# THE MOVEMENT OF A POINT SET IN CARTESIAN COORDINATES AND STUDIED IN THE MATHCAD ENVIRONMENT

***Asen Stoyanov***

*University of Mining and Geology "St. Ivan Rilski", 1700 Sofia*

**ABSTRACT.** The movement of a point M within the time interval $t_0 \div t_1\,(s)$ is specified by the law $x = x(t)$ $y = y(t)$. The trajectory and the position of point $M$ on it are determined for the moment of time $t = t_1(s)$. The radial and transverse components of the velocity and acceleration of the point are represented in a matrix form and all the kinematics characteristics of the movement are graphically depicted.

**Keywords:** transverse velocity, radial velocity, transverse acceleration, radial acceleration, MathCAD

**ДВИЖЕНИЕ НА ТОЧКА ЗАДАДЕНО В ДЕКАРТОВИ КООРДИНАТИ И ИЗСЛЕДВАНО В СРЕДА НА MATHCAD**

***Асен Стоянов***

*Минно-геоложки университет "Св. Иван Рилски", 1700 София*

**РЕЗЮМЕ.** Движението на точка М е зададено в интервала от време $t_0 \div t_1\,(s)$ със закона $x = x(t)$ и $y = y(t)$. Траекторията и положението върху нея на т. $M$ са определени за момента от време $t = t_1(s)$. Радиалните и трансверзалните компоненти на скоростта и ускорението на точката са представени в матрична форма, като всички кинематични характеристики на движението са графично изобразени.

**Ключови думи:** трансверзална скорост, радиална скорост, трансверзално ускорение, радиално ускорение, MathCAD

## Introduction

The article examines a classical problem related to the kinematics of a point. The movement of the point is set in a coordinate way. The trajectory formed by its movement within the examined interval of time is part of a hyperbola.

When the motion of the point is set at polar coordinates, its velocity and acceleration are defined as the geometric sum of the radial and transversal components – see Fig.2. B) and C) The article shows how these operations are performed using the MathCAD math package.

## Solution to a problem with the MathCAD package

The material point $M$ moves within the time interval $t_0 \div t_2$ according to the law $x = \dfrac{-3}{t+2};\ y = 3.t + 6$ (Yablonsky, 1978) – see Figure 1.

For the moment $t = t_1$, it is necessary to determine:

- The position of point $M$ on the trajectory – see Fig.1.A);
- The velocity, the full, tangential, and normal acceleration of point $M$ – $\vec{v}, \vec{a}, \vec{a}_\tau$ $u$ $\vec{a}_n$ – see Fig.2.A);
- The transverse $\vec{v}_{tr}$ and the radial $\vec{v}_{rad}$ components of the velocity $\vec{v}_M$ – see Fig.2.B);
- The transverse $\vec{a}_{trx}$ and the radial $\vec{a}_{rady}$ components of the acceleration $\vec{a}_M$ – see Fig.2.C);
- The radius of curvature $\rho(t)$ of the trajectory at moment $t_1$ – see Fig.2.D).

The calculations and graphical visualisation have to be performed using the MathCAD package.

