GRAPHICAL DETERMINATION OF 2D FRAME REACTIONS UNDER THE ACTION OF CONCENTRATED FIXED LOADS AND SLOWLY MOVING EVENLY DISTRIBUTED LOAD

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ABSTRACT. A study of the equilibrium of a 2D structure of the frame type was performed. One of the applied loads (evenly distributed load) moves very slowly along the roadway of the frame. The intermediate joint at point C of the frame is replaced by an N-release. This non-standard constructive solution helps to analyse the change in the bending moment at point C for arbitrary position of the moving load. The study ends with an analysis of the results obtained.

Key words: slowly-moving evenly distributed load with constant intensity, frame, MathCAD

ГРАФИЧНО ОПРЕДЕЛЯНЕ НА РЕАКЦИИТЕ НА 2D РАМКА, ПОД ДЕЙСТВИЕТО НА КОНЦЕНТРИРАНИ НЕПОДВИЖНИ ТОВАРИ И БАВНО ДВИЖЕЩ СЕ РАВНОМЕРНО РАЗПРЕДЕЛЕН ТОВАР

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РЕЗЮМЕ. Проведено е изследване на равновесието на 2D структура от типа рамка. Един от приложените товари (равномерно разпределен товар) се премества много бавно по пътното платно на рамката. Междинната става в т.C на рамката е заменена с N-апарат. Това нестандартно конструктивно решение помага да се анализира промяната на огъващия момент в т.C за произволно положение на движещия се товар. Изследването завършва с анализ на получените резултати.

Ключови думи: подвижен равномерно разпределен товар, рамка, MathCAD

Introduction

The article presents an alternative method for calculating a bridge structure of a frame type, which carries an evenly distributed load moving very slowly along the roadway of the frame.

The classical method for solving these problems requires knowledge related to the influence lines for the external and internal support reactions of the statically determinate structure.

When the calculations are not automated, the variation of the reactions in the different positions of an evenly distributed load requires too much routine and monotonous work, which in turn is a prerequisite for making mistakes.

The shown example is solved with matrix operations in graphical form through the mathematical package MathCAD.

The presented programme can be used to study the same frame using Q-release in the intermediate joint. In addition, in order to optimise the 2D structure, combinations of N-release and Q-release in the external supports are possible as well as changes in its dimensions.

Solution of the problem by MathCAD package

Fig.1 presents the solution for the frame if it supports the shown external load.

The evenly distributed load is mobile. It moves very slowly along the roadway (ED) of the frame. The geometric dimensions and load of the structure are as follows:

$$a = 2\,m;\ b = 5\,m;\ c = 4\,m;\ d = 2\,m;\ e = 6\,m;$$

$$f = 4\,m;\ l = 2{,}8\,m;\ \alpha = \frac{\pi}{3};\ \beta = \frac{\pi}{4};\ P_1 = 20\,kN;$$

$$P_2 = 50\,kN;\ M = 25\,kN.m;\ q = 60\,\frac{kN}{m}.$$

Solution:

Let a coordinate „x” be accounted from the left end of the distributed load (Fig.1).

