THE APPLICATION OF SEARCH METHODS FOR SOLVING OPTIMISATION PROBLEMS IN GEODESY

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ABSTRACT. As a rule, excessive data is used to solve any problem in practice. When it happens, the problem has several solutions (or, in some cases, a possibly infinite number thereof). As a result, a problem arises of how to optimise the solution process.
The purpose of the optimisation problem is to find a solution in accordance with any objective function (such as the criterion of efficiency or quality). Most practical problems are nonlinear in nature, i.e. the objective function and /or the connections between the parameters are nonlinear. The following algorithms of search optimisation have been developed for solving optimisation nonlinear problems: genetic algorithms, the method of simple search with a variable step size, the parabolic optimisation method. The paper describes the results of running search algorithms in Visual Basic for Applications (VBA) for solving optimisation problems in geodesy.
Search methods are very effective in solving optimisation nonlinear problems due to their advantages:
- a great variety of mathematical algorithms which have already been developed;
- the possibility of combining these algorithms with each other and with other methods of nonlinear programming;
- ease of programming;
- independence from the accuracy of the preliminary values of the parameters defined;
- there is no need to formulate error-correction equations or constraint equations or use a system of normal equations and solve them.
Search methods are convenient in programming and a large number of already existing methods along with the development of new search algorithms makes it possible to adapt them to solving any problems. This article discusses the possibility of using search methods in solving optimization nonlinear geodetic problems using the example of approximation of the results of measurements of a circle. To significantly speed up the search process, the algorithm for determining the minimum parabola is considered.

Keywords*:* optimisation; convex programming; search methods; parabolic optimisation

РЕШАВАНЕ НА ОПТИМИЗАЦИОННИ ЗАДАЧИ В ГЕОДЕЗИЯТА ЧРЕЗ МЕТОДИ НА ТЪРСЕНЕТО

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РЕЗЮМЕ. Обикновено решаването на практическите задачи се извършва в условията на излишък от данни. Когато това се случи, задачата има няколко решения, (а в някои случаите са безкрайно много на брой). В резултат на това възниква въпросът как да се оптимизира процеса на търсене на решение. Целта на оптимизацията е да се намери решение в съответствие с всяка целева функция (като критерия за ефективност или качество). Повечето практически оптимизационни задачи са нелинейни по своята същност, т.е. целевата функция и/или връзките между параметрите са нелинейни. Разработени са множество алгоритми за търсенето на оптимално решение при нелинейни оптимизационни задачи: генетични алгоритми, метод на просто търсене с променлив размер на стъпката, метод за параболична оптимизация. В статията са описани резултатите от работа с алгоритми за търсене, реализирани във Visual Basic for Applications (VBA) за решаване на оптимизационни задачи в геодезията. Методите за търсене са много ефективни при решаване на нелинейни задачи за оптимизация поради следните предимства:
- голямо разнообразие от математически алгоритми, които вече са разработени;
- възможността за комбиниране на тези алгоритми помежду им и с други методи на нелинейно програмиране;
- лесни за програмна реализация;
- независимост на точността от предварителните стойности на параметрите;
- не е необходимо да се формулират уравнения за корекция на грешки или уравнения на ограниченията или да се използва система от нормални уравнения, които да се решават.
Методите за търсене са удобни за използване, а големият брой вече съществуващи методи, заедно с разработването на нови алгоритми за търсене, дават възможност за адаптирането им към решаване на всякакви проблеми. В тази статия се разглежда възможността за използване на методи за търсене при решаване на оптимизационни нелинейни геодезически задачи, използвайки примера на апроксимацияна резултатите от измерванията на окръжност. За значително ускоряване на процеса на търсене се разглежда алгоритъм за минимизация по парабола.

Ключови думи: оптимизация; изпъкнало програмиране; методи на търсене; оптимизация по парабола

Introduction

Modern measuring tools allow to get a lot of information about an object, for example, when scanning millions of points. Due to the overabundance of data, the required parameters can be obtained many times (many solutions). Optimisation is understood as the process of choosing the best solution of the problem from all possible (Turchak L.I., 1987). Currently, optimisation problems arise in various fields of scientific and industrial activity. The purpose of optimisation is to find a solution in accordance with any objective function (criterion of efficiency, quality, accuracy, reliability, etc.).

Among non-linear optimisation of surveying and geodetic applications, we can mention:
- strain prediction;
- finding communication parameters between different coordinate systems;
- reconciliation of non-linear engineering structures;
- combining digital images;
- equalisation of planned and spatial networks, etc.

The methods of solving optimisation problems are quite diverse and have a branched classification (Kuznetsov A.V., 1994). At present, there is a possibility for widespread implementation of search methods for solving such problems. These methods are very effective because of their advantages, including:
- wide variety of already developed mathematical algorithms;
- possibility of combining these algorithms with each other and with other methods of nonlinear programming;
- easy software implementation;
- independence from the accuracy of the preliminary values of the determined values (you can take values that are far from true, and the decision process is not violated);
- need to create an equation of the amendments or the equation of the relationship, to move to a system of normal equations and solve them;
- there is no need even for the first analogue derivatives in the linearisation of the calculation process.