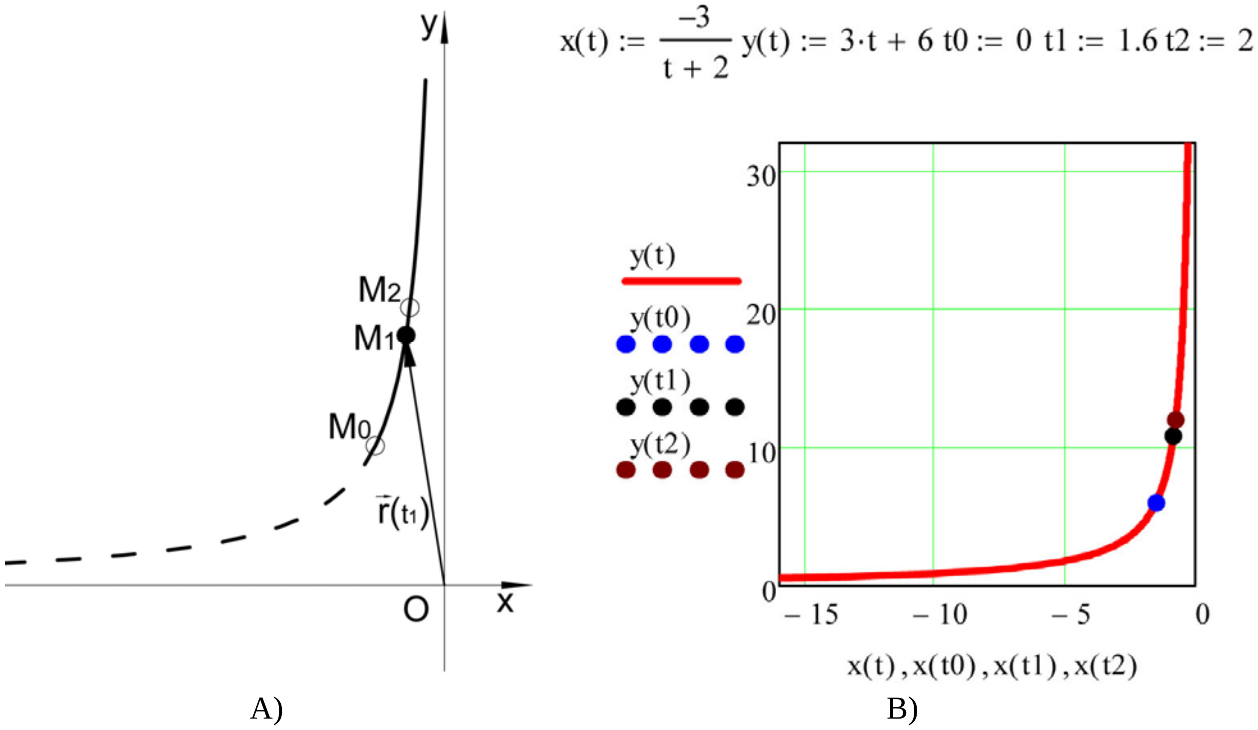


**Fig.1. The trajectory of point $M$ and its positions for moments $t_0, t_1$, and $t_2$, interpreted with: A) Excel и AutoCAD; B) MathCAD;**

The problem is solved within MathCAD environments using the following algorithm:

- The output data is entered – see Fig.1.B);
- In the studied section, the trajectory of point M is visualised graphically for the time interval $t_0 \div t_2$ – see Fig.1.B);
- The law of the change of the velocity of point $M$ is determined analytically and graphically;
- The law of the change of the acceleration of point $M$ is determined analytically and graphically;
- The laws of the change of the tangential and normal acceleration of point $M$ are determined analytically and graphically;
- The law of the change of the radius-vector $\vec{r}(t)$ of point $M$ is determined analytically;
- The law of the change of the transverse and radial component of the velocity of point $M$ is determined analytically and graphically;
- The law of the change of the transversal and radial component of the acceleration of point $M$ is determined analytically and graphically;
- The law of the change of the radius of curvature of the trajectory $\rho(t)$ is determined.

