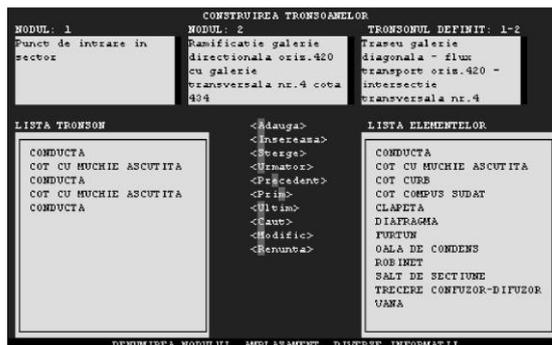


Fig. 1. The window for definition of transom

The parameters of state from nodes and transoms can be visualized, pressure drops, flow and the variation of the temperature for each element of the network, as well as the resistances of every element in network. Also we can have clear situation of the exergetic balance of network, on sorts of loss, and the exergy lost on each type of network element: pipelines, diaphragms, elbows etc.

After the initial data input and validation of these, calculus can start from the menu "Calcul" (Calculus) in which were three options "Parcurgere fără calcul" (Inspecting the network), "Calcul" (Calculus), "Optimizare rețea" (Network optimization).

For verification of algorithms a network with two rings having a common edge fig. 2 was considered.

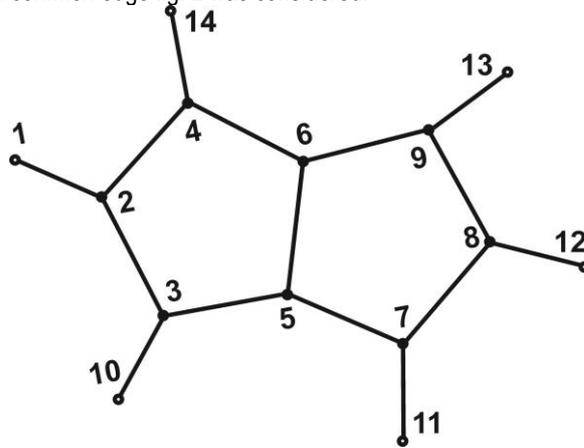


Fig. 2. Ring shaped network with common transoms

After defining these, from menu "Calcul" (Calculus) choose "Parcursere fără calcul" (Visiting nodes). Results obtained were represented in fig. 3.

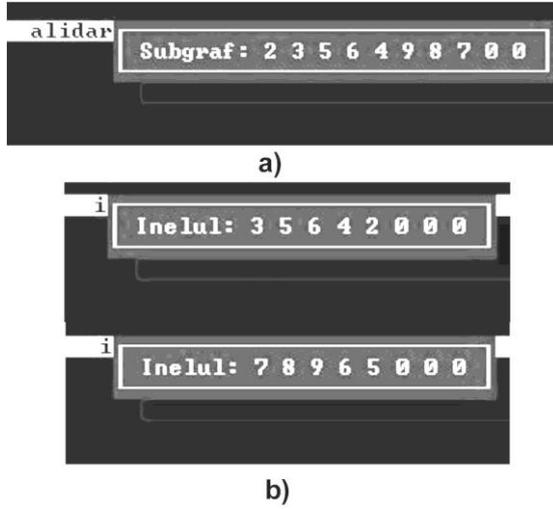


Fig. 3. Results for ring shaped network with a common edge

In fig. 4, a ring shaped network is presented. It can be solved from top to bottom using the method of cycling or iterative method (Sârbu, 1997), that consist in calculating flow rate corrections for transoms until the divergence of the pressure drop for the ring is null.

Equations that can be used for the algorithm of simple iteration method (Sârbu, 1997):

- flow rate conservation in nodes:

$$f_j = \sum_{\substack{i=1 \\ i \neq j}}^N Q_{ij} + q_j = 0 \quad (j=1, \dots, N - N_{RP}) \quad (6)$$

in which  $f_j$  is the residual flow for node  $j$ ,  $Q_{ij}$  the transit flow of transom  $ij$  having sign (+) when enters node  $j$ , and (-) when leaving node  $j$ ;  $q_j$  – concentrated flow rate of the node  $j$  having sign (+) when entering node and (-) when consumed in node;

