DETERMINING THE NATURAL FREQUENCY ON A STEPPED SHAFT WITH A TRANSITIONAL CURVED SECTION

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ABSTRACT. The article discusses the topic of determining the natural frequency in a specific shaft. The shaft consists of three segments. The second one is fulfilled with a rounded radius. In the process of operation, the shaft acts by its natural vibrations. An approximate method is applied for their determination. According to the method, each segment of the shaft is divided into portions along its length. The computational scheme is a free beam, propped up with two camps at its ends. It is loaded with concentrated forces whose values are equal to the weights of the individual portions. Their application points are in the middle of the width of the portions selected by the package engineer. The displacements in the points of this simple beam are calculated by the differential equation of the elastic line. The solution to this equation is realised through its numerical integration. For the curved segment, an algorithm is applied to determine the radii, weights and stiffness. For the entire shaft, the reaction forces and bending moments in the application points of the forces are obtained. The inclines of the elastic line and the displacements in these points are calculated by numerical integration. The presented solution to determining the natural frequency of shaft with a transitional section confirms the validity of the analytical expressions of the above values.

Keywords: stepped shaft with a curved segment, natural frequency, approximated method, differential equation of the elastic line.

ОПРЕДЕЛЯНЕ НА ЧЕСТОТАТА НА СОБСТВЕНИ ТРЕПТЕНИЯ НА СТЪПАЛЕН ВАЛ С ПРЕХОДЕН УЧАСТЪК

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РЕЗЮМЕ. В статията се разглежда въпросът за определяне на собствените трептения в конкретен вал. Той се състои от три участъка, като вторият е изпълнен със закръгление с определен радиус. В процеса на работа върху вала действат собствени трептения. За тяхното определяне е приложен приблизителен метод. Според него всеки участък от вала се разделя на сегменти по дължината му. Изчислителната схема е греда, подпряна с два лагера в краищата й. Тя е натоварена със съсредоточени сили, чиито стойности са равни на теглата на сегментите. Приложните им точки са в средите на избраните от конструктора ширини на сегментите. Преместванията в точки от простата греда се определят чрез диференциалното уравнение на еластичната линия. Решението на това уравнение се реализира посредством численото му интегриране. За преходния участък е приложен алгоритъм за определяне на радиусите, теглата и коравините на сегментите. За целия вал са получени опорните реакции и огъващите моменти в приложните точки на силите. Чрез числено интегриране са получени наклоните на еластичната линия и преместванията в същите точки. Представеното решение за определяне на честотата на собствените трептения във вал с преходен участък потвърждава верността на аналитичните изрази за описаните по-горе величини.

Ключови думи: стъпален вал, собствени трептения, приблизителен метод, диференциално уравнение на еластична линия.

Introduction

The natural frequency of a stepped shaft is the object of investigation by specialists calculating the shafts. The approximate method is well known. It applies numerical integration to resolve the differential equation of the elastic line (Feodosev, 1965). Refining this method, an algorithm for the transition section of the shaft is added (Trifonova-Genova et al., 2017). This algorithm consists of eleven steps. They include formulae by which the current diameter and the stiffness of each section are obtained. The main objective of this article is to apply this algorithm to a concrete stepped shaft with a curvilinear transition.

Methods

1. Formulation of the problem

A stepped cylindrical shaft is presented at Figure 1 (Anchev et al., 2011). The lengths of three sections are: $l_1 = 12\ cm$, $l_2 = 0.5\ cm$ and $l_3 = 5.5\ cm$. The radius of the transition zone is: $r = 0.5\ cm$, the diameters of the first section and the beginning of the second and the third section of the relevant shaft are, respectively, $D_1 = 5.6\ cm$, $D_{2,o} = 5.0\ cm$, and $D_3 = 4.0\ cm$.

2. Method for determining the of internal forces

To determine the natural frequency, the approximate method is applied (Feodosev, 1965). According to it, the shaft is divided into portions with sized widths in the first and the third sections: $\Delta x_1 = \Delta x_3 = 1\,cm$, and in the second section - $\Delta x_2 = 0.1\,cm$.