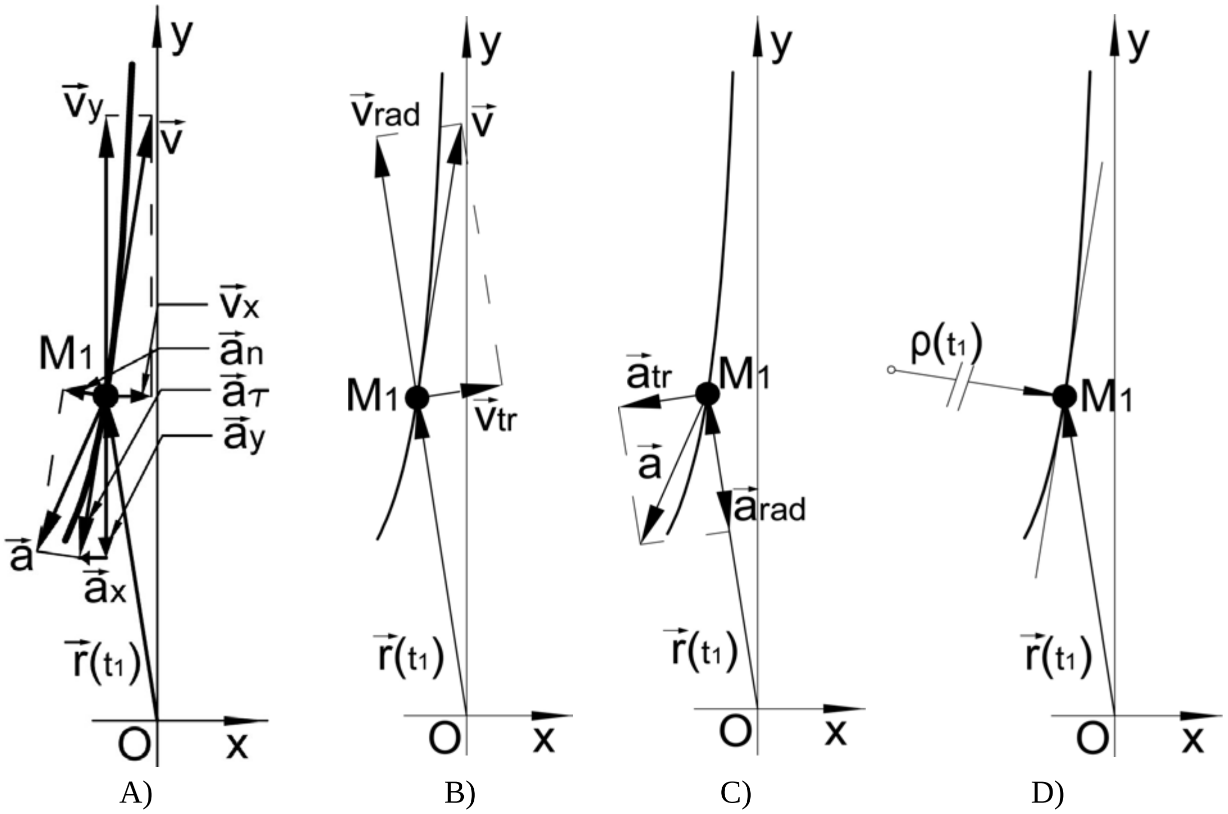


**Fig. 2. Kinematics characteristics of point $M$ for the moment $t_1$**

$$vx(t) := \frac{d}{dt}x(t) \quad vy(t) := \frac{d}{dt}y(t) \quad v(t) := \sqrt{vx(t)^2 + vy(t)^2} \quad ax(t) := \frac{d}{dt}vx(t) \quad ay(t) := \frac{d}{dt}vy(t)$$

$$a(t) := \sqrt{ax(t)^2 + ay(t)^2} \quad a\tau(t) := \frac{vx(t)\cdot ax(t) + vy(t)\cdot ay(t)}{v(t)} \quad an(t) := \sqrt{a(t)^2 - a\tau(t)^2} \quad \rho(t) := \frac{v(t)^2}{an(t)}$$

$$r(t) := \sqrt{x(t)^2 + y(t)^2} \quad vradx(t) := x(t)\cdot\frac{vx(t)\cdot x(t) + vy(t)\cdot y(t)}{r(t)^2} \quad vrady(t) := y(t)\cdot\frac{vx(t)\cdot x(t) + vy(t)\cdot y(t)}{r(t)^2}$$

$$vrad(t) := \sqrt{vradx(t)^2 + vrady(t)^2} \quad vtrx(t) := \frac{-x(t)\cdot y(t)\cdot vy(t) + y(t)^2\cdot vx(t)}{r(t)^2}$$

$$vtry(t) := \frac{x(t)^2\cdot vy(t) - x(t)\cdot y(t)\, vx(t)}{r(t)^2} \quad vtr(t) := \sqrt{vtrx(t)^2 + vtry(t)^2} \quad v(t) := \sqrt{(|vtr(t)|)^2 + vrad(t)^2}$$

