# ANALYTICAL EXPRESSIONS FOR STRESSES IN STEEPLY STRATIFIED ROCK MASS AROUND AN OPENING IN THE SHAPE OF AN ELLIPSE

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**ABSTRACT.** The article discusses the question of analytically determining stresses around an opening. Its cross-section is an ellipse. The rock mass consists of homogenous, parallel and steep layers. They are isotropic or have the plane of isotropy that is parallel to the stratification. The thickness of layers is greater than the dimensions of the opening. The influence of stresses which is due to the passage of the work extends into the rectangular area around the opening. The specified class of tasks is solved by the methods of two theories: the theory of elasticity and the mechanics of the stratified random.
The approach applied here is to determine the analytical expressions of stresses at random points of two layers. In these expressions, the stresses in a generalised field in polar coordinates are involved.
The obtained analytical results are applied to a real rock mass with layers considered as homogeneous and isotropic environments. A diagram of the normal tangential stresses along the contour of the opening is given.

**Keywords:** stratified random, theory of elasticity, analytical solution, stress state, opening in the shape of an ellipse.

**АНАЛИТИЧНИ ИЗРАЗИ ЗА НАПРЕЖЕНИЯТА В СТРЪМНОНАПЛАСТЕН МАСИВ ОКОЛО ИЗРАБОТКА С ФОРМА НА ЕЛИПСА**

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**РЕЗЮМЕ.** В статията се разглежда въпросът за аналитично определяне на напреженията около изработка. Нейното напречно сечение е елипса. Масивът се състои от хомогенни, успоредни и стръмни пластове. Те са изотропни или притежават равнина на изотропия, която е успоредна на напластяването. Дебелини на пластовете са по-големи от размерите на изработката. Влиянието на напреженията, дължащо се на прокарване на изработката, се простират в правоъгълна област около отвора. Този вид задачи се решават с методите на теорията на еластичността и механика на напластените среди.
Тук е приложен е подход за определяне на аналитичните изрази за напреженията в произволни точки от два пласта. В тези изрази участват напреженията в обобщена среда в полярни координати. Физико-механичните характеристики на тази среда са функции на същите на всеки пласт.
Получените аналитични резултати са приложени за реален масив с пластове, разгледани като еднородни и изотропни среди. Дадена е диаграмата на нормалните тангенциални напрежения по контура на изработката.

**Ключови думи:** напластените среди, теория на еластичността, аналитично решение, напрегнато състояние, изработка с форма на елипса.

## Introduction

The analytical and numerical methods are applied in calculating the stresses around a circular opening drawn in a stratified rock mass (Trifonova-Genova, 2012; Trifonova-Genova, June 2012; Minchev, 1960). The rock mass consists by homogeneous layers which are parallel to one another and to the axis of the opening. The layer thickness is commensurate with the dimensions of the opening. These thicknesses participate in the analytical expressions for stresses. These expressions are suitable for low sloping layers.

In the case when the layers are steep and their thickness is greater than the dimension of the circular opening, other expressions are offered (Trifonova-Genova, 2017; Trifonova-Genova, 2018).

The present work focuses on obtaining the analytical expressions for stresses around an opening in the shape of an ellipse.

## Methods

### Formulation of the problem

A horizontal opening with elliptic cross section is drawn at a sufficiently large depth $H$. The influence on the distribution of stresses does not reach the boundary of the half-plane. The latter is accepted as an infinite linear deformable environment. The width of the opening is $2b$ and the height is $2a$. The influence of the opening on the distribution of stresses encompasses the rectangular area with a width of $12b$ and a height of $12a$. The beginning of the $Oxz$ coordinate system is chosen in the center of the opening. The layers are isotropic or transversally isotropic with the plane of isotropy parallel to the stratification. The plane forms an angle $\alpha$ with the horizontal axis.

### Stresses in a generalized field

By the theory described in (Minchev, 1968), the stratified field is replaced by a generalized field with a volume of weight designed as $\gamma^{(o)}$. When the layers are isotropic, $E^{(o)}$ is Young's modulus and $\mu^{(o)}$ is Poisson's ration (Trifonova-Genova, 2017). If the layers are transversally isotropic, the field is transversally isotropic with two types of physical and mechanical characteristics. The former are in the direction parallel to the plane of isotropy ( $E_1^{(o)}$, $\mu_1^{(o)}$ ), but the latter are in the direction perpendicular to the plane of isotropy ( $E_2^{(o)}$, $\mu_2^{(o)}$ ). These characteristics are expressed for each layer respectively (Trifonova-Genova, 2012; Minchev, 1968).

The task assigned is reduced to the task of the plane theory of elasticity with an opening that is loaded to infinity with the stresses of the undisturbed rock. The complex variable theory is used to obtain the stresses in the generalised field in a polar coordinate system. This theory describes the stresses and the displacements by the complex potentials. The view of them for an isotropic rock mass can be found in (Minchev, 1960; Bulychev, 1982).