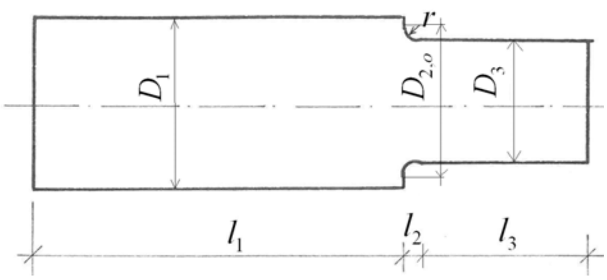

Fig.1. Stepped shaft with a curved segment

Current diameter $D_{2,i}$ in the second segment (section)

For determining this diameter for each segment of this section, the algorithm of (Trifonova-Genova et al., 2017) is applied. For given values of arrows, the chords and the diameters are calculated. The results are given in Table 1.

Table 1.
Diameters of a segment with a transitional curved section

Point of segment	f_i mm	a_i mm	$D_{2,o}$ mm
13	0.5	4.4	46
14	1.5	7.5	43
15	2.5	8.7	41
16	3.5	9.5	40
17	4.5	10.0	40

Stiffness of the segment of the shaft

The Young's modulus of shafts is taken by (Kisiov, 1978) ($E = 1.98.10^4 \kappa g / cm^2$). With a known diameter of each segment of the shaft, the moment of inertia is calculated. The stiffness of the segments in the three sections are given in Table 2.

Table 2.
Stiffness of the segments of the shaft

Section	points	$EJ_y\,10^6\,[kgcm^2]$
1	1-12	95.536
2	13	43.516
2	14	33.227
2	15	27.464
2	16	24.881
2	17	24.881
3	18	24.869
3	19-23	24.869

Weight of the segments

The volumetric weight of the material of the shaft is $\gamma = 7.8.10^{-3} kg / cm^3$, but the weight of the segments is given in Table 3.

Table 3.
Forces

Section	points	$P_i^I\ [kg]$
1	1-12	0.1921
2	13	0.0128
2	14	0.0111
2	15	0.0105
2	16	0.0100
2	17	0.0098
3	18	0.0480
3	19-23	0.0981

The segment with the principal point number 18 has a width of $0.5\,cm$. This is the reason why the weight is half of the weight of the segments to follow.

The reactions of a support

The computational scheme is a free beam loaded by weights. The values of the first iteration are given in Table 3. The reactions of support are calculated with the methods of statics (Vulkov, 2004; Vulkov et al, 2013). Their values are: $A^I = 1.635\,kg$, $B^I = 1.262\,kg$.

Bending moment diagram

The moments in the individual points are determined with the methods of the resistance of the materials (Vulkov, 2011). The values of first iteration are given in Table 4.

Table 4.
Bending moments

point	M_i^I $[kgcm]$	point	M_i^I $[kgcm]$
A	0	13	5.758
1	0.818	14	5.689
2	2.261	15	5.620
3	3.512	16	5.550
4	4.571	17	5.478
5	5.437	18	5.275
6	6.112	19	4.696
7	6.595	20	3.827
8	6.885	21	2.860
9	6.984	22	1.794
10	6.890	23	0.631
11	6.604	B	0
12	6.126		

3. Natural frequency

Natural frequency is obtained through the following relations. In the beginning, the following proportion of the bending moments for each point is calculated from Table 4 to the stiffness from Table 2. This proportion is multiplied by the width of each segment. With the method of numerical integration, the slope of an elastic line is obtained (Feodosev, 1965).

The displacements w_i are represented by an equation which has two coefficients. They are obtained by the boundary conditions: $w_1(0)=0$, $w_n(l)=0$. For the concrete shaft, these coefficients are: $C_1^I = -0.5827.10^{-6}$ and $C_2^I = 0$. The displacement for the concrete position x_i of point i in the computational scheme is determined.

Finally, two multiplications are composed. The first includes the displacement and the force at each point. The second involves the displacements elevated to the second degree and the force. The values are given in Table 5.

