PENTAGON-DODECAHEDRAL AND ICOSAHEDRAL ARTIFACTS IN ANTIQUITY: 3D FIVE-FOLD SYMMETRY APPLIED TO CULTURAL HERITAGE

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ABSTRACT. Regular pentagon-dodecahedral and icosahedral artifacts (two of the five regular Platonic solids) from the Antiquity period, made of mineral substance or metal (bronze, gold), are found in many museums and collections. It is assumed that these objects, along with other similar regular geometric objects copy the crystal habits of minerals in nature. Their function is subject of discussion. An overview is made for the pentagon-dodecahedral artifacts and all the known icosahedral artifacts are listed. The pentagon-dodecahedral artifacts (symbol of the universe) occur primarily as hollow bronze (in Europe) or gold (in Southeast Asia) objects with circular holes on the 12 faces and small spherical knobs on their vertices. The icosahedral artifacts (symbol of water) are most often made of transparent quartz (rock crystal), faience or serpentinite (their faces often inscribed with Greek letters). Report of the five-fold symmetry and the corresponding presence of 12 faces or vertices in the artifacts target their possible interpretation to astronomical (calendar and zodiacal) symbolism and function.

ПЕНТАГОНДОДЕКАЕДРИЧНИ И ИКОСАЕДРИЧНИ АРТЕФАКТИ ОТ АНТИЧНОСТТА: 3D ПЕТОРНА СИМЕТРИЯ ПРИЛОЖЕНА КЪМ КУЛТУРНОТО НАСЛЕДСТВО

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РЕЗЮМЕ. Правилни пентагондодекаедрични и икосаедрични артефакти (две от петте правилни тела на Платон) от античната епоха, направени от минерално вещество или метал (бронз; злато) са установени в редица музеи и колекции. Приема се, че тези артефакти, наред с други подобни правилни геометрични обекти имитират кристалните форми на минералите в природата. Тяхната функция е предмет на дискусия. Направен е обзор на пентагондодекаедричните изделия и е съставен списък на икосаедричните такива. Пентагондодекаедричните артефакти (символ на вселената) се срещат главно като кухи бронзови (в Европа) или златни (в югоизточна Азия) изделия с кръгли отвори по 12-те стени и малки топчета по върховете. Икосаедричните артефакти (символ на вода) се срещат най-често направени от прозрачен кварц (планински кристал), фаянс или серпентинит (стените често надписани с гръцки букви). Петорната симетрия и геометрично съответното наличие на 12-те стени или върха при артефактитите насочват тяхната възможна интерпретация към астрономична (календарна и зодиакална) символика и функция.

Introduction: five-fold symmetry in minerals and the Platonic solids

Crystals with pseudo five-fold symmetry or five-fold cluster composition are reported among different minerals (Kostov, Kostov, 1988; 1999). Quasicrystals with five-fold symmetry were synthesized (Shechtman et al., 1984) and the first mineral species with such symmetry was found in nature – the mineral icosahedrite (Bindi et al., 2011).

Five-fold pattern symmetry can be observed on prehistoric Neolithic pottery from the near East and the Balkans and later on – on Minoan and Mycenean seals. The Greek philosopher Plato (427-347 B.C.) described in his dialogue "Timaeus" (Plato, 53c) regular polyhedra, which are later named "the five Platonic bodies" – five regular polyhedra with equal in form and size faces: tetrahedron (4 faces; it looks like a three-sided pyramid), cube or hexahedron (6 faces), octahedron (8 faces), pentagon-dodecahedron (12 faces) and icosahedron (20 faces). Regular solids are three-dimensional forms all of whose edges have the same length and all of whose faces have the same shape. These geometric bodies are linked in ancient philosophy to the corresponding 5 elements of nature: fire, earth, air, universe (cosmic soul) and water (Fig. 1). In Plato's dialogue "Phaedo" the earth is described as a ball "made of twelve pieces of skin" (Plato, 110b). Plato believed the five solids to be the atomic shape of the corresponding elemental fire, air, earth, water and universe (cosmic spirit). According to the XIII book of Euclid's "Elements", three of the five regular solids must be attributed not to Plato, but to the Pythagoreans, and the rest two of them – to Theaetetus (c. 415-369 B.C.). Both regular pentagon-dodecahedral and icosahedral forms as artifacts in antiquity may copy typical for pyrite crystals forms: irregular pentagon-dodecahedron {210} in the first case, or the

