DIRECT PROBLEM AND THE LIE GROUP ANALYSIS IN THE NON-LINEAR STOCHASTIC EARTH'S SURFACE SUBSIDENCE MECHANICS

Mihail Vulkov

University of Mining and Geology "St. Ivan Rilski", 1700 Sofia, e-mail: mvulkov@abv.bgl

ABSTRACT: The problem is in the field of applied geo-mechanics. The investigation is focused on strata and ground movement over mined-out areas. Using the Lie group analysis of the main equation of the nonlinear stochastic geo-mechanics, a solution to determine the earth's surface subsidence is obtained .The partial differential equation of the nonlinear stochastic geo-mechanics is transformed into an ordinary one. The solution is compared with other existing solutions. The obtained relations may be used to plan and manage mining operations.

Keywords: surface subsidence mechanics, nonlinear stochastic model, Lie group analysis, direct problem

ПРИЛОЖЕНИЕ НА ГРУПОВИЯ АНАЛИЗ НА ЛИИ В МЕХАНИКА НА МУЛДАТА - ПРАВА ЗАДАЧА

Михаил Вълков

МГУ „Св. Иван Рилски",1700 София, e-mail mvulkov@abv.bg

РЕЗЮМЕ: Въз основа на груповия анализ на Лии, приложен към основното уравнение на нелинейната стохастична геомеханика, е предложено решение на задачата за предвиждане на последиците от провеждането на подземни минни работи върху земната повърхност (права задача в механика на минната мулда). Анализирани са предимствата и недостатъците на разгледания метод. Направени са изводи.

Ключови думи: механика на мулдата, групов анализ, пряка задача

Introduction

This paper is in the field of mining geo-mechanics. The investigation is focused on the mining subsidence when mining out underground ore bodies. As a basis, the nonlinear stochastic theory is proposed.

The main differential equation is obtained as a nonlinear parabolic one with the assumption that the rock mass is a stochastic medium consisting of elastic parts.

The basic goal of Lie group analysis of equations is to study the results of the application of the allowed from the equation group on the variety of its solutions.

Using the allowed from the equation group, it is possible to arrange an algebraical structure of the multitude of its solutions. This knowledge may be applied to:

- the description of the general structure of the family of all solutions of the equation;
- the determination of the types of solutions easier to be found in difference to the general solution;
- the construction of new solutions on the basis of the existing solutions, etc.

The problem

To introduce the model of the nonlinear stochastic geo-mechanics, it is assumed that the following axioms are satisfied:

- The rock mass displacements are studied from the viewpoint of the stochastic processes;
- The geo-material is stochastic medium built from elastic parts;
- The characteristics of the mining field allow the model of the new stochastic medium, introduced in (Vulkov 2006), to be applied.

By studying the plane model of cages, shown in Fig.1, and assuming that the rock mass is a stochastic medium built from elastic particles, the general equation for calculating the rock mass subsidence in the influence zone is determined in (Vulkov 1989, 2006).

On the basis of this model and according to the probability mechanisms, the following equation is obtained:

$$\left[A_{11}(P)P_x\right]_x+\left[A_{12}(P)P_x\right]_z+\left[A_{22}(P)P_z\right]_z+B_1(P)P_x+B_2(P)P_z=0 \quad (1)$$

where $P=P(x,z)$ is the probability that a particle of the stochastic medium appears in a point with coordinates (x,z), $P_x=\frac{\partial P}{\partial x}$, $P_z=\frac{\partial P}{\partial z}$, $A_{11}(P)$, $A_{12}(P)$, $A_{22}(P)$, $B_1(P)$ and $B_2(P)$ are rock mass characteristics for the subsiding process.