The stresses on the contour of the opening are:

$$\sigma_r^{(o)} = 0;$$
$$\tau_{r\theta}^{(o)} = 0; \qquad (1)$$
$$\sigma_\theta^{(o)} = -\gamma^{(o)} H \left[ \lambda_1 + \frac{\lambda m + (m_1 + \lambda m_2)\cos 2\theta_*}{1 + m^2 + 2m\cos 2\theta_*} \right],$$

where

$$m_1 = \frac{1+m^2}{2}; \quad m_2 = \frac{1-m^2}{2}; \quad \lambda = \frac{\mu^{(o)}}{1-\mu^{(o)}};$$
$$\lambda_1 = 1 + \lambda; \quad m = \frac{a-b}{a+b}; \quad \theta_* = \theta - 90^\circ.$$

Here, $r_o$ and $\theta$ are polar coordinates of points on the plane.

**Stresses in layers**

The layers around the opening in the shape of an ellipse are two (Fig.1). Their thickness is greater than the dimensions of the opening. The rectangular area of influence is loaded by vertical and horizontal stresses (Trifonova-Genova, 2018). There are stresses in the corresponding layers in undamaged rock.

The transformation formulae from an article by Trifonova-Genova (2018) and the opposite of them are used there. The stresses in each layer are expressed by adduced stresses of a generalised field. In the polar coordinate system, these expressions are:

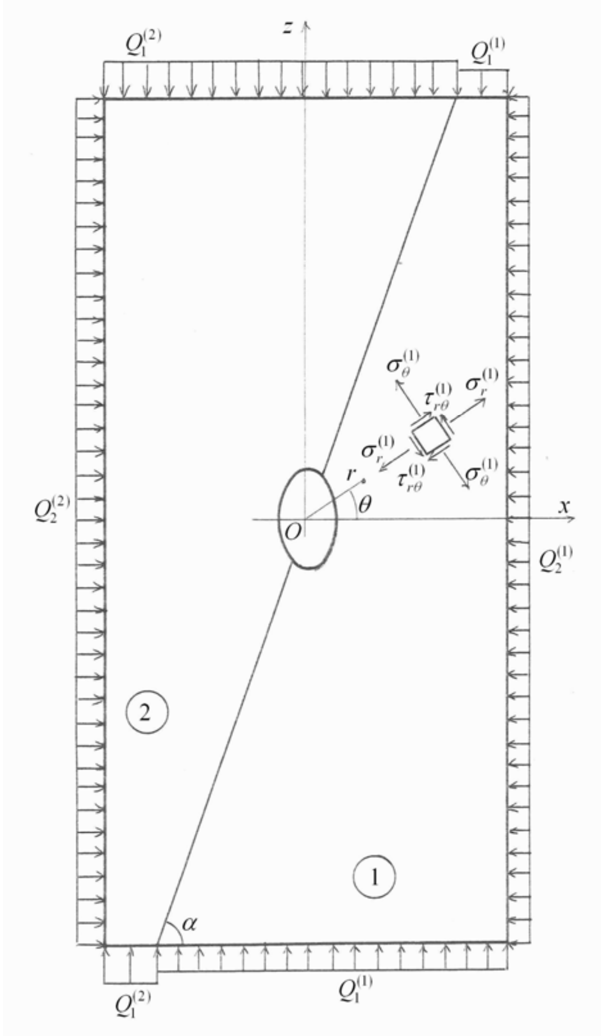


**Fig.1. Calculation scheme for determining stresses**

$$\sigma_r^{(i)} = T_{11}^{(i)}\sigma_r^{(o)} + T_{12}^{(i)}\sigma_\theta^{(o)} + T_{13}^{(i)}\tau_{r\theta}^{(o)};$$
$$\sigma_\theta^{(i)} = T_{21}^{(i)}\sigma_r^{(o)} + T_{22}^{(i)}\sigma_\theta^{(o)} + T_{23}^{(i)}\tau_{r\theta}^{(o)}; \qquad (2)$$
$$\tau_{r\theta}^{(i)} = T_{31}^{(i)}\sigma_r^{(o)} + T_{32}^{(i)}\sigma_\theta^{(o)} + T_{33}^{(i)}\tau_{r\theta}^{(o)}; \quad i = 1.2,$$