$$vx(t1) = 0.231 \quad vy(t1) = 3 \quad v(t1) = 3.009 \quad \rho(t1) = 70.61 \quad an(t1) = 0.128 \quad a\tau(t1) = -9.893\times 10^{-3}$$

$$a(t1) = 0.129 \quad vtrx(t1) = 0.46 \quad vtry(t1) = 0.036 \quad vradx(t1) = -0.229 \quad vrady(t1) = 2.964$$

$$vrad(t1) = 2.973 \quad vtr(t1) = 0.462 \quad aradx(t) := x(t)\cdot\frac{ax(t)\cdot x(t) + ay(t)\cdot y(t)}{r(t)^2}$$

**Fig.3. Analytical expressions for determining the kinematics characteristics of point $M$ and their values at moment $t = t_1(s)$ (Бертяев, 2005; Brent Maxfield, 2009; Доев и др., 2016).**

$$\mathrm{arady}(t) := y(t)\cdot\frac{ax(t)\cdot x(t) + ay(t)\cdot y(t)}{r(t)^2} \qquad \mathrm{arad}(t) := \sqrt{\mathrm{aradx}(t)^2 + \mathrm{arady}(t)^2}$$

$$\mathrm{atrx}(t) := \frac{-x(t)\cdot y(t)\cdot ay(t) + y(t)^2\cdot ax(t)}{r(t)^2} \qquad \mathrm{atry}(t) := \frac{x(t)^2\cdot ay(t) - x(t)\cdot y(t)\, ax(t)}{r(t)^2}$$

$$\mathrm{atr}(t) := \sqrt{\mathrm{atrx}(t)^2 + \mathrm{atry}(t)^2} \qquad a(t) := \sqrt{(|\mathrm{atr}(t)|)^2 + \mathrm{arad}(t)^2} \qquad a(t1) = 0.129$$

**Fig.3. Analytical expressions for determining the kinematics characteristics of point $M$ and their values at moment $t = t_1(s)$ (Бертяев, 2005; Brent Maxfield, 2009; Доев и др., 2016).**