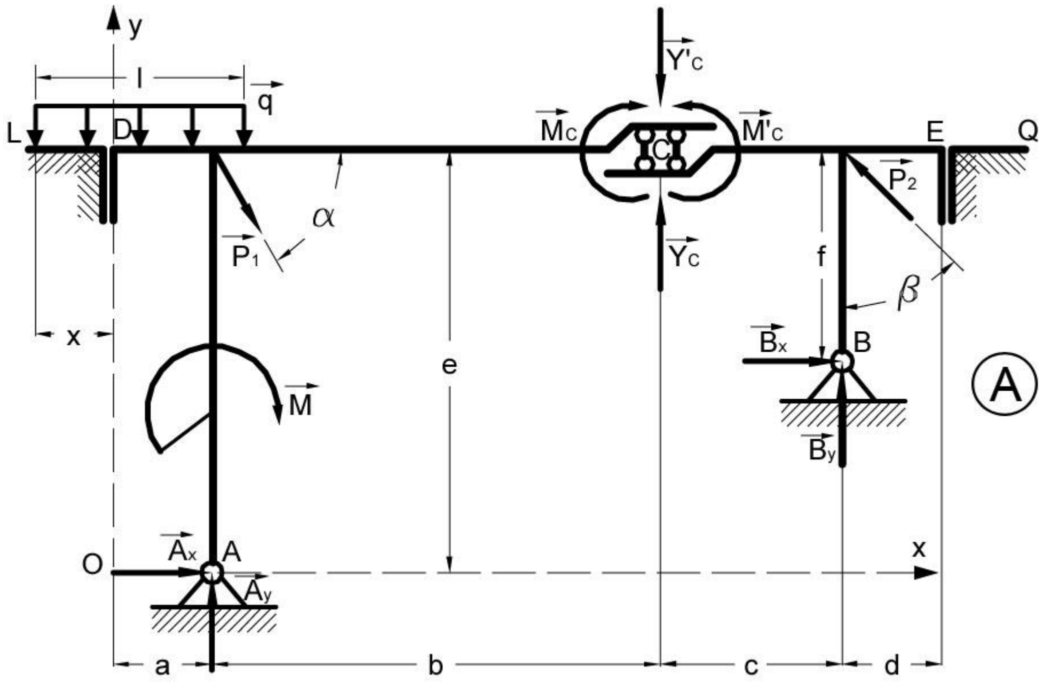

Fig. 1. Calculation scheme. The distributed load as it is moving in the section $-l \le x < 0$ - case A)

We dismember the frame and draw the free-body diagram Fig.1.

The unknown reactions are determined by the six equilibrium equations:

$$\sum M_{Ci} = 0;\ \textit{for a left side}$$
$$A_x.e - A_y.b + P_{1y}.b - M +$$
$$+q.(l+x).[a+b-0{,}5.(x+l)] = 0; \tag{1}$$

$$\sum M_{Bi} = 0;\ \textit{for all construction}$$
$$A_x.(e-f) - A_y.(b+c) - M - P_{1x}.f +$$
$$+P_{1y}.(b+c) + q.(l+x).[a+b+c-0{,}5.(x+l)] +$$
$$+P_{2x}.f = 0; \tag{2}$$

$$\sum M_{C'i} = 0;\ \textit{for a right side}$$
$$B_x.f + B_y.c + P_{2y}.c + M'_C = 0; \tag{3}$$

$$\sum M_{Ai} = 0;\ \textit{for all construction}$$
$$-B_x.(e-f) + B_y.(b+c) + P_{2y}.(b+c) +$$
$$+P_{2x}.e - P_{1x}.e - M +$$
$$+q.(l+x).[a-0{,}5.(l+x)] = 0; \tag{4}$$

$$\sum P_{yi} = 0;\ \textit{for a left side}$$
$$A_y + Y_C - P_{1y} - q.(l+x) = 0; \tag{5}$$

$$\sum M_{Ai} = 0;\ \textit{for a left side}$$
$$Y_C.b - M_C - P_{1x}.e - M +$$
$$+q.(l+x).[a-0{,}5.(l+x)] = 0. \tag{6}$$

These equations refer to the case shown on Fig.1 and are in effect for $x \in [-l, 0)$.

For the cases in Fig.2 other equilibrium equations are valid. They express the dependence between reactions and the position of the distributed load on the members of the structure.

Hence, it becomes clear that the solution of such a problem "by hand" is a difficult and monotonous process.

Generally, each system from linear equations can be presented in a matrix form –

$$A.R = P_i(x) \tag{7}$$

Where:

- A - a square matrix from the coefficients in front of the unknowns;
- $P_i(x)$ - a vector from the free members of the system (the known values move to the right parts of the equations);
- R - The vector with the unknowns.

In this case it is convenient to use any of the following software applications: Matlab, MathCAD or Maple (Doev et al. 2016; Bertyaev 2005; Stoyanov 2016; Ivanov 2011, 2017). Here the problem is solved with the MathCAD package.

The solution of the problem is demonstrated on Figures 3 and 4.