where

$$T_{11}^{(i)} = 0.25 s_2^2 B_1^{(i)} + c^4 B_2^{(i)} + s^4 + 0.5 s_2^2;$$
$$T_{12}^{(i)} = c^4 B_1^{(i)} + 0.25 s_2^2 B_2^{(i)} - 0.25 s_2^2;$$
$$T_{13}^{(i)} = s_2 c^2 \left[ B_1^{(i)} - B_2^{(i)} + 1 \right];$$
$$T_{21}^{(i)} = 0.25 s_2^2 B_{21}^{(i)} + s^4 B_1^{(i)} - 0.25 s_2^2;$$
$$T_{22}^{(i)} = 0.25 s_2^2 B_1^{(i)} + s^4 B_2^{(i)} + c^4 + 0.5 s_2^2;$$
$$T_{23}^{(1)} = s_2 s^2 \left[ B_1^{(i)} - B_2^{(i)} + 1 \right];$$
$$T_{31}^{(i)} = -s_2 c^2 B_1^{(i)} + c^4 B_2^{(i)} + s^4 + 0.5 s_2 (c_2 - s_2);$$
$$T_{32}^{(i)} = c^4 B_1^{(1)} + 0.25 s_2^2 B_2^{(i)} + 0.5 s_2 s^2;$$
$$T_{33}^{(i)} = s_2 c^2 \left[ B_1^{(i)} - B_2^{(i)} + 0.5 (1 + c_2^2) \right].$$

In these expressions, the following trigonometric functions are designed with a lower case letter:

$$c^2 = \cos^2 \beta; \quad s^2 = \sin^2 \beta;$$
$$s_2 = \sin(2\beta); \; c_2 = \cos(2\beta); \quad (3)$$
$$c^4 = \cos^4 \beta; \quad s_2^2 = \sin^2(2\beta); \quad \beta = \theta - \alpha.$$

The coefficients $B_1^{(i)}$ and $B_2^{(i)}$ in expressions (2) are constants, linked to Young's modulus and Poisson's rations. For the isotropic layers, these coefficients are:

$$B_1^{(1)} = A_2.A_{**}.A_*; \quad B_1^{(2)} = -A_1.A_{**}.A_*;$$
$$B_2^{(i)} = A.A_*.E^{(i)}; \quad A_* = \left[A_1 E^{(1)} + A_2 E^{(2)}\right]^{-1}; \quad (4)$$
$$A_{**} = \mu^{(1)} E^{(2)} - \mu^{(2)} E^{(1)},$$

where $E^{(i)}$ is Young's modulus and $\mu^{(i)}$ is Poisson's ration.

When the layers are transversal and isotropic, these coefficients are determined by:

$$B_1^{(1)} = \frac{A_2 A_{**}^t E_1^{(1)} E_1^{(2)}}{E_2^{(1)} E_2^{(2)} A_*^t}; \; B_1^{(2)} = \frac{-A_1 A_{**}^t E_1^{(1)} E_1^{(2)}}{E_2^{(1)} E_2^{(2)} A_*^t};$$
$$B_2^{(1)} = \frac{A E_1^{(1)}}{A_*^t}; \quad B_2^{(2)} = \frac{A E_1^{(2)}}{A_*^t}; \quad (5)$$
$$A_{**}^t = -\mu_2^{(2)} E_2^{(1)} + \mu_2^{(1)} E_2^{(2)}; A_*^t = A_2 E_1^{(2)} + A_1 E_1^{(1)}.$$

The physical and mechanical characteristics of the layers in expressions (5) are two types. There are the ones that are in the direction parallel to the plane of isotropy and are denoted by an index lower than 1 ($E_1^{(i)}$, $\mu_1^{(i)}$). The others are in the direction perpendicular to the plane of isotropy and are denoted by an index lower than 2 ($E_2^{(i)}$, $\mu_2^{(i)}$).

The total area under consideration around the opening in expressions (4) and (5) is designated as $A$. The areas of the layers are designated as $A_1$ and $A_2$ (Fig.1).

The stresses in polar coordinates to the contour of the opening are obtained by the stresses in a generalised field:

$$\sigma_r^{(i)} = 0;$$
$$\tau_{r\theta}^{(i)} = 0; \quad (6)$$
$$\sigma_\theta^{(i)} = \left\{\left[B_1^{(i)} c^2 + B_2^{(i)} s^2\right].s^2 + c^4 + 0.5 s_2^2\right\} \sigma_\theta^{(o)}.$$

**Numerical example**

An opening in the shape of an ellipse has a width of $2b = 3m$ and a height of $2a = 4.8m$. It draws at a depth of $H = 320m$. The inclination of the plane between two steep layers is $\alpha = 70°$. The equation of the straight line corresponding to the plane is $z = 2.747x - 3$. The total rectangular area of influence is $A = 518.4\, m^2$ (Fig.1). The areas of the first and the second layers correspond to $A_1 = 217.3\, m^2$ and $A_2 = 301.1\, m^2$.