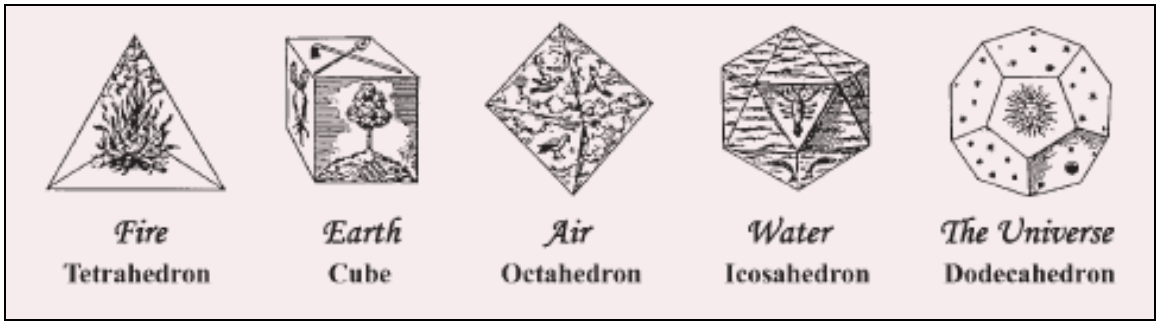

Fig. 1. The five Platonic solids in Johannes Kepler's "Harmonices Mundi", Book II (1619), according to ancient Greek philosophy (used in his geometric cosmology)

combination of octahedron {111} and irregular pentagon-dodecahedron {210} – in the second case (Kostov, 1993). The rest Platonic bodies as artifacts may also have some corresponding to crystal habits of minerals shape, for example: tetrahedrite for the tetrahedron; magnetite, fluorite and diamond – for the octahedron; pyrite, galena, halite and fluorite – for the cube or hexahedron.

In Scotland (mainly in Aberdeenshire) are found stone rounded ornamented (cut with small knobs) balls with geometric patterns, some of them corresponding to the Platonic bodies and also with a 5-fold symmetry (including dodecahedron with 12 faces, a form with two opposite hexagons, each surrounded by six pentagons with 14 faces and icosahedron with 20 faces). For example, the Ashmolean Museum at Oxford University exhibits five different rounded stones with regularly spaced bumps. All these balls are dated from the Late Neolithic to Bronze Age (~II-I mill. B.C.; predating Plato) and their number is estimated between four-five hundred specimens (Mann, 1914; Marshall, 1976/77; 1983). The high points of each bump mark the vertices of each of the regular polyhedra. It has been conjectured that such artifacts were used: (1) in some sort of game or for amusement, (2) for magical or religious ceremonies, (3) mounted or tied to rods and used as weapons (stone mace) or (4) as trade weights.

Pentagon-dodecahedral artifacts

In several mainly west and central Europeans countries (United Kingdom, Belgium, France, Germany, Luxembourg, The Netherlands, Switzerland, Italy, Austria, Hungary and Croatia, with the greatest number discovered in the west of Germany and in France), during archaeological research, are found bronze hollow dodecahedra (from a crystallographic point of view – regular pentagon-dodecahedra) related in age to Roman times, their number estimated above 100 samples (de Saint-Venant, 1907; Saint-Michel, 1951; Déonna, 1954; Duval, 1981; Artmann, 1993; Hill, 1994; Nouwen, 1994; Guggennberger, 2000; 2013; with corresponding therein references; Fig. 2; Table 1A). They have circular opening usually of different diameter on each face of the artifact, as well as small spherical knobs on their vertices. Also, in most cases on the faces are observed small incised concentric circles, arranged around the circular hole. The size of the artifacts varies 4-11 cm. Their purpose and utilization is under discussion: for example – heads of scepters, or sticks of command; heads of maces; linings sprinklers; chandeliers for candles of various sizes; gauge measuring cylinders of various sizes; caliber for coin blanks; instrument for playing, a kind of dice; with an as-part against a species of rod tapered tip graduated on which would been drawn hallmarks; toys, without specifying; masterpiece for control (measure) (de Saint-Venant, 1907; cit. after Déonna, 1954). Additionally some other hypothesis are listed: connection to the zodiac (Artmann, 1993); optical rangefinder for the army or dioptron (Sparavigna, 2012a; 2012b), clinometer (Meréaux-Tanguy, 1975) candle- or flower stands, finger ring-size gauges or even a toy to throw and catch on a stick or some sort of oracle.

