AN ECONOMIC AND MATHEMATICAL MODEL FOR DETERMINING OPTIMAL MINING WITH PRE-DETERMINED QUALITY INDICATORS IN A MINING ENTERPRISE

Velichka Hristova, Kristina Ilieva-Stoycheva, Milen Petkov

Kardzhali Branch of the University of Mining and Geology "St. Ivan Rilski" Sofia, 6600 Kardzhali; velichka.hristova@mgu.bg

ABSTRACT. Ore mining in a mining enterprise has different contents of useful and harmful components. For further processing, the ore quality is required to be within a certain range, and it is necessary to plan the mining process. This article proposes an economic and mathematical model describing the planning and management of ore mining according to pre-defined indicators. The model is applied to the conditions of a mining enterprise in Bulgaria.

Keywords: model of mathematical economics, linear programming, mining, mining enterprise

ИКОНОМИКО-МАТЕМАТИЧЕСКИ МОДЕЛ ЗА ОПРЕДЕЛЯНЕ НА ОПТИМАЛЕН ДОБИВ ПРИ ЗАДАДЕНИ ПОКАЗАТЕЛИ ЗА КАЧЕСТВО В МИННО ПРЕДПРИЯТИЕ

Величка Христова, Кристина Илиева-Стойчева, Милен Петков

Филиал Кърджали на Минно-геоложки университет "Св. Иван Рилски" София, 6600 Кърджали

РЕЗЮМЕ. Добивът на руда в минно предприятие е с различно съдържание на полезни и вредни компоненти. За следващото му преработване се изисква качество на рудата в определен диапазон и е необходимо планиране на добива от рудници или участъци. В настоящата статия е предложен икономико-математически модел, описващ планиране и управление на добив на руда по зададени показатели. Моделът е приложен за условията на минно предприятие в България.

Ключови думи: икономико-математически модел, линейно програмиране, рудодобив, минно предприятие

Introduction

The specificity of work in a mining enterprise is the ore mining from separate areas of the mine or separate mines with different contents of useful and harmful constituents of the ore. The further processing of the ore requires a metal content within a certain range "in the input" of an enrichment plant. Therefore, it is necessary to plan the mining from each mine or area in such a way so that the total flow corresponds to specified requirements for mining and/or processing.

In mining enterprises these problems in the planning and management of mining are solved with linear optimisation. Linear models, in which the objective function and all its constraints are linear, adequately describe the processes in the mode of average indicators (Lalova et al., 1980; Reklaitis et al., 1983).

Model

The creation of an economic and -mathematical model for determining the optimal mining from several mines with specified capacities of each one and the requirements for the ore quality is based on linear programming and refers to the class of mixed problems.

The control parameters are:

- x_i - volume of mining of i – th mine $/i = \overline{1,n}/$;
- α_i - content of the 1st component in i – th mine;
- β_i - content of the 2nd component in i – th mine;
- γ_i - content of the 3rd component in i – th mine;
- Q_i^{max}, Q_i^{min} – respectively maximum and minimum volume of mining of i – th mine;
- c_i - mining costs in the in i – th mine;
- Q_{pl} – planned volume of ore mining;
- α_{pl} - planned content of the 1st component;
- β^{min}, β^{max} - minimum and maximum permitted content of the 2nd component;
- γ^{max} - maximum permitted content of the 3rd harmful component.

The model is with minimising the objective function:

$$\sum_{i=1}^{n} x_i \,.\, c_i \to min. \qquad (1)$$

The limitations are:

- by the productivity of each mine:

$$Q_i^{max} \geq x_i \geq Q_i^{min}. \qquad (2)$$

- by the total volume of work:

$$\sum_{i=1}^{n} x_i \geq Q_{pl}. \qquad (3)$$

- by the ore quality:

$$\frac{\sum_{i=1}^{n} x_i.\alpha_i}{\sum_{i=1}^{n} x_i} = \alpha_{pl}; \quad (4)$$

$$\beta^{max} \geq \frac{\sum_{i=1}^{n} x_i.\beta_i}{\sum_{i=1}^{n} x_i} \geq \beta^{min}; \quad (5)$$

$$\frac{\sum_{i=1}^{n} x_i.\gamma_i}{\sum_{i=1}^{n} x_i} \leq \gamma^{max}. \quad (6)$$

- by the positive values of the volume of mining for each mine:

$$x_i \geq 0, \ \ i = \overline{1,n}. \quad (7)$$

Application of the model for the conditions of Varba-Batantsi AD

Varba-Batantsi AD and Gorubso-Madan AD are enterprises located in South Central Bulgaria. Their activity is mining of lead-zinc ores from three concession areas: Petrovitsa, Krushev dol and Varba-Batantsi situated in the Madan ore field. The owners of the first enterprise are KCM 2000 Group AD and Minstroy Holding AD, with equal participation, while the majority ownership of the second enterprise is held by Varba-Batantsi AD (Sabev, Yordanov, 2014).

By common characteristic, the Madan ore deposits are polymetallic with a basic content of lead and zinc, and accompanying precious components such as silver, gold, copper, etc.

Vein and metasomatic ore bodies in Petrovitsa, Krushev dol and Varba-Batantsi deposits are mainly composed of sulphide - galena, sphalerite, pyrite, chalcopyrite, etc. and non – metallic - quartz, calcite, rhodochrosite, etc. (Georgiev et al., 2007).