Development of a method of optimisation of a parabola

In the previous publication on this topic (Zubov A.V., Eliseeva N.N., 2017) the algorithm of the simplest search with variable step is considered. It consists of a sequential multiple calculation of the objective function and each time one or more variables are changed in one direction or another until its minimum is reached.

The algorithm was used to solve the problem of approximation of measurements of a circle (Zubov A.V., Eliseeva N.N., 2017), and the following results are obtained: $x = 100.036\ m$, $y = 10.488\,m$, $r = 1.674\,m$, $f_{end} = 0.012436$, $n = 224$ (x, y - the coordinates of the centre of the circle, r - circle radius, f_{end} - the final value of the objective function $[VV] = \min$, n - the number of cycles required to solve the problem).

The answers were tested in MathCAD, the difference of the results did not exceed 0.5 mm (Zubov A.V., Eliseeva N.N., 2017). Thus, the correctness of the algorithm of simple search and the reliability of the results of calculations are confirmed.

The main disadvantage of this method is the large number of iterations. This article describes an algorithm for significantly accelerating the search engine optimisation method, especially at the initial stage of approximation.

Figure 1 shows a graph of the change of the function $[VV]$ from the argument x in some area. It is seen that this is a parabola, i.e. a graph of convex quadratic functions of the form (1):

$$y = a \cdot x^2 + b \cdot x + c, \qquad (1)$$

where x, y - are variables; a, b, c - are given numbers $(a \neq 0)$.

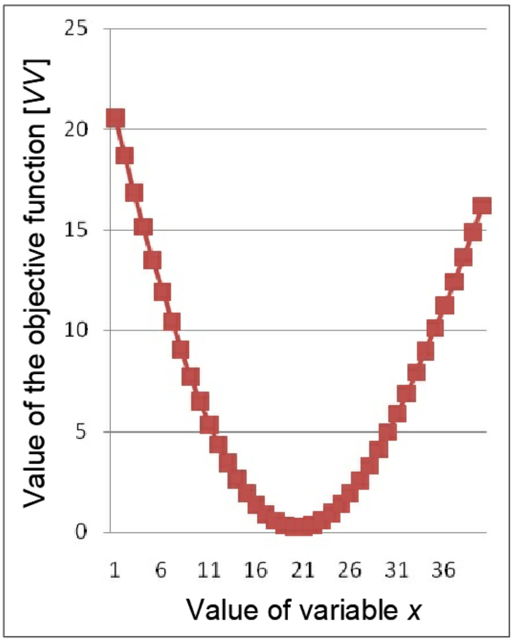

Fig. 1. Graph of the target function change from the variable *x*

The following theory is proposed: it is possible to get to the minimum of the objective function in one «global step» by constructing optimisation parabolas for each variable.

Let's check this theory with the example of an already considered problem of approximation of results of measurements of a circle. In determining the method of the least squares coordinates of the circle's centre (x and y) and its radius r on the coordinates of the points measured on the circle (x_i and y_i), the objective function has the form (2):

$$F(x, y, r) = \sum_{i=1}^{n} \left[\sqrt{(x_i - x)^2 + (y_i - y)^2} - r \right]^2 = \min. \qquad (2)$$

Any function of linear or nonlinear form can be taken as a target function, for example, the sum of the squares of the corrections to the measurement results $[V^2] = \min$ or the sum of the correction modules $[|V|] = \min$. It is important that it is a reliable criterion of efficiency in solving the optimisation problem.

Let's set the initial value of the parameter x_0 and a small enough step to change this parameter k. Arguments y and r remain unchanged. Calculate three values of the objective function $F(x_{-1}, y, r)$, $F(x_0, y, r)$, $F(x_{+1}, y, r)$ for the arguments $x_{-1} = x_0 - k$, x_0 and $x_{+1} = x_0 + k$. The optimisation parabola for the parameter x is shown in Figure 2.

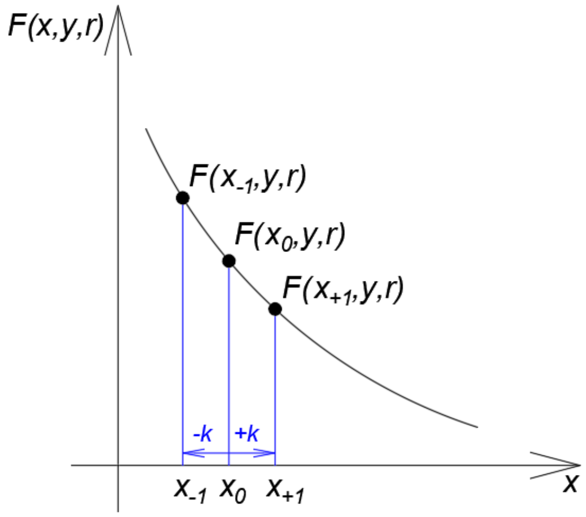

Fig. 2. Fragment optimisation of the parabola for *x*

The conclusion of the «global step» will begin with the compilation of a system of three equations:

$$\begin{cases} a \cdot x_{-1}^{2} + b \cdot x_{-1} + c = F_{-1}; \\ a \cdot x_{0}^{2} + b \cdot x_{0} + c = F_{0}; \\ a \cdot x_{+1}^{2} + b \cdot x_{+1} + c = F_{+1}. \end{cases}$$