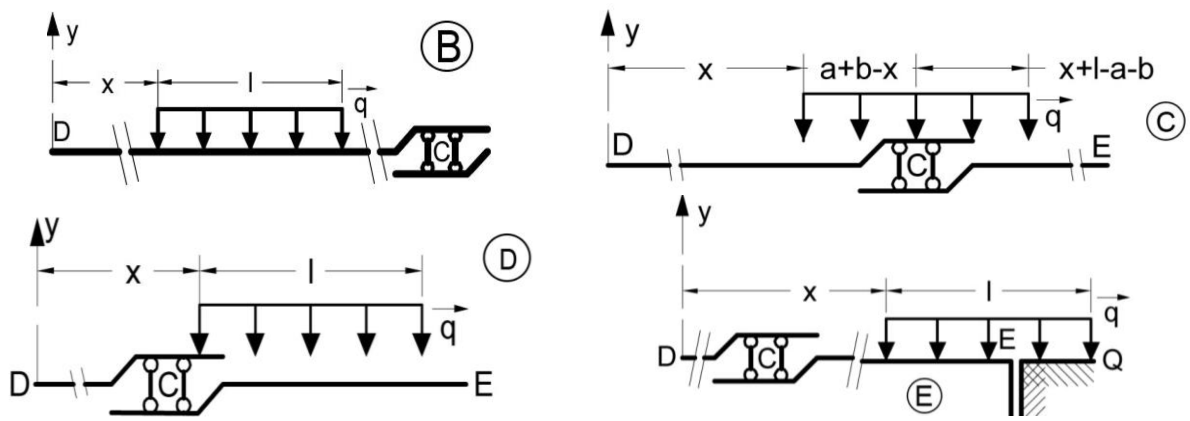

Fig. 2. A movement of the distributed load on the roadway of a structure: case B) - $x \in [0, a+b-l]$; **case C) -** $x \in (a+b-l, a+b)$; **case D) -** $x \in [a+b, a+b+c+d-l]$; **case E) -** $x \in (a+b+c+d-l, a+b+c+d)$;

$$a := 2 \quad b := 5 \quad c := 4 \quad d := 2 \quad e := 6 \quad f := 4 \quad l := 2.8$$

$$\alpha := \frac{\pi}{3} \quad \beta := \frac{\pi}{4} \quad P1 := 20 \quad P2 := 50 \quad M := 25 \quad q := 60$$

$$A := \begin{bmatrix} e & -b & 0 & 0 & 0 & -1 \\ e-f & -(b+c) & 0 & 0 & 0 & 0 \\ 0 & 0 & f & c & 0 & 1 \\ 0 & 0 & -(e-f) & (b+c) & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & b & -1 \end{bmatrix} \qquad \begin{array}{l} P1x := P1\cdot\cos(\alpha) \\ P1y := P1\cdot\sin(\alpha) \\ P2x := P2\cdot\sin(\beta) \\ P2y := P2\cdot\cos(\beta) \\ |A| = 2.376\times 10^{3} \end{array}$$

$$R1(x,l) := q\cdot(x+l) \quad k1(x,l) := [a+b-.5\cdot(x+l)] \quad k2(x,l) := [a+b+c-.5\cdot(x+l)]$$

$$k3(x,l) := [a-.5\cdot(l+x)]$$

$$R2(l) := q\cdot l \quad k4(x,l) := (a+b-x-.5\cdot l) \quad k5(x,l) := (a+b+c-x-.5\cdot l)$$

$$k6(x,l) := (a-x-.5\cdot l)$$

$$R3(x) := q\cdot(a+b-x) \quad k7(x) := (a+b-x)\cdot.5 \quad k9(x) := [a-x-.5\cdot(a+b-x)]$$

