COMPUTER MATHEMATICS SYSTEMS IN HIGHER MATHEMATICS TRAINING OF STUDENTS AT TECHNICAL UNIVERSITIES

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ABSTRACT. The subject of this paper is related to some applications of computer mathematics systems (CMS) in higher mathematics teaching. The mathematical preparation of students at the technical universities is characterised by its practical orientation. It is formed on the basis of interdisciplinary integration, which is based on general scientific principles and methods, one of them being the *modelling method* combining the mathematical subjects with the natural scientific and vocational training of students at technical universities. The paper presents a diagram of the process of solving engineering problems using ICT. The role and place of CMS in this process is explored. The levels of command of CMS – low, intermediate and high – are analysed. A system of mathematical problems which can be solved with appropriate CMS (Maple, MathCad, Excel and Mathematica) is proposed.

Keywords: computer mathematics systems (CMS), higher mathematics teaching, students at technical universities

СИСТЕМИТЕ НА КОМПЮТЪРНАТА МАТЕМАТИКА В ОБУЧЕНИЕТО ПО ВИСША МАТЕМАТИКА НА СТУДЕНТИТЕ ОТ ТЕХНИЧЕСКИ ВУЗ

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РЕЗЮМЕ. Предмет на настоящата работа са някои приложения на системите на компютърната математика (СКМ) в обучението по висша математика. Математическата подготовка на студентите в технически ВУЗ се характеризира със своята практическа насоченост. Тя се формира на базата на интердисциплинарната интеграция, която се основава на общонаучни принципи и методи, един от които е *методът на моделирането*, обединяващ математическите дисциплини с естественонаучната и професионална подготовка на студентите от технически ВУЗ. В работата е представена схема на процеса на решаване на инженерни задачи с помощта на ИКТ. Разгледана е ролята и мястото на СКМ в този процес. Направена е характеристика на нивата на овладяване на СКМ от студентите – ниско, средно и високо. Предложена е система от математически задачи, чието решение е с подходящи СКМ (Maple, MathCad , Excel и Mathematica).

Ключови думи: системи на компютърна математика (СКМ), обучение по висша математика, студенти от технически ВУЗ

Introduction

Modern social development is characterised by global informatisation. The rapid development and improvement of ICT leads to an increase in the information density of the public and professional activity of people. The skills to work with information, to create and study mathematical models, to perform mathematical calculations using mathematical packages and applied computer programmes, as well as the good command of the means of information and communication technologies are important components in the structure of the professional readiness and ability of the future graduates from a technical university.

Integration of training is based on general scientific principles and methods, one of which is the *modelling method* combining mathematical disciplines with the natural and vocational training of students at technical universities and thus, substantially enhancing fundamentality of technical higher education. The development of computer technologies is of particular importance in the training of the future engineers as it helps in the modelling of technological processes. This is due first to the increased amount of information that students obtain and it leads to the need of qualitative changes in the training content; secondly, to the integration of sciences requiring the ability to consistently apply knowledge from different university disciplines.

Exposition

The use of programming tools and ICT precedes the construction of mathematical models, so the students at the technical universities are obliged to study mathematics. It is necessary for them to learn to study mathematical models because this is required for their future professional activity.

The process of solving engineering problems with the implementation of modern ICT is presented in Figure 1 (Shishakov, Trohova, 2005).

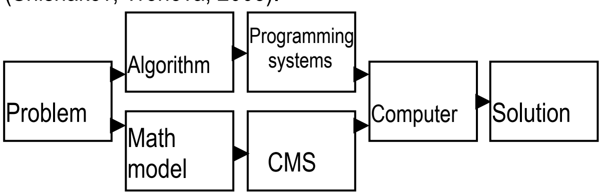

Fig. 1. Structure of the process of solving engineering problem with ICT

As can be seen from Figure 1, the solution can be obtained in two ways: the first one – by creating a programme in a relevant programming language; the second one – by using ready-made programming tools and ICT without direct programming. The rapid development of ICT with a convenient graphical interface makes it possible to significantly increase the number of engineering problems solved with them. The right choice of software tools and ICT used in the training of students at technical universities enhances their professional preparation and allows for the effective formation of mathematical culture and literacy based on the integration of mathematics and informatics.

The integrative link between mathematics and informatics is based on the content of their subject areas. In the process of learning mathematics, logical and algorithmic thinking of students is formed, the ability to compile mathematical models of phenomena and processes, skills for estimating results, prognosis of results, etc. are developed. The training in informatics provides a basis for understanding of the information character of the phenomena studied, enables the formulation and solution of problems in an effective visual form.