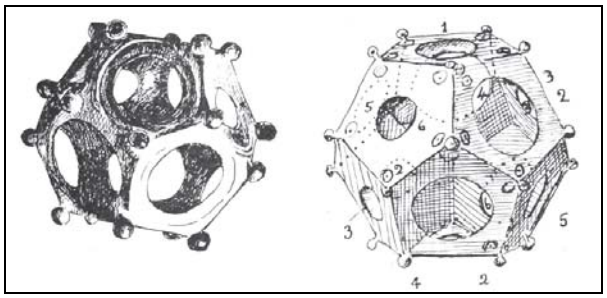

Fig. 2. Examples of "Roman dodecahedron" (de Saint-Venant, 1907, pl. 1: left – from Reims N38; right from Membrey N26)

The earliest dodecahedra, dated about 500 B.C., made from stone (soapstone – steatite), are supposed to be of an Etruscan origin (Lindemann, 1896, cit. after Artmann, 1993). In a single case, in Switzerland (Geneva), a silver pentagon-dodecahedron with lead core has been found and described, the faces inscribed with the 12 zodiacal names (Cervi-Brunier, 1985).

Metallic gold pentagon-dodecahedral artifacts are discovered in south-east Asia. The polyhedral gold bead from Khao Sam Kaeo, in eastern peninsular Thailand, is stylistically identical to those from Oc Eo in the Mekong Delta (Vietnam), and to numerous similar beads from other Pyu sites in Myanmar (Bennett, 2009). Several of the gold ornaments recorded in private collections around Khao Sam Kaeo, including the characteristic pentagonal polyhedral gold beads, have also been found in local private collections and also bear strong similarities to the finds from Oc Eo (because of the 12 faces, they are supposed to be linked to some sort of astronomical or astrological significance; Spagnoli, 2004).

In some cases, the dodecahedra are made of solid rock or mineral, as in the case with the artifact in the Varna Archaeological Museum in Bulgaria (probably a Roman die; together with another cubic and typical to Roman times bone die on display; Haralambieva, 2006).

Table 1. *Regular pentagon-dodecahedral (A) and icosahedral (B) as artifacts in Antiquity (data from references and Internet)*

