THE PRODUCTION FUNCTION OF A MINING ENTERPRISE AS AN ANALYTICAL TOOL FOR THE PRODUCTION OPTIMISATION

Kristina Ilieva-Stoycheva, Gabriela Siderova, Perihan Yusein

Kardzhali Branch of the University of Mining and Geology "St. Ivan Rilski" Sofia, 6600 Kardzhali; Ilieva_krisi@abv.bg

ABSTRACT. The paper presents the production optimisation by applying the two-factor Cobb–Douglas production function. The function is derived from a database of a functioning mining enterprise in the country and the isoquant map is analysed. The obtained ex-post production function shows the replacement capabilities of the enterprise and it is a tool for analysing, planning and forecasting the business.

Keywords: Cobb–Douglas production function, mining enterprise, isoquants.

ПРОИЗВОДСТВЕНАТА ФУНКЦИЯ НА МИННО ПРЕДПРИЯТИЕ КАТО АНАЛИТИЧЕН ИНСТРУМЕНТ ЗА ОПТИМИЗАЦИЯ НА ПРОИЗВОДСТВОТО

Кристина Илиева-Стойчева, Габриела Сидерова, Перихан Юсеин

Филиал Кърджали на Минно-геоложки университет "Св. Иван Рилски" София, 6600 Кърджали, Ilieva_krisi@abv.bg

РЕЗЮМЕ. Статията разглежда оптимизацията на производството чрез прилагане на двуфакторната производствена функция на Коб-Дъглас. Функцията е изведена на база данни от действащо минно предприятие в страната и е анализирана картата на изоквантите. Получената производствена функция ex_post показва заместващите възможности на фирмата и е инструмент за анализ, планиране и прогнозиране на дейността.

Ключови думи: производствена функция на Коб-Дъглас, минно предприятие, изокванти.

Introduction

The production model in the most synthesised form can be presented as a system that processes different types of resources into finished products. The dependence between the volume of finished products and the costs for the different types of resources needed for this production is called a production function.

Modern growth models and theories have different hypotheses about production functions (Walters, 1963). The difference between the ex-ante production function and the ex-post production function is important. The first collects a full set of replacement capabilities for the enterprise when choosing the mode of production, and the second shows the available options with already selected mode of production.

Another significant difference is the object of the production function - an individual enterprise or a separate sector.

The paper examines the ex-post production function for a mining enterprise in Bulgaria.

Production function

The resources for production may be: labour costs; energy costs; concentrate costs, transport costs, etc. The most popular two-factor production function is the Cobb-Douglas function with costs of the materials and labour costs, which was published in an article by the American scientists Charles Cobb and Paul Douglas in a study of the American economy in 1928 (Cobb, Douglas, 1928). Volume production F is dependent:

$$F(K, L) = A.K^{\alpha}.\mathrm{L}^{\beta}\,, \qquad (1)$$

on the condition:

$$\alpha + \beta = 1\,, \qquad (2)$$

where:

A – production coefficient;

K – costs of basic capital investments;

L – labour costs;

α – elasticity coefficient of the costs of capital investments;

β – elasticity coefficient of labour costs.

One production function is neoclassical if it meets the following requirements:

1. There is a positive effect of an increase in production factors – each production factor contributes to the production and the increase in the quantities used from this factor leads to higher production.
2. The law of diminishing returns is in force - the increase of each resource leads to an increase in production, but with an increase in the quantity of the resource at fixed volumes of the other resources, the effect is a decrease in the rate of increase relative to it.

3. There are constant returns in terms of scale – if the inputs are increased λ times, the production will also increase so many times, as the production function is positively linear homogeneous of first degree:

$$F(\lambda.x_1, \dots \lambda.x_n) = \lambda.F(x_1, \dots, x_n), \ \forall\lambda > 0\,. \quad (3)$$

4. The conditions of Inada are met – if the i−th production factor is insufficient, it becomes very valuable and its marginal product becomes very large /i.e. a small increase in the quantity used leads to a large increase in production/; if the production factor is already used in large quantities, the use of a little more of it almost does not change the production:

$$\lim_{x_i \to 0} F_{x_i} = +\infty\,; \quad \lim_{x_i \to +\infty} F_{x_i} = 0. \quad (4)$$

Isoquants are curves of production indifference and identify all effective combinations of two production factors that produce the same quantity of output (Fig. 1).