The idea of using computer mathematics systems (CMS) as a tool for integrating the students' learning-mathematical activities is in line with the basic principles of the competence approach. The training for working with modern CMS develops skills for creating and solving mathematical models of practical tasks. Using computers as means of learning and organising research creates new opportunities for interaction between students and teachers in the learning process as well as among students themselves; allows each student to increase their intellectual potential.

The new educational paradigm which is based on the integration of education on the basis of the fundamentalisation and introduction of key competences implies new educational goals, new principles for training content selection and systematisation, not so much for broadening of the scope of general scientific and professional knowledge, but rather for defining the interrelationship between them and other ways of their formation. In the near future, CMS will start to determine the new characteristics of mathematical activity and "the command of CMS will become a special key competence that will be directed towards preparing of students in the context of the new information technologies. In addition, we can talk about the skills for applying CMS as key competences of mathematical activity (both practical and theoretical), reflecting contemporary integrative tendencies in conducting mathematical research and extending the scope of application of mathematical methods (Ivanyuk, 2008).

Taking into account the structure and the general principles of activity in CMS integrative environment, the following levels of (Table 1) command and application of CMS in the learning-mathematical activity of students at technical universities can be distinguished.

Table 1. *Characterisation of levels of command of CMS by students at technical universities*

Level of PC	Characterisation of level
Low	The activity at this level is characterised by mastering of general skills for working with CMS (starting, inputting of data and outputting of results, their saving, etc.) and the ability to apply internal built-in functions of selected system packages when solving standard mathematical tasks, use of their reference materials.
Intermediate	The activity implies an experience in using internal functions for solving standard tasks, skills to apply built-in functions at the stage of searching for ideas for solving non-standard tasks, i.e. in performing learning-mathematical activity in modified and unknown situations. Autonomous creation of external functions for solving standard mathematical problems.
High	In addition to the previous level, the activity is characterised by the existence of experience for autonomous creation of external functions when performing learning-mathematical activity in modified and unknown situations.

We will look at the possibilities of modern CMS and how they can be applied in higher mathematics training of students at technical universities.

Let us illustrate two applications of *Maple* in analytical geometry training – for canonisation of curves of the second degree and for rotational surfaces.

Problem 1. Find the canonical equation of the following centre curve:

$F(x,y) = a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{13}x + 2a_{23}y + a_{33} = 0$.

Draw the curve.

It is known how laborious the process of canonisation of a second degree curve is in the classical way, whereas with the help of *Maple* it is achieved with only a few commands, which is evident from the proposed solution:

F:=a11*x^2+a22*y^2+2*a12*x*y+2*a13*x+2*a23*y+a33;

a11:=5;a22:=2;a12:=2;a13:=-12;a23:=-6;a33:=18; $a11 := 5$, $a22 := 2$, $a12 := 2$, $a13 := -12$, $a33 := 18$

s1:=simplify(1/2*a11+1/2*a22+1/2*(a11^2-2*a11*a22+a22^2+4*a12^2)^ (1/2)); $s1 := 6$

s2:=simplify(1/2*a11+1/2*a22-1/2*(a11^2-2*a11*a22+a22^2+4*a12^2)^(1/2)); $s2 := 1$

p1:=simplify(2/(8*a12^2+2*a11^2-4*a11*a22-2*a11*(a11^2-2*a11*a22+a22^2+4*a12^2)^ (1/2)+2*a22^2+2*a22*(a11^2-2*a11*a22+a22^2+4*a12^2)^(1/2))^(1/2)*a12);

$$p1 := \frac{2\sqrt{5}}{5}$$

p2:=simplify(-2*(1/2*a11-1/2*a22-1/2*(a11^2-2*a11*a22+a22^2+4*a12^2)^(1/2))/(8*a12^2 +2*a11^2-4*a11*a22-2*a11*(a11^2-2*a11*a22+a22^2+4*a12^2)^(1/2)+2*a22^2+2*a22* (a11^2-2*a11*a22+a22^2+4*a12^2)^(1/2))^(1/2));

$$p2 := \frac{\sqrt{5}}{5}$$

The coordinates of the centre are:

y0:=-(-a11*a23+a12*a13)/(-a11*a22+a12^2);x0:=-(a12*a23-a13*a22)/(-a11*a22+a12^2);

$$y0 := 1,\ x0 := 2$$

We find k:=eval(F,[x=x0,y=y0]); $k := -12$

The translation is x=x0+x1;y=y0+y1;