The mining of lead-zinc ore is carried out underground and amounts to 353074 tonnes per year from the three mines for 2018. The metal content of lead-zinc ore is low and direct metallurgical processing is unprofitable. The ore is subjected to a preliminary processing by applying a flotation method in an enrichment plant in the town of Rudozem /owned by Varba-Batantsi AD/. The daily production capacity of the enrichment plant is processing of 1765 tonnes of ore. The result from the processing is as followslead concentrate with lead content 70%, zinc concentrate with zinc content 52-54% and waste. Lead extraction is 94-96%, and zinc - about 82%. The produced lead and zinc concentrate is transported to KCM 2000 Group AD - Plovdiv where they are metallurgically processed and the final technological waste is deposited in a tailings dam.

The economic and mathematical model is calculated with data from the Consolidated financial statements of the mining enterprise for 2018 (Consolidated financial statements of Varba-Batantsi AD, 2018). The enrichment plant processes lead-zinc ore from the three mines with an annual processing volume of 353074 tonnes. The requirement is that the content of lead should be in the range: $1.9 \div 2.6\%$, and the zinc content: 2.2%.

According to the presented model, the annual planned mining volumes of the plant are searched at specified quality in Table 1 with minimal costs.

Table 1. *Parameters of the ore*

Indicators	Mine 1 "Petrovitsa"	Mine 2 "Krushev Dol"	Mine 3 "Varba-Batantsi"
Mining costs per 1 tonne of ore, [thousand BGN]	105	111	129
Maximum volume of mining, [thousand tonnes]	126.567	127.341	99.166
Content of Pb in ore, [%]	2.54	3.25	2.45
Content of Zn in ore, [%]	1.71	2.95	1.88

The annual volume of mining for each mine is: $x_1, \ x_2, \ x_3$.
The objective function to minimise mining costs is:

$$Z = 105.x_1 + 111.x_2 + 129.x_3 \rightarrow \text{min}, [\text{thousand BGN}], \quad (8)$$

At limitations:

- by maximum capacity of each mine:

$$x_1 \leq 126.567; \ x_2 \leq 127{,}341, [\text{thousand tonnes}]. \quad (9)$$

- by total volume of work:

$$x_1 + x_2 + x_3 = 353{,}074, \ [\text{thousand tonnes}]. \quad (10)$$

- by the quality of the ore:
  - ✓ minimum and maximum percentages of lead in the ore:

$$\frac{2.54.x_1+3.25.x_2+2.45.x_3}{x_1+x_2+x_3} \geq 1.9, \ [\%]; \quad (11)$$

$$\frac{2.54.x_1+3.25.x_2+2.45.x_3}{x_1+x_2+x_3} \leq 2.6, \ [\%]. \quad (12)$$

  - ✓ maximum zinc content in the ore:

$$\frac{1.71.x_1+2.95.x_2+1.88.x_3}{x_1+x_2+x_3} \leq 2.2, \ [\%]. \quad (13)$$

- by positive values of the volume of mining for each mine:

$$x_1 \geq 0, \ \ x_2 \geq 0, \ \ x_3 \geq 0, \ [\text{thousand BGN}]. \quad (14)$$

The solution of the optimisation problem in PTC Mathcad worksheet is presented in Figure 1. In the solved block, the objective function and constraints with Boolean operators are entered and built-in function Minimise is used to minimise the function through the method of the least squares (Maxfield, 2014).

The solution of the objective function is 41574 thousand BGN with an optimised mining respectively from the first mine - 126567 tonnes; second mine − 51963 tonnes and from the third mine - 174544 tonnes.

In case that the mining enterprise requires the use of uniformity criteria or minimum fluctuations in quality, the problem is changed and refers to non-linear programming.

$$\text{ORIGIN} := 1$$

$$Z(x1, x2, x3) := 105 \cdot x1 + 111 \cdot x2 + 129 \cdot x3$$

$$x1 := 1 \qquad x2 := 1 \qquad x3 := 1$$

Given

$$\frac{2.54 \cdot x1 + 3.25 \cdot x2 + 2.45 \cdot x3}{x1 + x2 + x3} \geq 1.9$$

$$\frac{2.54 \cdot x1 + 3.25 \cdot x2 + 2.45x3}{x1 + x2 + x3} \leq 2.6$$

$$\frac{1.71 \cdot x1 + 2.95 \cdot x2 + 1.88 \cdot x3}{x1 + x2 + x3} \leq 2.2$$

$$x1 + x2 + x3 = 353.074$$

$$0 \leq x1 \qquad 0 \leq x2 \qquad 0 \leq x3$$

$$x1 \leq 126.567 \qquad x2 \leq 127.341$$

$$R := \text{Minimize}(Z, x1, x2, x3)$$

$$R = \begin{pmatrix} 126.567 \\ 51.963 \\ 174.544 \end{pmatrix}$$

$$Z(R_1, R_2, R_3) = 41573.611$$

Fig. 1. Worksheet of the problem in PTC Mathcad

Conclusions

The mining plan in a mining enterprise with several mines or areas under the conditions of a corresponding common flow with specified requirements and effective processing in an enrichment plant is a problem of the mining practice of a Bulgarian company.

The paper presents an algorithm and a solution of the economic and mathematical model for the average quantity mode of linear programming in PTC Mathcad programming environment. The model simulation results can be used to make future more effective solutions.

Acknowledgements. The paper was implemented with the support of Research Project FK-018/11.03.2019 of UMG "St. Ivan Rilski" Sofia.

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