USING SIMULATION MODELLING IN TEACHING THE GAME THEORY ELEMENTS

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ABSTRACT. The paper presents the author's attempt to use simulation modelling in the teaching of zero-sum matrix games. A matrix game with a 2×2 dimension is selected, which has a solution in mixed strategies. The conditions of the game are such that the solution obtained by applying the corresponding formulas seems paradoxical and questionable for the students. The correctness of the solution is confirmed by conducting a simulation experiment in the Microsoft Excel environment. At each stage of the teaching process, a survey is conducted. The statistical processing of survey data shows that the demonstration of simulation modelling results has the greatest impact on the learning of the curriculum.

Keywords: simulation modelling, game theory

ИЗПОЛЗВАНЕ НА СИМУЛАЦИОННО МОДЕЛИРАНЕ ПРИ ПРЕПОДАВАНЕ НА ЕЛЕМЕНТИ ОТ ТЕОРИЯТА НА ИГРИТЕ

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РЕЗЮМЕ. В настоящата работа е представен опит за използване на симулационно моделиране при преподаване на учебен материал от областта на матричните игри. Избрана е матрична игра с размерност 2×2, която може да се реши с прилагане на съответния аналитичен апарат, но полученият резултат предизвиква съмнение у студентите. Верността на решението се потвърждава с провеждане на симулационен експеримент в средата на Microsoft Excel. Оказва се, че експерименталното потвърждения на теорията оказва благотворно решаващо влияние върху усвояването на учебния материал.

Ключови думи: симулационно моделиране, теория на игрите

Introduction

Teaching is not just a transfer of information from the teacher to the learner. The memorisation of statements and facts is not enough. It is necessary to create conviction in their truth. Hence, the teaching material should be presented in such a way as to be credible. The attitude of the learner is very important. If his interest is awaken appropriately, teaching will achieve its goals.

The development of modern information technologies in the transition from the industrial to the information society (Georgiev, 2015) creates the opportunity to apply various innovative methods, which are capable of provoking a positive reaction of the learner belonging to the highly interactive generation (Atanasov, 2018; Atanasov, Ivanova, 2019). Unfortunately, however, the blackboard is still being applied, and the electronic materials are not being fully used (Kalev, 2019).

The above is also true for the teaching of mathematical disciplines. Computers are mostly used to present static training material (for example, as a Power Point presentation). They replace the classic blackboard without using their full potential. There is a positive experience with the use of software products that enable the rapid solution of classical problems, such as using Excel to solve the task of dynamic programming (Milkova, Yordanova, 2014). The authors use a computer to facilitate calculations for solving a deterministic problem. However, it should be noted that this is not a new way of solving. Manual solving and computer solving use the same algorithm. This saves learning time. It is possible to solve more examples and to confirm the validity of the theory, but the theory is confirmed in the same way. The teaching pattern is from the theory (as a formal algorithm) to the practice (executing the formal algorithm with specific data). In the technical disciplines, for example, it is possible to conduct physical model experiment or 3D modelling with application software such as CAD/CAM (Kalev, 2011) to support the theoretical statements. This is a more difficult in abstract mathematical disciplines.

The acquisition knowledge process is a process of denying wrong ideas. Some of them are too persistent. Often the denial contradicts the ‘common sense’. In such cases, the best way to achieve new knowledge is to present examples and analogies. The more diverse they are, the more plausible the new statement seems to be. Such are the views of Polya (1968). He describes patterns of reasoning in which the belief that a statement is true depends on how it is confirmed by its implications. If they are many and different, the statement is much more credible. Hence, the similar applications of the theory are not so credible.

Polya gives remarkable examples in the area where deterministic problems are dealt with. The task is more difficult in the area of stochastic problems. In this case, the best way is to conduct an experiment. Classical probability theory patterns (drawing balls from an urn, coin flipping, etc.) require a time that limited curriculum does not allow. Also, these are boring repetitive actions and could repel the young people who are accustomed to dynamic modern life. To avoid this contradiction, suitable software and technical means may be used. They are now widely available. Thus, teaching can be even attractive (Shabanova et al., 2017). There are available tools that allow the creation of models to obtain new knowledge (Mihaylov, 2017).

Methods and Results

The hypothesis has been formulated that the presentation of an appropriate example or experiment is essential to persuade learners in the truth of theoretical statements that have a higher degree of abstraction.

To verify the hypothesis, an experiment was conducted with 12 students studying elements of game theory. The Microsoft Excel simulation modelling capabilities were used to represent the probability model. These capabilities are not developed enough, but are sufficient to achieve the goal.