$$x = 2 + x1,\ y = 1 + y1$$

To draw, we use the programme:
with(plots):
t1:=textplot([-0.1,-0.2,O],align={LEFT,DOWN},colour=red):
t2:=textplot([x0-0.2,y0+0.2,M0],align={LEFT},colour=green, thickness=3):
t3:=plot([t,0,t=-1..4.5],thickness=1):
t4:=plot([0,t,t=-1..4],thickness=1):
t5:=plot([x0+t,y0,t=-1..4],thickness=2,color=green):
t6:=plot([x0,y0+t,t=-1..3],colour=green,thickness=2):
t7:=plot([x0+p1*t,y0+p2*t,t=-2..4],colour=blue,thickness=2):
t8:=plot([x0-p2*t,y0+p1*t,t=-4..4.5],colour=blue,thickness=2):
t9:=implicitplot(F=0,x=-1..4,y=-4..4.5,colour=red,thickness=2):
t10:=textplot([4+0.2,-0.1,x],align={DOWN},colour=red):
t11:=textplot([-0.2,3.8,y],colour=red,align={DOWN}):
t12:=textplot([x0+1+1.5,y0-0.1,x1],colour=green,align={DOWN}):
t13:=textplot([x0-0.2,y0+1+2,y1],colour=green,align={DOWN}):
t14:=textplot([x0+p1+1.5,y0+p2+0.5,X],colour=blue,align={DOWN}):t15:=textplot([x0-p2-0.9, y0+p1+2.5,Y],colour=blue,align={DOWN}):
display([t1,t2,t3,t4,t5,t6,t7,t8,t9,t10,t11,t12,t13,t14,t15]);
The rotation is:
x1:=simplify(p1*X-p2*Y);y1:=simplify(p2*X+p1*Y);

$$x1 := \frac{2\sqrt{5}\,X}{5} - \frac{\sqrt{5}\,Y}{5},\ y1 := \frac{\sqrt{5}\,X}{5} + \frac{2\sqrt{5}\,Y}{5}$$

The canonical equation is (Fig. 2):
K:=s1*X^2+s2*Y^2+eval(F,[x=x0,y=y0])=0;

$$K := 6\,X^2 + Y^2 - 12 = 0$$

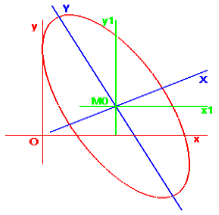

Fig. 2.

Let us take, for example, the course of mathematical analysis containing concepts such as limits, derivatives, integrals of different types, ranks and rows, differential equations. A large number of rules for finding limits, derivatives, calculating integrals, etc. have to be learned, the tables of the derivatives of the basic elementary functions and the main integrals have to be remembered and this is a huge amount of information. This information is very quickly forgotten and it is necessary to use reference books later on. Such calculations in real activity are not the main purpose of engineers. Their purpose is to solve a practical problem and the calculations are only an intermediate step on the way to this solution. By using CMS, much time can be saved and many calculations errors can be avoided (Izvorska, 2017).

Examples of this are the following problems.

Problem 2. Draw and animate the graph of the following function for large values of x. Use the graph to find the limit of the function at x tending to infinity (Fig. 3). Calculate this limit.

with(plots): plot(exp(-x/100)* cos(x), x=0..500 ,numpoints=100);
animate(plot,[exp(-x/100)* cos(x), x=0..t], t=0..500, frames=50);

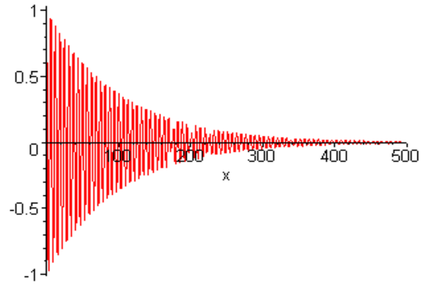

Fig. 3.

We will demonstrate the applications of *Maple* when studying integrals.

Problem 3. Solve

> int(exp(sin(x)),x);

$$\int e^{\sin(x)}\, dx$$

It can be seen that this integral cannot be expressed through functions known to *Maple.* We will graphically present some approximation of the integral. Through the *Maple*-function $\mathrm{taylor}(\mathrm{f}(x), x = a, n)$ we obtain a Taylor row for the function *f(x)* around the point *a* with precision to the *n*-th member.