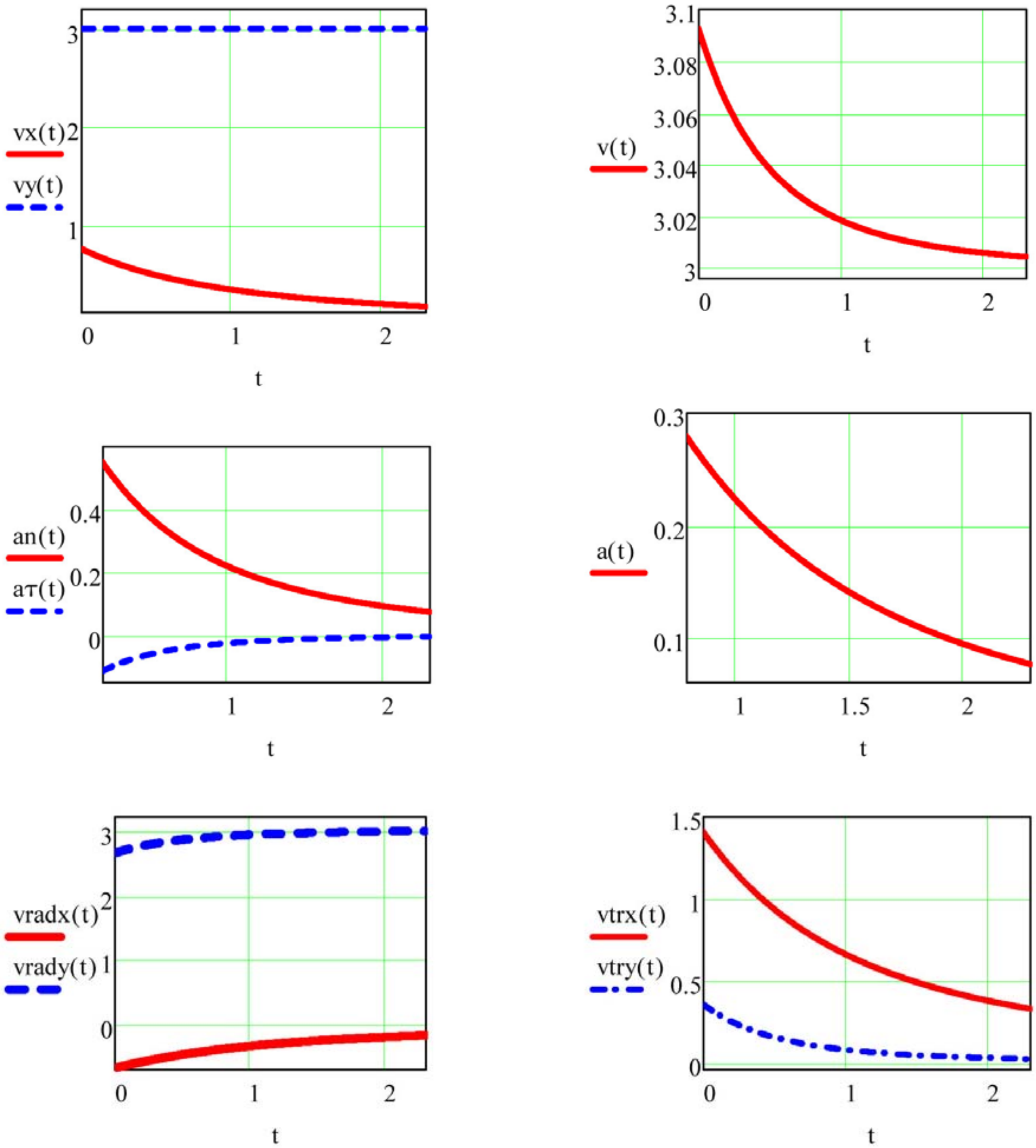


**Fig. 4. Graphs expressing the dependence of the kinematic characteristics of point $M$ on time $t\,(s)$ (Brent Maxfield, 2009; Доев и др., 2016)**