$$R4(x,l) := q\cdot(x+l-a-b) \quad k8(x,l) := (x+l-a-b)\cdot.5$$

$$P11(x,l) := \begin{bmatrix} -P1y\cdot b + M - R1(x,l)\cdot k1(x,l) \\ M + P1x\cdot f - P1y\cdot(b+c) - R1(x,l)\cdot k2(x,l) - P2x\cdot f \\ -P2y\cdot c \\ -P2y\cdot(b+c) - P2x\cdot e + P1x\cdot e + M - R1(x,l)\cdot k3(x,l) \\ P1y + R1(x,l) \\ P1x\cdot e + M - R1(x,l)\cdot k3(x,l) \end{bmatrix}$$

$$P12(x,l) := \begin{bmatrix} -P1y\cdot b + M - R2(l)\cdot k4(x,l) \\ M + P1x\cdot f - P1y\cdot(b+c) - P2x\cdot f - R2(l)\cdot k5(x,l) \\ -P2y\cdot c \\ -P2y\cdot(b+c) - P2x\cdot e + P1x\cdot e + M - R2(l)\cdot k6(x,l) \\ P1y + R2(l) \\ P1x\cdot e + M - R2(l)\cdot k6(x,l) \end{bmatrix}$$

Fig. 3. Solution of a frame with the MathCAD package

$$P13(x,l) := \begin{bmatrix} -P1y \cdot b + M - R3(x) \cdot k7(x) \\ M + P1x \cdot f - P1y \cdot (b+c) - P2x \cdot f - R2(l) \cdot k5(x,l) \\ -P2y \cdot c + R4(x,l) \cdot k8(x,l) \\ -P2y \cdot (b+c) - P2x \cdot e + P1x \cdot e + M - R2(l) \cdot k6(x,l) \\ P1y + R3(x) \\ P1xe + M - R3(x) \cdot k9(x) \end{bmatrix}$$

$$P14(x,l) := \begin{bmatrix} -P1 \cdot b + M \\ M + P1x \cdot f - P1y \cdot (b+c) - P2x \cdot f - R2(l) \cdot k5(x,l) \\ -P2y \cdot c + R2(l) \cdot (-k4(x,l)) \\ -P2y \cdot (b+c) - P2x \cdot e + P1x \cdot e + M - R2(l) \cdot k6(x,l) \\ P1y \\ P1x \cdot e + M \end{bmatrix}$$

$R5(x) := q \cdot (a+b+c+d-x) \quad k10(x) := .5 \cdot (a+b+c+d-x) - d$

$k11(x) := -.5 \cdot (a+b+c+d-x) + d + c \quad k12(x) := -.5 \cdot (a+b+c+d-x) + b + d + c$

$$P15(x,l) := \begin{bmatrix} -P1y \cdot b + M \\ P1x \cdot f + M - P1y \cdot (b+c) - P2x \cdot f - R5(x) \cdot k10(x) \\ -P2y \cdot c + R5(x) \cdot k11(x) \\ -P2y \cdot (b+c) - P2x \cdot e + P1x \cdot e + M + R5(x) \cdot k12(x) \\ P1y \\ P1x \cdot e + M \end{bmatrix}$$

$L1(x,l) := A^{-1} \cdot P11(x,l) \quad L2(x,l) := A^{-1} \cdot P12(x,l) \quad L3(x,l) := A^{-1} \cdot P13(x,l)$

$L4(x,l) := A^{-1} \cdot P14(x,l) \quad L5(x,l) := A^{-1} \cdot P15(x,l) \quad x := -2.8, -2.799 .. 13$

$$L(x,l) := \left| \begin{array}{ll} L1(x,l) & \text{if } -l < x < 0 \\ L2(x,l) & \text{if } 0 \le x \le a+b-l \\ L3(x,l) & \text{if } b+a-l < x < b+a \\ L4(x,l) & \text{if } a+b \le x \le a+b+c+d-l \\ L5(x,l) & \text{if } a+b+c+d-l < x < a+b+c+d \end{array} \right.$$

$Ax(x,l) := L(x,l)_0 \quad Ay(x,l) := L(x,l)_1 \quad Bx(x,l) := L(x,l)_2 \quad By(x,l) := L(x,l)_3$

$YC(x,l) := L(x,l)_4 \quad MC(x,l) := L(x,l)_5$

Fig. 4. Continuation of the solution of the frame with the MathCAD package

A similar example is discussed in (Doev et al., 2016).