> taylor(%,x=0,8);

% is a system variable that remembers the last result

$$x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{40}x^5 - \frac{1}{90}x^6 - \frac{1}{1680}x^7 + \frac{1}{720}x^8 + \mathrm{O}(x^9)$$

We will convert the row into a polynomial (no residual member):

> convert(%,polynom);

$$x + \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{40}x^5 - \frac{1}{90}x^6 - \frac{1}{1680}x^7 + \frac{1}{720}x^8$$

Let us draw the graph of this polynomial (Fig. 4):

> plot(%,x=-4..4);

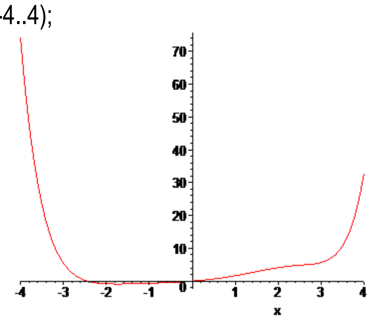

Fig. 4.

Problem 4. *3-D graphs.* Let us consider the function f, whose graph is the „mountain" in Figure 5. After differentiating and integrating the function with x, we get the graphs in Figures 6 and 7, respectively.

f:=exp(-abs(x-sin(y)))*(1+0.2*cos(x/2))*(1+0.4/(0.3+y^2));
plot3d(f,x=-6..6,y=-6..6);

$$f := e^{(-|x - \sin(y)|)} \left(1 + 0.2\cos\left(\frac{x}{2}\right)\right)\left(1 + \frac{0.4}{0.3 + y^2}\right)$$

plot3d(diff(f,x),x=-6..6,y=-6..6); plot3d(int(f,x),x=-6..6,y=-6..6);

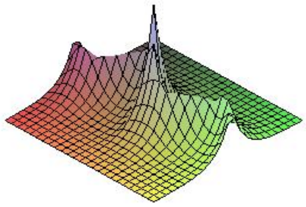

Fig. 5.

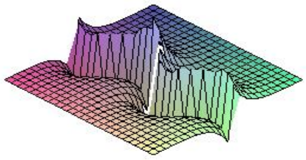

Fig. 6.

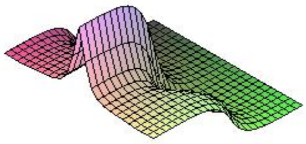

Fig. 7.

Applying Maple in higher mathematics training makes it possible to improve the quality and the efficiency of students' mathematical preparation, to apply a differentiated approach to learning taking into account their individual characteristics. In addition, the interaction between student and teacher in a dialogue regime facilitates the process of information exchange.

Combination methods are the basis of many problems from the probability theory, mathematical statistics and their applications. There are different methods of solution of combinatorial problems – the method of exhaustion of the possible variants; methods based on the application of the rules of addition and multiplication of probabilities by which basic combinatorial ideas are acquired, and the method of finding the number of compounds with or without repeats (variations, permutations and combinations) using formulae.

We will illustrate these methods with problems that can be solved using CMS.

Problem 5. From 10 red and 8 white roses a bouquet has to be made so that there are 2 red and 3 white roses in it. In how many ways can this be done?

Let's first choose 2 red roses out of the 10. This can be done in C_{10}^{2} ways. Regardless of the choice of the red roses, from the 8 white roses 3 can be chosen in C_{8}^{3} ways. Using the multiplication rule, 2 red and 3 white roses can be chosen in

$C_{10}^{2}.C_{8}^{3}=\dfrac{10.9}{1.2}.\dfrac{8.7.6}{1.2.3}=2520$ ways.

In *MathCad,* the combinations are obtained as follows:

```
n:=10                n:=8
m:=2                 m:=2
C:=combin(n,m)       C:=combin(n,m)
C = 45               C = 56
```

Problem 6. There are employees of different ages in a computer manufacturing company. The young employees are 24, middle-aged ones – 82 and the people of retirement age – 16. The probability of sending a young employee to a qualification course is 0.52; a middle-aged one – 0.54; and a person of retirement age – 0.36. What is the likelihood of a randomly selected employee to be sent to a qualification course?