- the energy conservation on ring:

$$\Delta h_m = \sum_{\substack{ij \in m \\ ij=1}}^T \varepsilon_{ij} \cdot h_{ij} - f_m = 0 \quad (m=1, \dots, M) \quad (7)$$

where  $\Delta h_m$  is the divergence of pressure drop for the ring  $m$ ;  $h_{ij}$  longitudinal pressure drop for transom  $ij$ ;  $\varepsilon_{ij}$  the orientation of transom (+1) when calculating in the same direction with the air flow, (-1) otherwise and (0) for  $ij \notin m$ ;  $f_m$  – the piezometric level induced by the potential elements of ring  $m$ , for simple closed rings  $f_m = 0$ .

$$Q_{ij} = Q_{ij}^{(0)} + \sum_{\substack{m=1 \\ ij \in m}}^M \varepsilon_{ij} \cdot \Delta Q_m, \quad (ij=1, \dots, T) \quad (8)$$

where  $\Delta Q_m$  is the correction flow rate for ring  $m$ ,  $Q_{ij}^{(0)}$  is the initial flow rate of the transoms.

The residual pressure drop on each simple ring for turbulent flow is given by the relation:

$$\Delta h_m = \sum_{(m)} \varepsilon_{ij} \cdot S_{ij} \cdot Q_{ij}^2 \quad (9)$$

in which  $S_{ij}$  is the modulus of hydraulic resistance of transom  $ij$

The correction flow rate is given by:

$$\Delta Q_m = - \frac{\Delta h_m}{\sum_{(m)} S_{ij} \cdot |Q_{ij}|} \quad (10)$$

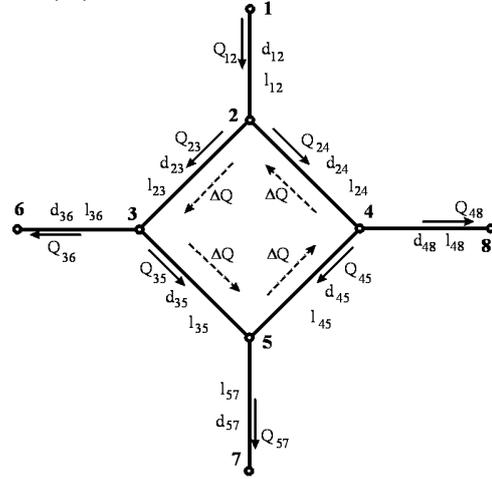


Fig. 4. Ring shaped network, iterative method

Using the algorithm for calculating the network from the consumers to compressor, fig. 5, the flow rate of the transoms can be obtained from Weissbach-Darcy equation. The direction of air flow is from high pressure to low pressure, while in the simple iteration method is given by the sign of the flow rate.

Initial data for calculus: length of the transoms:  $l_{12}=50$  m,  $l_{23}=20$  m,  $l_{24}=50$  m,  $l_{36}=70$  m,  $l_{35}=50$  m,  $l_{57}=40$  m,  $l_{45}=40$  m,  $l_{48}=30$  m; the diameter of transoms:  $d_{12}=0.15$  m,  $d_{24}=0.1$  m,  $d_{35}=0.1$  m,  $d_{45}=0.05$  m,  $d_{23}=0.1$  m,  $d_{36}=0.075$  m,  $d_{48}=0.075$  m. The temperature of compressed air  $T=293$  K and the friction coefficient  $\lambda=0.022$  is assumed constant for the entire network.

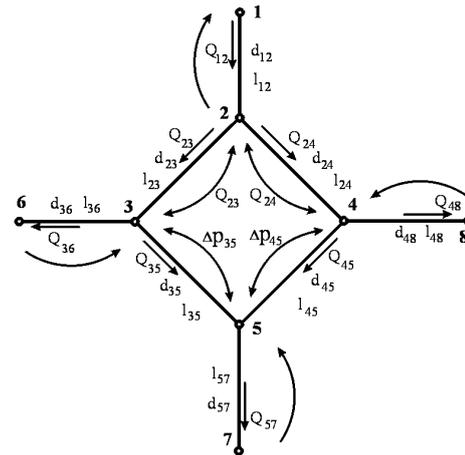


Fig. 5. Ring shaped network, bottom to top

Also the assumption of no flow loss is made, and the density of the compressed air is calculated for the medium pressure of the transom. For top to bottom calculus, the flow rate of the compressor is  $Q_1=1.26$  kg·s<sup>-1</sup>, at the pressure of  $p_1=620,000$  Pa, and the divergence of pressure drop for the ring is calculated with the precision 0.001.