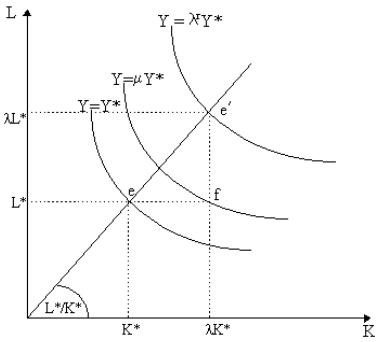

Fig. 1. Isoquants of production function

Main features of the production function are:

- average capital and labour productivity:

$$A_1 = A.K^{\alpha-1}.L^{\beta};\ A_2 = A.K^{\alpha}.L^{\beta-1}\,. \quad (5)$$

- marginal capital and labour productivity:

$$M_1 = A.\alpha.K^{\alpha-1}.L^{\beta};\ M_2 = A.\beta.K^{\alpha}.L^{\beta-1}\,. \quad (6)$$

- partial elasticities and total elasticity:

$$E_1 = \alpha\,;\ E_2 = \beta\,;\ E = \alpha + \beta\,. \quad (7)$$

- technological replacement rate:

$$R_{12} = (\alpha.L)/(\beta.K)\,. \quad (8)$$

The Cobb Douglas production function does not take into account technical progress as a factor of economic growth along with labour and capital costs. This was later done in Robert Solow's neoclassical economic growth model with a modified Cobb-Douglas production function.

The output of a production function by data is the basis for further analytical calculations; an assessment of the effectiveness and expediency of using additional resources in production; forecasting the volume of output produced at different amounts of resources; management decisions about the forthcoming development of the object of research.

Methodology of calculating production function

The study conducted on the production function of a mining enterprise includes the following stages:

- selection and justification of function indicators;
- determination of primary data for the study;
- defining methods for approximation of the function;
- selecting a programme environment for output function;
- verifying the adequacy of the model and the quality of the obtained function (coefficient of determination, correlation coefficient, Durbin-Watson criterion, Fisher's, Student's);
- conducting the computational experiments;
- logical and mathematical analysis of the qualities of the obtained function.

The nonlinear multiple regression method was selected in PTC Mathcad and applied to the Cobb-Douglas function with defined coefficients. In the general case, the coefficients $a_0, a_1, \dots, a_k$ are calculated by the condition for a minimum of the function through the method of least squares:

$$F(a_0, a_1, \dots, a_k) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2\,, \quad (9)$$

where:

$\hat{y}_i = S\left(a, x_{i,1}, x_{i,2}, \dots, x_{i,k}\right), i = 1, \dots, n$ are the values of the empirical dependencies at set values of the independent variables.

To minimise this function, the built-in Minimise function in PTC Mathcad is selected (Maxfield, 2014). The programme's solve block capabilities are used to set constraints of the coefficients searched $a_0, a_1, \dots, a_k$ with Boolean operators and solving the problem of conditional minimizing of the function.

Production Function of Varba-Batantsi AD

Varba-Batantsi AD has been operating since 2011 with main subject of activity - mining and processing of lead-zinc ores and production of lead-zinc concentrates. In 2012 the mining enterprise acquired the shares of Gorubso Madan AD (Sabev, Yordanov, 2014). The concession contracts for mining of lead-zinc ore from the Krushev dol and the Petrovitsa deposits belong to Gorubso Madan AD, and Varba-Batantsi AD has a concession contract for mining from the Varba-Batantsi deposit.

Using data from the consolidated annual financial statements of Varba-Batantsi AD for capital [thousand BGN], labour [number of workers] and production volume [thousand BGN] for six years from 2013 to 2018 according to the methodology a two-factor production function in the PTC Mathcad programming environment is derived (Fig. 2).

The function F is minimised in a solve block of the programme by introducing the additional condition with Boolean operators and the calculated coefficients result a_0, a_1, a_2 is derived in a vector form.