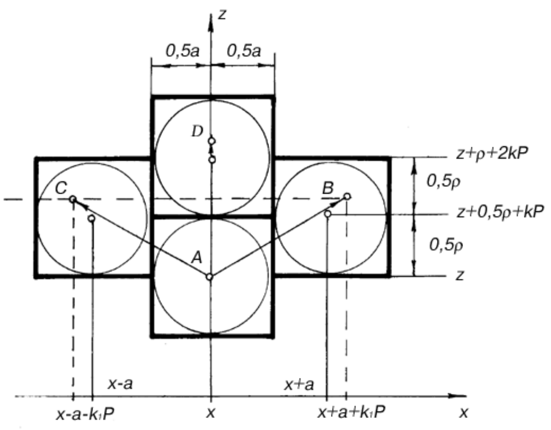

Fig. 1.

By mining out of a horizontal coal seam and by horizontal earth's surface, it is fulfilled:

$$A_{12}(P) = A_{22}(P) = B_1(P) = 0$$
$$B_2(P) = 1 .$$

In this case, equation (1) can be simplified to the form:

$$\left[A_{11} P_x\right]_x + P_z = 0 \qquad (2)$$

The solutions to equation (2) are the objects of this study.

The calculation of the earth's surface displacements, caused by the underground mining of mineral deposits, in given limits, is an important practical problem.

The boundary conditions of the studied problem can be written as follows:

$$P(x,0) = \varphi_o(x) \qquad -\infty \le x \le \infty \qquad (3)$$

$$P(-k,z) = \varphi_1(z) \qquad P(k,z) = \varphi_2(z)$$

$$0 \prec x \le H \qquad (4)$$

The Lie group analysis of the main equation

L.V.Ovsyannicov (1959, 1978) made the Lee group analysis of the equation similar to the main equation of the non-linear stochastic geo-mechanics. The group invariant solutions to that equation may be classified after the type of the non-linear coefficient, which characterizes the heterogeneity of the rock mass:

- when the coefficient $A(w)$ is an arbitrary function, then equation (3) allows three independent operators:

$$\xi_1 = \frac{\partial}{\partial x},\ \xi_2 = \frac{\partial}{\partial z},\ \xi_3 = x\frac{\partial}{\partial x} + 2z\frac{\partial}{\partial z}; \qquad (5)$$

- bigger number of operators may exist only when

$$A(w) = \exp(w) \text{ and } A(w) = w^{2n} . \qquad (6)$$

By constructing the main equation of the non-linear stochastic geo-mechanics, the nonlinear coefficient is obtained in a polynomial form (Vulkov 1989). So, the cases (6) are not interesting for this study.

Generally, the different solutions to equation (2) of ground of one parametric under groups are shown in table 1 (Ovsyannicov 1959).

Table 1

№	Case	$A_{11}(P)$ - arbitrary function
1	I	ξ_1
2	II	ξ_2
3	III	ξ_3
4	IV	$\xi_1 + \xi_2$

The invariant solutions in cases I, II, and IV are not interesting to use in solving geo-mechanical problems connected with the formation of the mining trough.

The attention in this study is concentrated on case III with invariants

$$\eta = \frac{x^2}{z} \text{ and } P(x,z) = V(\eta) \qquad (7)$$

Solving the problem

Let us consider the plane problem for determining the subsidence trough equation in an infinite stripe - Fig. 1.

Using the assumption of J. Litwiniszyn (1974) for proportionality of the probability $P = P(x,z)$ and the vertical displacement of the rock mass $w = w(x,z)$ in the influence zone, the problem (2)- (4) can be written in the following form:

$$\left[A(w) w_x\right]_x + w_z = 0 \quad -\infty \le x \le \infty;\ 0 \prec x \le H; \qquad (8)$$

$$w(x,0) = \varphi_o(x) \qquad -\infty \le x \le \infty \quad z = 0 \qquad (9)$$

$$w(0,H) = 0{,}5 w_o,\ x = 0,\ z = H \qquad (10)$$

where $w = w(x,z)$ is the vertical displacement of a point P of the influence zone with coordinates (x,z); $A(w)$ is the coefficient which characterizes the heterogeneity of the rock mass; $\varphi = \varphi(x)$ is a function describing the subsidence of the immediate seam top; *H* is the depth of the coal seam; w_o is the maximal vertical displacement of the immediate seam arising by the existing conditions.