The values of Young's modulus ($E^{(1)}, E^{(2)}$), Poisson's rations ($\mu^{(1)}$, $\mu_2^{(2)}$), and the volume of weight ($\gamma^{(1)}$, $\gamma^{(2)}$) for two isotropic layers are determined and can be viewed in Table 1, rows four and five (Trifonova-Genova, 2012). The parameters of the generalised field are given in row six in the same table.

Table 1
*Physical and mechanical characteristics of layers*

| $i$ | $E^{(i)}$ | $\mu^{(i)}$ | $\gamma^{(i)}$ |
|---|---|---|---|
| dimension | $MPa$ | | $MN\,/\,m^3$ |
| multiplier | $10^3$ | | $10^{-2}$ |
| 1 | 0.148 | 0.154 | 0.28 |
| 2 | 0.595 | 0.237 | 0.25 |
| $o$ | 0.4075 | 0.2 | 0.263 |

The intersection points of the straight line between the two layers and the ellipse are added to the fixed numbers of points on the contour. For them, the polar angular coordinates and the trigonometric functions take part in the expressions of normal tangential stresses in the first and the second layers and the generalised field. The coefficient of lateral pressure $\lambda^{(o)}$ of the generalised field is equal to 0.25. The vertical stress in the same field is $Q_1^{(o)} = \gamma^{(o)} H = 8.32 MPa$.

The results of the calculations for the stresses without dimension are given in Table 2. It must be noted that the values of stresses in each layer depend considerably on the values of Young's modulus. Then, in layer 2, the stresses are greater than the same in the generalised field. But in layer 1, these values are smaller than the same in the generalised field. Comparing the values of these stresses, we have established a peak in the diagrams of the boundary between the two layers.

The diagrams of the normal tangential stresses are shown in Figure 2. Here, the stresses in the generalised field are marked with a dotted line and the stresses in the layers with a thick line. The peak in the diagrams of the boundary between the two layers can be seen in points two and eleven.

**Key findings**

Given analytical expressions of stresses in each layer are applied in a rock mass in which the layers are steep and isotropic or transversally isotropic.

The diagram of normal tangential stresses in a rock consisting of two layers is obtained for an opening in the shape of an ellipse.

Table 2.
*Normal tangential stresses in a generalized field in two layers*

| № | $\theta\,[^\circ]$ | $\dfrac{\sigma_\theta^{(o)}}{Q_1^{(o)}}$ | $\dfrac{\sigma_\theta^{(1)}}{Q_1^{(o)}}$ | $\dfrac{\sigma_\theta^{(2)}}{Q_1^{(o)}}$ |
|---|---|---|---|---|
| 1 | 0 | 1.713 | 0.876 | |
| 2 | 31.148˚ | 1.532 | 1.409 | -1.621 |
| 3 | 60 | 0.928 | | -0.927 |
| 4 | 90 | 0.258 | | -0.258 |
| 5 | 120 | 0.928 | | -1.062 |
| 6 | 150 | 1.546 | | -2.212 |
| 7 | 180 | 1.714 | | 2.319 |
| 8 | 210 | 1.546 | | -1.646 |
| 9 | 240 | 0.928 | | -0.927 |
| 10 | 270 | 0.258 | | -0.258 |
| 11 | 275.538˚ | 0.292 | -0.374 | -0.294 |
| 12 | 300 | 0.928 | -0.742 | |
| 13 | 330 | 1.546 | 0.623 | |
| 14 | 360 | 1.714 | 0.876 | |

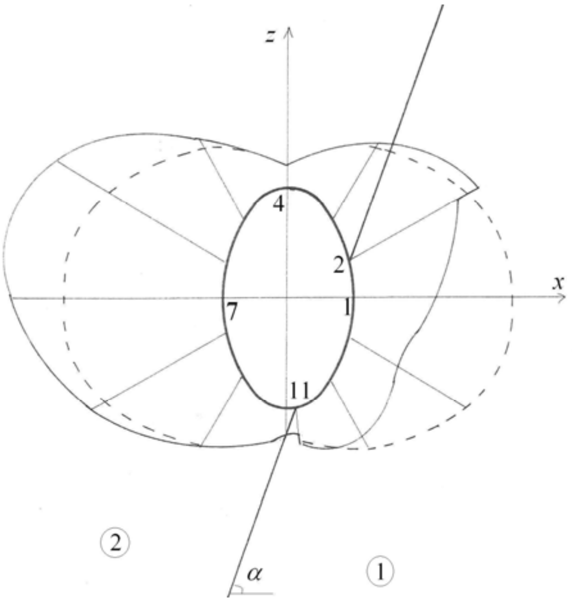


**Fig.2. Diagram of normal tangential stresses along the contour of the opening**

## Conclusion

The method described in this article has the following advantages:

-It is very easy to apply;

-It uses affordable means of calculating (a computer, a calculator, the Exell software product, and others);

-It can be adapted to openings drawn in the rock with other shapes of the cross-section.

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