For bottom to top calculus are given: the flow rates of consumers:  $Q_6=0.36 \text{ kg}\cdot\text{s}^{-1}$ ,  $Q_7=0.72 \text{ kg}\cdot\text{s}^{-1}$ ,  $Q_8=0.18 \text{ kg}\cdot\text{s}^{-1}$ , and the pressures  $p_6=604,000 \text{ Pa}$ ,  $p_7= 587,000 \text{ Pa}$ ,  $p_8=615,000 \text{ Pa}$ .

Results given in table 1 show that the results obtained are similar; the relative error is approximately  $10^{-3}$ .

Tabel 1  
*Results of calculus*

Flow rate [ $\text{kg}\cdot\text{s}^{-1}$ ]	Iterative method (after 6 iterations)	Bottom to top
Q <sub>23</sub>	0.9314	0.9305
Q <sub>35</sub>	0.5714	0.5696
Q <sub>54</sub>	0.1486	0.1482
Q <sub>42</sub>	0.3286	0.3295

Conclusively, using the algorithm presented for solving the compressed air network from consumer to compressor can reduce the amount of hardware resources. Building solutions progressively, in the memory of the computer only data needed for performing calculus is found.

Another advantage of going from the consumer to the compressor is that in many cases, iterations are avoided. In the example above the assumptions (no heat exchange, no flow loss and constant friction coefficient for all the ducts) were made for illustrating the problem, and showing the potential of the algorithm. In real life, in compressed air networks heat exchange and flow loss usually occurs, and the friction coefficient can vary for different ducts according to the type of flow.

Even so, using the algorithm presented can speed up the calculus for compressed air networks, which in real life are bigger and more complicated.

## References

Burducea, C., Leca, A. 1974. *Conducte și rețele termice (Pipelines and thermal networks)*. IN: Editura Tehnică, Bucharest, 106-270.

- Cristea, V., Athanasiu, I., Kalisz, E., Iorga, V. 1993: *Tehnici de programare (Techniques of programming)*. IN: Editura Teora, Bucharest, 104-173
- Dodescu, Gh., Toma, E. 1976. *Metode de calcul numeric (Numerical calculation procedures)*. IN: Editura Didactică și Pedagogică, Bucharest.
- Dosa, I. 1998: *Cercetări privind energetica proceselor termofluidodinamice din instalațiile pneumatice miniere (Research concerning thermo-fluid-dynamics processes from the pneumatic mining installations)*. IN: Doctoral dissertation, Petrosani, 134-176 p.
- Idlecik, I.E. 1984: *Îndrumător pentru calculul rezistențelor hidraulice (Guide for the calculus of hydraulic resistances)*. IN: Editura Tehnică, Bucharest.
- Ilicev, A.C. 1951: *Instalații pneumatice miniere (Pneumatic mining installations)*. IN: Editura Tehnică, Bucharest.
- Ionescu Texe, C., Zsako, I. 1990. *Structuri arborescente cu aplicațiile lor (Arborescent structures with their applications)*. IN: Editura Tehnică, Bucharest.
- Irimie, I. I., Matei, I. 1994. *Gazodinamica rețelelor pneumatice (Gas dynamics of pneumatic networks)*. IN: Editura Tehnică, Bucharest, 103-220.
- Kiselev, P.G. 1988: *Îndreptar pentru calcule hidraulice (Guide for hydraulic calculus)*. IN: Editura Tehnică, Bucharest.
- Larionescu, D. 1989: *Metode numerice (Numerical methods)*. IN: Editura Tehnică, Bucharest.
- Leca, A., Prisăcaru, I., Tănase, H.M., Lupescu, L., Raica, C. 1986: *Conducte pentru agenți termici (Pipelines for thermal agents)*. IN: Editura Tehnică, Bucharest.
- Rosculeț, M. 1984: *Analiză matematică (Mathematical analysis)*. IN: Editura Didactică și Pedagogică, Bucharest.
- Salvadori, M.G., Baron, L.M. 1972: *Metode numerice în tehnică (Numerical methods in technics)*. IN: Editura Tehnică, Bucharest.
- Simionescu, I. 1995: *Metode numerice în tehnică – Aplicații în FORTRAN (Numerical methods in technics – Applications in FORTRAN)*. IN: Editura Tehnică, Bucharest.
- Sârbu, I. 1997. *Optimizarea energetică a sistemelor de distribuție a apei (Energetic optimization of water distribution systems)*, IN: Editura Academiei Române, București, 37-78.
- Vrânceanu, Gh., Gh., Mititelu, Șt. 1984: *Probleme de cercetare operațională (Problems of operational research)*. IN: Editura Tehnică, Bucharest, 5-148.

*Recommended for publication by the Editorial staff*