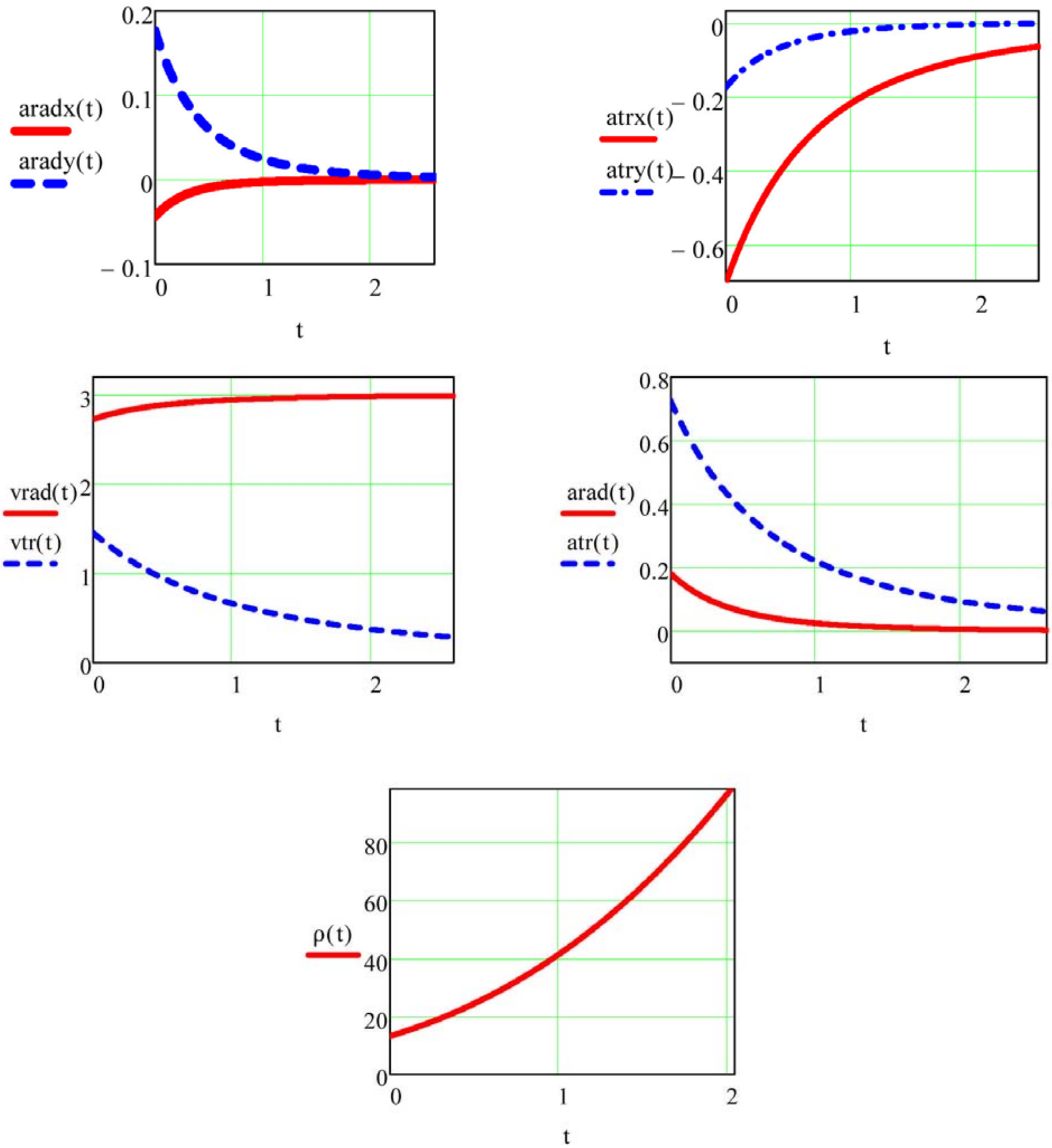


**Fig. 4. Graphs expressing the dependence of the kinematic characteristics of point $M$ on time $t\,(s)$ (Brent Maxfield, 2009; Доев и др., 2016)**

## Conclusion

An idea for a short solution is given by using the MathCAD package. The example complements the kinematics teaching aids in Bulgaria.

A similar problem has been solved by Doev and Doronin (Доев и др., 2016).

This article makes the solution more complete and detailed. The kinematics characteristics of the point are clearly detailed in Fig. 2. In addition to the graphical changes of the velocity and acceleration, Fig. 4. shows the changes in their components as well.

Finally, the task can be solved by hand as well. The partially automated solution overcomes the occurrence of mathematical difficulties and makes it possible to track the changes in kinematics magnitudes. It is compact and adaptable to the changes in the input data of the problem.

## References

Бертяев, В. Теоретическая механика на базе MathCAD практикум. Санкт–Петербург, „БХВ–Петербург", 2005. – 739 с. (Bertyaev, V. Teoreticheskaya mehanika na baze MathCAD praktikum. Sankt-Peterburg, "BHV-Peterburg",

2005. - 739 p.)

Brent Maxfield Essential MathCAD for Engineering, Science, and Math. 30 Corporate Drive, Suite 400, Burlington, MA 01803, USA, 525 B Street, Suite 1900, San Diego, California 92101-4495,USA, 84 Theobald's Road, London WCIX 8RR, UK, Copyright 2009. - 501 p.

Доев, В. С., Ф. А. Доронин. Сборник заданий по теоретической механике на базе MathCAD. Санкт Петербург, Москва, Краснодар, Лань, 2016. - 585 с. (Doev, V. S., F. A. Doronin. Sbornik zadaniy po teoreticheskoy mehanike na baze MathCAD. Sankt-Peterburg, M., Krasnodar, Lan, 2016. - 585 p.)

Яблонский, А. А. Сборник заданий для курсовых работ по теоретической механике, М., Высшая школа, 1978. (Yablonskiy, A. A. Sbornik zadaniy dlya kursovaih rabot po teoreticheskoy mehanike, M., Visshaya shkola, 1978.)