Analysis of the results obtained

The position of the distributed load does not influence the magnitude of the reactions A_x *and* B_x - see Fig.5.

The module of the support reaction A_y increases to $x = 0$, and after that decreases to $x = 10{,}19\,m$ according to a linear law. The same reaction A_y changes its size according to a quadratic law in the section $10{,}19 \le x \le 13\,m$ and it reaches a minimum module at $x = 11\,m$ Fig.5.

The support reaction B_y changes by module and direction. In a section $-2{,}8 \le x \le 0\,m$ the change of a module is according to a quadratic law, and after that according to linear laws, respectively. In the coordinates $x = 2{,}84\,m$ and $x = 12{,}42\,m$ the direction of the support reaction B_y changes and has a zero value (Fig.5).

The module of the internal support reaction C_y also changes. For a coordinate $x = 4{,}15\,m$ it gets a maximum value, and for $x = 6{,}95\,m$ - respectively minimum. Besides, it changes its direction four times (Fig.5).

The change of the module of the reactive moment at a point C is associated with three local extremums. They appear near $x = -0{,}63\,m$, $x = 5{,}28\,m$ and $x = 11{,}02\,m$ respectively (Fig.5).

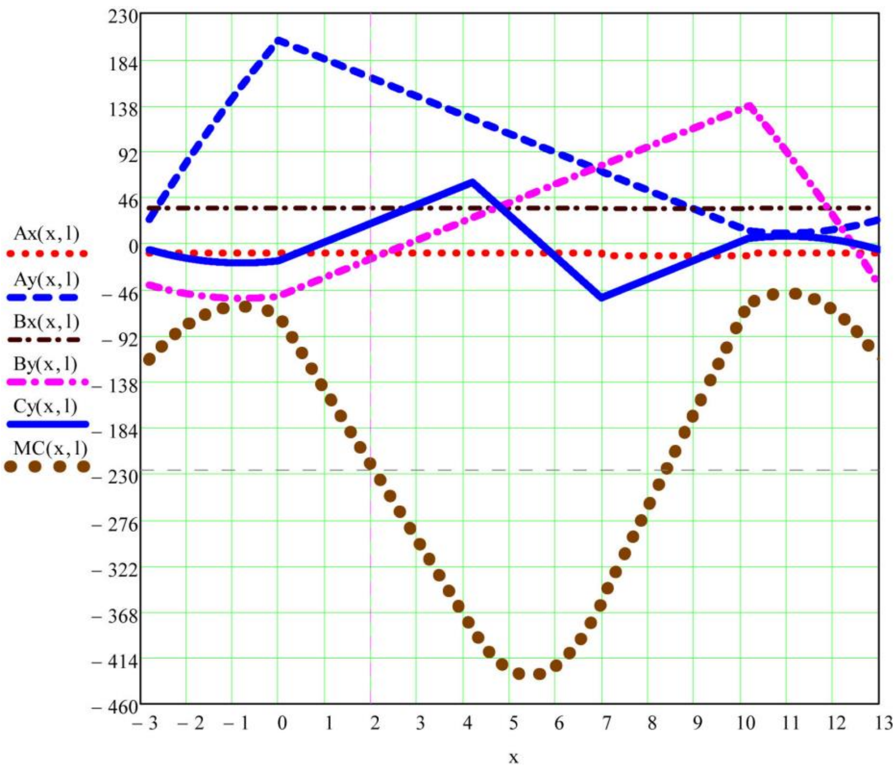

Fig. 5. Amendments of the external and internal reactions depending on the position of the evenly distributed load along the roadway of the frame

Conclusion

The study offers an opportunity to optimise the structure in geometrical attitude, as well as to choose the support reaction devices.

It demonstrates the easy use of MathCAD's graphical interface to analyse the results of the solution.

The shown example can be used in the engineering practice to automate the solution for a wide range of problems. It can be applied in problems related to transport bridges and more specific in the automation of their design (Minin 2013; Hristova et al. 2018).

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