The following knowledge presentation pattern has been designed:

1. Formulating a problem whose obvious solution is wrong.
2. Explanation of a theory statement which gives true solution, but this solution seems non-plausible.
3. Conducting a convincing experiment that confirms the theory.

An anonymous survey was conducted at each stage.

It should be noted that the announcement that an experiment and a multi-stage survey will be conducted during the classes and the dealing of the questionnaires caused excitement among the students.

The game called Odd or Even was presented to the student. The rules of the game are:

The players *X* и *Y* announce simultaneously and independently of each other one of the numbers 1 or 2. If the sum is even, then *X* pays to *Y* this sum. If the sum is odd, then *Y* pays to *X* this sum.

This game is classical Zero-Sum-Game. The payoff matrix is

$$H_{2\times 2} = \begin{Vmatrix} -2 & 3 \\ 3 & -4 \end{Vmatrix} .$$

The first question from the questionnaire was if game rules gave advantage to one of the players. Possible answers and the number of students who have responded to each of them are:

A) The first player has an advantage - one student.
B) The second player has an advantage - three students.
C) Neither of the two players has an advantage, the game is fair-play - eight students.

In the second stage it was shown how to solve the game. The optimal *X* strategy $P^* = (p_1^*, p_2^*)$, the optimal *Y* strategy $Q^* = (q_1^*, q_2^*)$ and expected game value V were calculated using formulas (Neumann and Morgenstern, 1953, pp. 172-173):

$$p_1^* = \frac{h_{22} - h_{21}}{h_{22} - h_{21} + h_{11} - h_{12}} =$$

$$= \frac{-4-3}{-4-3-2-3} = \frac{7}{12}$$

$$p_2^* = \frac{h_{11} - h_{12}}{h_{22} - h_{21} + h_{11} - h_{12}} =$$

$$= \frac{-2-3}{-4-3-2-3} = \frac{5}{12}$$

$$q_1^* = \frac{h_{22} - h_{12}}{h_{22} - h_{21} + h_{11} - h_{12}} =$$

$$= \frac{-4-3}{-4-3-2-3} = \frac{7}{12}$$

$$q_2^* = \frac{h_{11} - h_{21}}{h_{22} - h_{21} + h_{11} - h_{12}} =$$

$$= \frac{-2-3}{-4-3-2-3} = \frac{5}{12}$$

$$V = \frac{h_{11} \cdot h_{22} - h_{12} \cdot h_{21}}{h_{22} - h_{21} + h_{11} - h_{12}} =$$

$$= \frac{8-9}{-4-3-2-3} = \frac{1}{12}$$

The result shows that the game rules give advantage to the first player. The expected value is equal to 1/12.

The students were asked how they accepted this result. Possible answers and the number of respondents were:

A) I totally reject the solution result because of distrust – two students.
B) I rather reject the solution result because of distrust – one student.
C) I am confused. This is contrary to the common sense – eight students.
D) I rather accept the solution result with confidence – one student.
E) I totally accept the solution result to be true – no one.

Multiple simulations of this game were demonstrated. The Microsoft Excel function RAND() was used. When the random value was less than 7/12 this meant the player chose his first strategy. In the opposite case the second strategy was chosen. Twenty sets of simulations (1000 simulations per set) were performed. The finite result was arithmetic mean of set's results. The example of simulations is shown on Figure. 1. The fact that the difference between theoretical result and simulation is less than 0.1 can be easily seen.

$$\frac{\left|V - \tilde{V}\right|}{V} = 0{,}0996 .$$

	A	B	C	D	E	F	G	H	I	J	K	L	M
1					p1=	7/12							
2					p2=	5/12							
3	H=	-2	3		q1=	7/12							
4		3	-4		q2=	5/12		Set	Value				
5								1	0,18		V=	0,0833	(Theoretical)
6								2	0,059				
7		X		Y		V		3	0,234				
8	1	0,55	1	0,934	2	3		4	0,055		Ṽ=	0,0916	(Simulation)
9	2	0,47	1	0,065	1	-2		5	0,038				
10	3	0,27	1	0,119	1	-2		6	0,074				
11	4	0,63	1	0,325	1	3		7	0,153				
12	5	0,41	1	0,202	1	-2		8	-0,005				
13	6	0,14	1	0,139	1	-2		9	0,199				
14	7	0,56	1	0,454	1	-2		10	-0,069				
15	8	0,8	2	0,831	2	-4		11	0,07				
16	9	0,53	1	0,667	1	3		12	-0,009				
17	10	0,35	1	0,534	1	-2		13	0,113				
18	11	0,56	1	0,861	2	3		14	0,04				
19	12	0,08	1	0,531	1	-2		15	0,107				
20	13	0,86	2	0,686	1	-4		16	0,127				
21	14	0,84	2	0,317	1	3		17	0,201				
22	15	0,23	1	0,898	2	3		18	0,138				
23	16	0,33	1	0,414	1	-2		19	0,12				
24	17	0,29	1	0,035	1	-2		20	0,029				
25	18	0,79	2	0,845	2	-4							