ORIGIN := 0 n := 6

$$x := \begin{pmatrix} 1321 & 715 \\ 5712 & 816 \\ 9564 & 851 \\ 16070 & 851 \\ 32646 & 907 \\ 34844 & 827 \end{pmatrix} \qquad y := \begin{pmatrix} 28272 \\ 29344 \\ 29470 \\ 37732 \\ 57280 \\ 53335 \end{pmatrix}$$

$$y_c(K,L,a0,a1,a2) := a0 \cdot K^{a1} \cdot L^{a2}$$

$$F(a0,a1,a2) := \sum_{i=0}^{n-1} \left(y_i - y_c\left(x_{i,0}, x_{i,1}, a0, a1, a2\right)\right)^2$$

a0 := 1 a1 := 0.5 a2 := 0.5

Given

a1 + a2 = 1 a0 > 0

$$\begin{pmatrix} a0 \\ a1 \\ a2 \end{pmatrix} := \text{Minimize}(F, a0, a1, a2)$$

$$\begin{pmatrix} a0 \\ a1 \\ a2 \end{pmatrix} = \begin{pmatrix} 22.3745 \\ 0.2707 \\ 0.7293 \end{pmatrix}$$

Fig. 2. Worksheet for calculating the production function in PTC Mathcad programming environment

The matrix of the production function for the calculated coefficients and its isoquant map are shown in Figures 3 and 4.

$$F(K,L) := 22.3745 \cdot K^{0.2707} \cdot L^{0.7293}$$

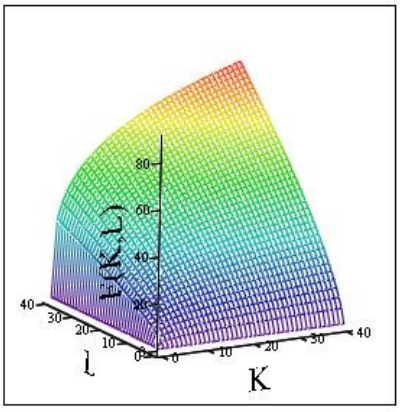

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Fig. 3. Production function

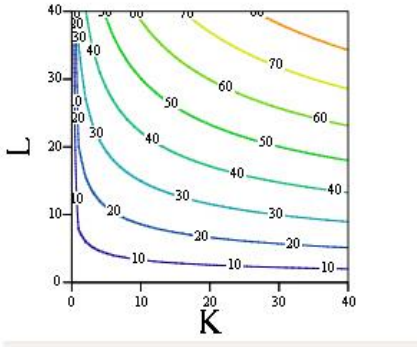

Fig. 4. Isoquants map

The main features of production function of the mining enterprise for the studied period with formulas from 5 to 8 are derived in Table 1.

Table 1. *Main features of the production function*

t	A_1	A_2	M_1	M_2	E_1	E_2	E	R_{12}
2013	14.2996	26.4193	3.8709	19.2676	0.2707	0.7293	1	0.2009
2014	5.4128	37.8897	1.4652	27.6330	0.2707	0.7293	1	0.0530
2015	3.8324	43.0706	1.0374	31.4114	0.2707	0.7293	1	0.0330
2016	2.6248	49.5667	0.7105	36.1490	0.2707	0.7293	1	0.0197
2017	1.6398	59.0232	0.4439	43.0456	0.2707	0.7293	1	0.0103
2018	1.4619	61.5940	0.3957	44.9205	0.2707	0.7293	1	0.0088

The analysis of the main features of the production function in a mining enterprise shows that, over time, capital productivity and technology replacement rate are decreasing and labour productivity is increasing. The values of elasticities are constants equal to the corresponding parameters of the production function.

Conclusions

The parameters of the Cobb-Douglas ex-post production function were evaluated using data for a six-year period from a functioning mining enterprise in Bulgaria.

On the basis of the outputs can be solved optimisation problems with budget constraints, maximising profits or maximising the total income between mines, and analysing management decisions.

The obtained production function is an analytical tool for optimal scenarios related to the future effective activity and investment policy of the mining enterprise.

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