Table 5.
Results of the first iteration

	x_i	w_i^I	$P_i^I w_i^I$	$P_i^I \left(w_i^I\right)^2$
		10^{-6}	10^{-6}	10^{-12}
	$[cm]$	$[cm]$	$[kgcm]$	$[kgcm^2]$
A	0	0	0	0
1	0.5	0.283	0.054	0.015
2	1.5	0.833	0.160	0.133
3	2.5	1.347	0.259	0.348
4	3.5	1.813	0.348	0.631
5	4.5	2.222	0.427	0.948
6	5.5	2.567	0.493	1.265
7	6.5	2.843	0.546	1.552
8	7.5	3.046	0.585	1.783
9	8.5	3.177	0.610	1.939
10	9.5	3.236	0.622	2.011
11	10.5	3.225	0.620	1.998
12	11.5	3.151	0.605	1.907
13	12.05	3.070	0.039	0.120
14	12.15	3.053	0.034	0.103
15	12.25	3.035	0.032	0.096
16	12.35	3.014	0.030	0.091
17	12.45	2.991	0.029	0.088
18	12.75	2.903	0.139	0.405
19	13.50	2.577	0.253	0.652
20	14.50	1.989	0.195	0.388
21	15.50	1.285	0.126	0.162
22	16.50	0.510	0.050	0.025
23	17.50	0.291	0.029	0.008
B	18.00	0	0	0
∑			6.228	16.671

After summing the last two columns, the square of the natural frequency is calculated: $\left(\omega^I\right)^2 = 366,494.10^6\ s^{-2}$. The natural frequency is $\omega^I = 19.144.10^3 s^{-1}$ by the first iteration of the approximate method (Kisyov, 1978).

The second iteration takes into account that, in process of vibration, the shaft is loaded not only with the force of weight but with the force of inertia as well. From the first approximation, the values of displacements and the square of the frequency are also used. For the second iteration, the force P_i^{II} is calculated according to the formula given in (Trifonova-Genova et al., 2017). After that, the reactions of support, the bending moments, the slope of the elastic line, and the displacements of the points are determined in the computational scheme, as well as both multiplications analogically to the second iterations. The values are given in Table 6.

Table 6.
Results of the second iteration

	x_i	w_i^{II}	P_i^{II}	$P_i^{II} w_i^{II}$	$P_i^{II} \left(w_i^{II}\right)^2$
		10^{-6}		10^{-6}	10^{-12}
	$[cm]$	$[cm]$		$[kgcm]$	$[kgcm^2]$
A	0	0	-	0	0
1	0.5	0.274	0.020	0.006	0.002
2	1.5	0.808	0.060	0.048	0.039
3	2.5	1.311	0.097	0.127	0.166
4	3.5	1.771	0.130	0.230	0.408
5	4.5	2.178	0.159	0.347	0.756
6	5.5	2.523	0.184	0.465	1.172
7	6.5	2.801	0.204	0.571	1.600
8	7.5	3.006	0.219	0.657	1.976
9	8.5	3.139	0.228	0.716	2.246
10	9.5	3.197	0.232	0.743	2.374
11	10.5	3.186	0.231	0.737	2.349
12	11.5	3.109	0.226	0.703	2.186
13	12.05	3.027	0.015	0.044	0.134
14	12.15	3.010	0.013	0.038	0.114
15	12.25	2.991	0.012	0.036	0.106
16	12.35	2.970	0.011	0.034	0.100
17	12.45	2.947	0.011	0.032	0.095
18	12.75	2.858	0.052	0.149	0.426
19	13.50	2.532	0.094	0.239	0.605
20	14.50	1.947	0.073	0.142	0.276
21	15.50	1.255	0.047	0.059	0.074
22	16.50	0.498	0.019	0.009	0.005
23	17.50	0.280	0.011	0.003	0.001
B	18.00	0	0	0	0
∑				6.135	17.210

The calculation from the first iteration are repeated and the natural frequency is obtained: $\omega^{II} = 18,701.10^3\ s^{-1}$. The value is compared to the value of the first iteration and a difference of 2.29% is obtained. This is sufficient for the practice accuracy. Therefore, the iterations are terminated and the value is finally chosen.

4. Key Findings

In this article, the approximate method is applied for calculating the value of natural frequency in a concrete shaft with a transitional curvilinear section. The algorithm is applied for this section and the values of diameters and weights are obtained. The successive parameters, like the reactions of the supports, the bending moments, the slopes, and the displacements of points in the free beam are given here. Finally, the natural frequency for two iterations is calculated.

Conclusion

The numerical results of this work confirm the formulae and the algorithm for stiffness in the curvilinear section. They give a more precise solution to the problem of determining the natural frequency in a shaft with a curvilinear section. This is the advantage of the proposed method. It can be extended to shafts with more sections.

No matter how weak the curvature in a curvilinear section is, it is desirable to take it into account in each real shaft.

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