Fig. 1. Matrix game simulations

It was explained to the students that in order to obtain closer to the theoretical result a large number of simulations must be performed. So, in the sets with only 1000 simulations, it is possible that the second player wins (as in set 8, 10 and 12 in Figure 1).

After simulations the students were asked again how they accepted the result. Now the distribution of respondents was:

A) I totally reject the solution result because of distrust – one student.
B) I rather reject the solution result because of distrust – no one.
C) I am confused. This is contrary to the common sense – no one.
D) I rather accept the solution result with confidence – six students.
E) I totally accept the solution result to be true – five students.

Discussion

The purpose of the first question in the survey is to provoke students to appreciate the matrix game from the point of view of 'common sense', which in this case is misleading. The majority answer that the game is fair. Their logic is understandable – the sum of all payoffs is zero. It is subconsciously assumed that the choice of strategies is in equilibrium, which is not an optimal solution.

This provocation leads to the fact that only one student accepts the solution of the game, obtained with formulas, with confidence. Everyone else is confused or openly express their mistrust.

However, the simulation experiment has led to a sharp change. Neutral confusion has already disappeared. Only one student still does not believe in the solution. Everyone else accepts it with confidence (some of them 'rather accepts').

	Y_1	Y_2	
X_1	$a=11$	$b=1$	$a+b=12$
X_2	$c=1$	$d=11$	$c+d=12$
	$a+c=12$	$b+d=12$	$n=24$

Fig. 2. Survey results

X_1 - result before the simulation;

X_2 - result after simulation;

Y_1 - students which reject the solution result or are confused;

Y_2 - students which rather accept the solution result with confidence or accept the solution result to be true.

The summarised results of the survey are shown in Figure 2.

Statistical analysis of the results should be done to assess the impact of simulations on the change in student's attitudes. Two hypotheses are formulated:

Null hypothesis H_0 – there is not a significant impact of the simulation on students' attitude and on teaching process.

Alternative hypothesis H_1 – there is a significant impact of the simulation on students' attitude and on teaching process.

The sample (number of students) is small, so the Fisher's Exact Test should be used (Fisher, 1954, 96-97).

Let $P(Y_1)$ и $P(Y_2)$ are the probabilities of events Y_1 и Y_2 and there is not an impact of conditions X_1 и X_2 on these events. Then the probability of obtaining the frequencies a,b,c,d (Fig. 2) is equal to

$$P = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{(a+b+c+d)!} \cdot \frac{1}{a!b!c!d!}.$$

The sums $(a+b)$, $(c+d)$, $(a+c)$ and $(b+d)$ should not be changed. It is possible to determine other frequencies whose departure is more extreme from independence. These are $a=12$, $b=0$, $c=0$, $d=12$ only. Finite probability that $P(Y_1)$ and $P(Y_2)$ are not dependent on X_1 and X_2 is the sum of the probability of observed frequencies (Fig. 2) and the probabilities of more extreme frequencies, i.e.:

$$P = \frac{12!\cdot 12!\cdot 12!\cdot 12!}{24!} \cdot \left[\frac{1}{11!\cdot 1!\cdot 1!\cdot 11!} + \frac{1}{12!\cdot 0!\cdot 0!\cdot 12!} \right] =$$

$$= \frac{145}{2704156} \approx 5{,}36 \cdot 10^{-5}$$

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Hence the probability that $P(Y_1)$ and $P(Y_2)$ are independent from X_1 и X_2 is a small value. The null hypothesis is unlikely and is rejected in favour of the alternative hypothesis. There is significant impact of the simulation on students' attitude and on the teaching process.

Conclusions

This study shows that experiments could be used to teach disciplines traditionally considered to be theoretical. It turns out that in this way the interest of the audience can be provoked, which has a positive effect on the perception of the educational content. The computer equipment and software are available (to be read 'cheap') and this is a great advantage. In addition, computers should not be used only for presentations but for demonstrating models and solving problems in different areas.

The results of the survey and the statistical analysis of the obtained data show that the simulation experiment played an important role in the perception of the study material in this class. Wider application of such methods would help to increase the learners' interest in the subjects taught and to acquire more stable knowledge.

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