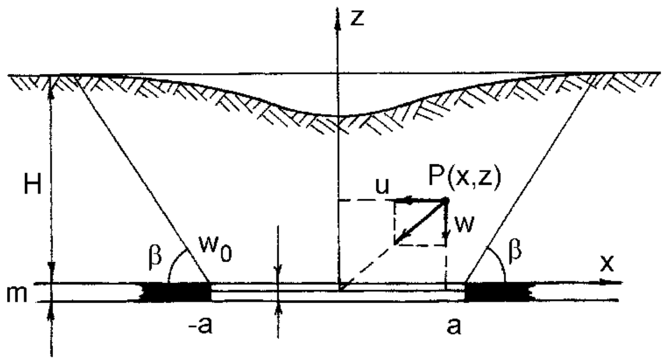

Fig. 2.

The problem for (8) – (10) can be simplified by applying the transformations (6). They transform the quasilinear parabolic partial equation into an ordinary one.

After introducing the following dimensionless variables in equation (8):

$$V = \frac{w}{w_o}; \qquad A(V) = \frac{A(w)}{A(w_o)} \tag{11}$$

and substituting (6) and (11) in problems (1) – (3), it is obtained:

$$-V_\eta = 4\left[A(V)V_\eta\right]_\eta \tag{12}$$

$$V(0) = 0,5; \tag{13}$$

$$V(\infty) = 0, \tag{14}$$

where $V_\eta = \frac{dV}{d\eta}$.

To solve the studied problem, the ordinary equation (12) is integrated once:

$$V - C = -4A(V)V_\eta. \tag{15}$$

After separating the variables, it is obtained:

$$d\eta = -\frac{4A(V)dV}{V-C}. \tag{16}$$

Equation (16) is integrated once and the result is the function $\eta = \eta(V)$:

$$\eta = -4\int \frac{A(V)}{V-C} dV + C_1 \tag{17}$$

where C_1 is an integration constant determined by the boundary conditions of the specific problem.

After transforming (17) in the form $V = V(\eta)$,the equation of the mining subsidence trough formed on the earth's surface is obtained.

Conclusion

In the paper, the direct problem connected with predicting the effects on the earth's surface resulting from underground mining activities is solved. This problem is reduced to Cauchy's problem for the nonlinear Fourier equation.

The partial differential equation of the nonlinear stochastic geo-mechanics is reduced into an ordinary one on the basis of the Lie group analysis of main differential equations of the non-linear stochastic mining subsidence theory.

References

Вълков М., Нови стохастични линейни и нелинейни модели в теорията на слягането на земната повърхност под влияние на подземни минни работи. Автореферат на дисертацията, София, 1989 г.(Vulkov, M., Novi stohastichni lineiyni i nelineyni modeli v teoriata na slyaganeto na zemnata povarhnost pod vliyanie na podzemni minni raboti, Avtoreferat na dicertatsiata, Sofia,1989.)

Вълков М., Стохастични модели в кинематиката на минната мулда, С., МГУ, 2006г. (Vulkov, M., Stohastichni modeli v kinematikata na minnata mulda, Sofia, MGU, 2006.)

Овсянников, Л. В., Групповые свойства уравнения нелинейной теплопроводности, ДАН, т.125, №3, 1959. (Ovsyannikov, L. V., Gruppovie svoistva uravneniya nelineinoi teploprovodnosti, DAN, 125, 3, 1959.)

Овсянников, Л. В., Групповой анализ дифференциальных уравнении, М., Наука, 1978 г. (Ovsyannikov, L. V., Gruppovoy analiz diferentsialynih uravneniy, Moskva, Nauka , 1978.)

Litwiniszyn J., 1974., Stochastic Methods in Mechanics of granular bodies. Wien, Heidelberg, New York, Springer Verlag.

Kratzsch H., Mining Subsidence Engineering, Berlin, Heidelberg, New York, Springer Verlag, 1983.

This article was reviewed by Assoc. Prof. Dr. Stanislav Topalov and Assoc. Prof. Dr. Violeta Trifonova-Genova.