Country/Region	Museum	Age	Notes	Material/Size
A				
Italy	-	1000-900 B.C. or ~500 B.C.	Etruscan; Monte Loffa, with markings on the faces	soapstone
Italy (2 samples)	Perugia, Museum of Antiquities	~5-4 c. B.C.	Etruscan; mounted on bronze sticks	bronze
Europe	>100 samples	1st - 4th c.	Roman period; hollow;	bronze
Switzerland	Geneva, Cathédrale Saint-Pierre	3rd-4th c.	Roman; Genf, inscribed zodiacal signs	silver; lead core; 297 grammes
USA	Arthur M. Sackler Museum; 1992.256.199	c. 300-500	Roman; inscribed letters	garnet; 5.6 cm
France	Troyes, Musée	Roman times	France; incised figures in Latin count (I to XII)	bone
Bulgaria	Varna, Historical Museum	Roman times	Roman die	stone; ~3 cm
SE Asia	-	5th c. B.C- 5th c.	Mekong River; other places	gold
France	Musée Guimet	2nd c.	Southern China; hollow beads	gold (2 samples)
B				
Germany	Bonn, Rheinisches Landesmuseum	Roman times	Arloff, hollow	bronze; ~8 cm
Sweden	Stockholm, Medelhavsmuseet	Ptolemaic Period	Egypt: Ptolemaic Period - Roman Period; inscribed with Greek letters	faience, brown (?); stone, glazed (?); stone
France	Paris, Louvre; I 1532	323-30 B.C.	Egypt: Ptolemaic Period - Roman Period; inscribed with Greek letters	steatite
USA	New York, The Metropolitan Museum of Art; 10.130.1157	2nd c. B.C.–4th c.	Egypt: Ptolemaic Period - Roman Period; inscribed with Greek letters	serpentine; 2.7 x 3 x 2.8 cm
USA	New York, The Metropolitan Museum of Art; 10.130.1158	2nd c. B.C.–4th c.	Egypt: Ptolemaic Period - Roman Period; inscribed with Greek letters	serpentine; 3.2 x 3.8 x 3.4 cm
USA	New York, The Metropolitan Museum of Art; 10.130.1159	2nd c. B.C.–4th c.	Egypt: Ptolemaic Period - Roman Period; inscribed with Greek letters	faience; 4.5 x 4.9 x 4.7
USA	New York, The Metropolitan Museum of Art; MMA 37.11.3	2-3 c.	Roman; inscribed with Greek letters	faience; green, 6.2 cm
Egypt	Kharga, New Valley Museum; inv. N843	1st c. (?)	Egypt; Dakhleh Oasis; inscribed with Egyptian divine names in Demotic letters	limestone; 5x6 cm
Egypt (2 icosahedra)	Cairo museum	-	Egypt; inscribed with Greek letters	steatite or faience
United Kingdom (4 icosahedra)	London, British Museum; EA 59732; AES 1911.0617.40	Greco-Roman period	Egypt; inscribed with Greek letters (1 pink; 1 greenish, 2 brown)	faience, serpentine, steatite, and/or calcite
[Christie's; 2003]	-	1st/2nd c.	Egypt; inscribed with signs	faience, green; 5.2 cm
United Kingdom	Birmingham, University of Birmingham; ECM552	-	Greek die; inscribed with Greek letters	stone, brown; 5.2x5.7 cm
United Kingdom	London, British Museum	1st c.	inscribed with Greek letters	stone; ~7.62 cm
United Kingdom	London, British Museum; GR 1923.0401.1184	1st/2nd c.	Roman; inscribed on 2 faces with X and I	rock crystal; 1.7x1.5 cm
France	Paris, Louvre	-	Egypt; inscribed with Latin letters and Roman numerals	rock crystal
France (Corsica)	Aléria, Museum	-	Roman	rock crystal
Italy	Arezzo, State Archaeological Museum	1st c.	Roman; feminine grave near Arezzo	rock crystal; ~2.8-3.0 cm
Turkey	Istanbul Art Museum	-	Greek or Roman	rock crystal
Serbia	Belgrade	3rd c.	Roman, Castrum Novae	rock crystal; 5.5x4.7cm
SE Asia	-	5th c. B.C- 5th c.	Mekong River; other places	gold

Rock crystal deformed (irregular) pentagon-dodecagonal spindle whorls (?) of the Merovingian period (6th century) are known from: Rhenen Donderberg grave 413 (Leiden Rh 413), fitted on to a Carolingian bone spindle rod from Dorestad, Leiden; Alfriston grave 65, c. 500-560, Lewes Castle Museum (hexagons and not pentagons on the two opposite sides); and from Jerusalem, "Tomb of the Prophets" (Merovingian?), BM 49/11-27/13. Most of the initial pentagon faces display additional facets forming trigonal and tetragonal faces, probably after a new faceting (see also Babelon, 1902; p. 11, Fig. 8, Mayence museum).

Icosahedral artifacts

Artifacts with five-fold symmetry displaying isosahedral habit (morphology) are known also in Antiquity. They are made mainly from rock crystal and faience (Western and Central Europe; Roman Empire) or serpentinite (Egypt; Ptolemaic Egypt – Roman times) (Table 1B).