Let the event A be "An employee is sent to a qualification course". All employees are 24 + 82 + 16 = 122. The young employees are 24 and if the event B1 is "a young employee has been chosen", then P(B1) = 24/122 = 0.2 (in *Excel* we have QUOTE *PB*1=24/122). The middle-aged employees are 82 and if the event B2 is "a middle-aged employee has been chosen", P(B2) = 82/122 = 0.67 (in *Excel* we have QUOTE *PB*2 =82/122). The employees of retirement age are 16 and if B3 is the event „an employee of retirement age has been chosen", then P(B3) = 16/122 = 0,13(in *Excel* we have QUOTE *PB3*=16/122). The probability a young employee to be sent to a qualification course is $P(A\,|\,B_1)=0.52$ (in *Excel* we have QUO TE $P(A\,|\,B_1)=0.52$); the likelihood a middle-aged employee to be sent is $P(A\,|\,B_2)=0.54$ (in *Excel* we have QUOTE $P(A\,|\,B_2)=0.54$); the probability a person of retirement age to be sent is $P(A\,|\,B_3)=0.36$ (in *Excel* we have QUOTE $P(A\,|\,B_3)=0.36$). Using the full probability formula, we get

$P(A)=P(A\,|\,B_1|)P(B_1)+P(A\,|\,B_2)P(B_2)+P(A\,|\,B_3)P(B_3)=0.51$

	A	B	C	D	E	F	G	H
1	B1=	24		P_B1(A)=	0,52		P(B1)=	0,20
2	B2=	82		P_B2(A)=	0,54		P(B2)=	0,67
3	B3=	16		P_B3(A)=	0,36		P(B3)=	0,13
4	N=	122						
5				P(A)=	0,51			

Problem 7. A manufacturer claims that the likelihood of a buyer's negative attitude towards a new good is not great. How many people should be interviewed so that with a probability of not less than 0.9 it can be argued that the relative frequency of the negative attitude towards a new commodity differs from the one stated by the manufacturer not more than 0.01.

The solution to the problem will be presented by using the system *Mathematica.*

We use the package Statistics`NormalDistribution`

In[1]:=<< *Statistics`NormalDistribution`*

Let us find the value of *n,* where the inequality is met

$\mathbf{P}\left\{\left|\dfrac{\xi}{n}-p\right|\le\varepsilon\right\}\ge\beta$, $\beta=0.9$, $\varepsilon=0.01$

In[2]:= $\beta=0.09$; $\varepsilon=0.01$

The sought value of *n* is found in the inequality

$n\ge\dfrac{1}{4}\,\dfrac{x_{\beta}{}^{2}}{\varepsilon^{2}}$

where x_β is a quantum of $(1+\beta)/2$ of the standard normal distribution

In[3]:= *ndist = NormalDistribution[0,1];*

In[4]:= $x_\beta = Quantile[ndist, \frac{1+\beta}{2}]$

Out[4]:= 1.64485

Let us find *n*.

In[5]:= $\frac{1}{4}\frac{x\beta^2}{\varepsilon^2}$

Out[5]:= 6763.86

Therefore $n = 6763.86$

In[6]:= *Clear[ndist,* x_β *,* ε *]*

The examples presented do not deplete the enormous possibilities of applying CMS in higher mathematics teaching. They only illustrate some of their applications.

The study of mathematical disciplines with CMS requires preliminary preparation including consecutive arrangement of lectures, seminars and laboratory classes in the weekly schedule of students on the one hand, and, on the other hand, the lecturer must prepare in advance a mathematical package of illustrations of basic concepts, discovering regularities when studying theorems, substitution of deductive evidence with geometric interpretations, suitable for visualization of counterexamples for concepts and theorems.

Conclusions

Summing up the studied problem, we can draw the following *conclusions*:

1. The knowledge acquired with the help of CMS has a universal character leading to the formation of special key competences − “command of CMS“. The motivation and interest in studying mathematics increase; there is a need for reflection and self-control; the learning results improve.

2. With the application of CMS, the traditional forms of organising higher mathematics training should not be radically changed. Application and adaptation of CMS in the existing lectures and practical seminars is needed.

3. Without acquiring skills for work with CMS, it is impossible to solve tasks with their help, as it is impossible to consciously use CMS without knowledge of the fundamentals of mathematics. Therefore, the productivity and effectiveness of the interactive dialogue between the learner and the computer is determined by: level of formation of general habits for work with CMS; the content and level of development of the learner for mathematical activity. Only lasting and deep mathematical knowledge allows the use of ICT in solving mathematical problems of applied character.

4. Training in mathematics of students at technical universities with the use of CMS should be implemented employing appropriate sets of mathematical problems. The methodology of such training implies: the proposed tasks should reflect the characteristics of the mathematical activity in the “man-computer” system, i. e. it should be computer-oriented; the need to use CMS must “ripen“ during the training and become a conscious need of the student; this need should be “increased” through gradual creation by the teacher of situations, necessary for this learning, which arise in the process of solving tasks.

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