Write down the determinants:

$$\Delta = \begin{vmatrix} x_{-1}^{2} & x_{-1} & 1 \\ x_{0}^{2} & x_{0} & 1 \\ x_{+1}^{2} & x_{+1} & 1 \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} F_{-1} & x_{-1} & 1 \\ F_{0} & x_{0} & 1 \\ F_{+1} & x_{+1} & 1 \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} x_{-1}^{2} & F_{-1} & 1 \\ x_{0}^{2} & F_{0} & 1 \\ x_{+1}^{2} & F_{+1} & 1 \end{vmatrix},$$

$$a = \frac{\Delta_1}{\Delta}, \quad b = \frac{\Delta_2}{\Delta}.$$

The minimum of the parabola is determined by the derivative (3), equating it to zero:

$$F'(x) = \lim_{\Delta x \to 0} \frac{\Delta F}{\Delta x} = 2 \cdot a \cdot X_{optimal} + b = 0, \tag{3}$$

where $X_{optimal}$ - value of the parameter x, which falls within the region of the parabola minimum.

From the last equation follows (4):

$$X_{optimal} = -\frac{b}{2 \cdot a} = -\frac{\Delta_2 \cdot \Delta}{\Delta \cdot 2 \cdot \Delta_1} = -\frac{\Delta_2}{2 \cdot \Delta_1}. \tag{4}$$

We introduce the value of h and write down the equation (5):

$$x_0 + h = X_{optimal}, \tag{5}$$

where h - the "big" (or "global") step for parameter x.

Thus, on the basis of equations (4) and (5) after the transformations, the formula for calculating the «global» step is derived (6):

$$h = \frac{k \cdot (F_{-1} - F_{+1})}{2 \cdot (F_{-1} - 2 \cdot F_0 + F_{+1})}. \tag{6}$$

Then "global" steps for other variables are calculated. Figure 3 shows the parabola of the optimisation parameter x.

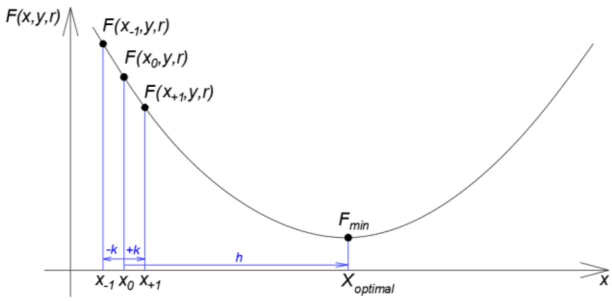

Fig. 3. Optimisation parabola in the parameter *x*

The developed algorithm was implemented in the Visual Basic for Applications programme and was called «Optimisation parabola method».

Thus, acting on the principle «from simple to complex», the transition from the method of simple search with a variable step to the method of optimisation parabola was made. The results obtained did not differ from those obtained by other methods, while the developed algorithm is not final and requires detailed study of other more complex practical problems. However, even at the initial stage, the main advantage of the optimisation parabola method is revealed, namely, the approximation to the minimum of the objective function for the first approximation.

Therefore, the developed method can be used at the initial stage of solving optimisation problems, namely to determine the minimum area of the objective function. Table 1 shows the results of solving the problem of approximating the circle by different methods. This algorithm can be used to create a programme for determining the roll of chimneys and cylindrical copra, when reconciling rotary kilns, etc.

Table 1. *Results of solving the problem of approximation by a circle*

The solution method of the optimisation of the parabola			The solution by the method of simple search with variable step	Solution in MathCAD using the Minimise function
Initial parameter values	The parameter values of the **firs**t **approximation**	Final parameter values		
$x_0 = 50\,m$ $y_0 = 5\ m$ $r_0 = 1\ m$	$x = 99.260\,m$ $y = 10.562\ m$ $r = 1.803\ m$	$x = 100.036\,m$ $y = 10.489\,m$ $r = 1.674\ m$	$x = 100.036\,m$ $y = 10.488\,m$ $r = 1.674\ m$	$x = 100.036\,m$ $y = 10.489\,m$ $r = 1.674\ m$
		The final value of the objective function		
		$f = 0.01243$	$f = 0.01244$	$f = 0.01243$

Conclusion

The optimisation parabola method can be applied to solve a wide range of nonlinear surveying problems.

The use of search methods is not limited to the implementation of existing algorithms. Round non-linear surveying and geodetic tasks are quite broad. Production tasks are not of the same type and sometimes they are even unique. Therefore, it is not always possible to solve them «by templates» by traditional methods. In turn, search methods are more convenient for software implementation, and a large number of existing and the development of new search algorithms can adapt them to solve any problems.

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