A unique icosahedron was found at Qaret el-Muzzawaqa in the 1980s and is now housed in the New Valley Museum at Kharga (probably dated to the 1st century). In contrast to other icosahedra, known from Graeco-Roman Egypt, this one is not inscribed with Greek or Latin letters or numbers, but with 20 Egyptian divine names in Demotic, thus adapting Egyptian concepts to a Greek form. The piece provides striking evidence for the mixing of cultural traditions in Dakhleh Oasis in the Roman Period. The polyhedron was presumably used in an oracular procedure intended to establish which deity would provide help to the petitioner (Minas-Nerpel, 2007).

Of interest is the rock crystal icosahedron, found in SE Europe at the Roman town Castrum Novae (dated from the reign of emperor Septimius Severus to the last third of the 3rd century) at Čezava (Vasić, 1992). The quartz icosahedron is interpreted as a magical object, related to the corresponding philosophical and mystical teachings (including the Neo-Pythagoreans) from the end of the 2nd and beginning of the 3rd century. Similar rock crystal artifacts are known from some other places and museums (Table 1B).

A single case of Roman hollow bronze icosahedron is known from Arloff (Bonn, Rheinisches Landesmuseum), also with small spherical knobs on the vertices, as well as concentric circles on the curvilinear triangular faces. Another similar in shape metallic object, but from Medieval times, is a gold icosahedral artifact from India, with gold letters on the faces (on display by Sotheby).

To the list of artifacts of regular pentagon-dodecahedral and icosahedral shape (Table 1) one can add also the case with an artifact from Ptolemaic Egypt with rhombic dodecahedral form {111}, also inscribed with Greek letters (as 2 letters are on one face, this point to the representation of numbers 1 to 12 by the letters in the Greek numeral system). The {111} form is typical for garnet crystals and in certain cases the shape of such artifacts may also copy minerals in nature. Typical cubes from bone are more common among Roman dice, but there are a few of them made of colourless glass (from Cyprus; Metropolitan Museum of Art; MMA 74.51.332) or quartz (rock crystal; British Museum 1923,0401.1187; 1923,0401.1198). Rock crystal dice as a combination of {100} and {111} forms are also known from the British Museum (1886,0401.1718). Four sided (tetrahedral) black dice with two of four corners marked in white are known from Mesopotamia.

Conclusions

The pentagram as a single side of a pentagonal dodecahedron is closely associated with the so-called golden section, which the ancient Greeks believed to have powerful mystical and aesthetic properties both. The dodecahedron was a form with enormous significance for the Neo-Pythagoreans as representation of the atomic shape of the universe (or cosmic spirit), with the twelve pentagonal faces corresponding to the signs of the zodiac. Most researchers accept the hypothesis that the ancient bronze dodecahedra were ritual or oracle tools, on occasion used as divinatory dice.

The existing and new lists of pentagon-dodecahedral and icosahedral artifacts, as well as new finds, are base for further on study on the utilization of these objects both from a practical and philosophical point of view: comparison of European and Asian cases; five-fold symmetry symbolism in different religious systems; calendar and zodiacal importance.

The main material for the pentagon-dodecahedral hollow artifacts is metal (bronze in the European area in Roman times and gold in South-East Asia). As an analog to the symbolism of the five Platonic bodies, the element "wood" is among the 5 major elements in Chinese philosophy and Feng Shui systems with corresponding green colour.

The main material for the icosahedral artifacts is quartz (transparent rock crystal), faience and probably serpentine. The icosahedral form corresponds to the element "water" in both Greek and Eastern symbolic systems. Water is symbolized by clear quartz (rock crystal).

The close resemblance of irregular pentagon-dodecahedral yellow pyrite crystal with metallic luster to the bronze or gold regular dodecahedral artifacts is evidence that ancient people tried to copy nature. Thus, the clear water is symbolized by rock crystal, and the metallic shining pyrite crystals – by bronze alloys or gold. The utilization of discussed artifact is open for discussion with stress on their probable astronomical (calendar and zodiacal) symbolism and significance.

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