DETERMINATION OF A 3D STRUCTURE'S EXTERNAL REACTIONS UNDER THE ACTION OF APPLIED LOADS WITH COMPLEX CONFIGURATION

Asen Stoyanov

University of Mining and Geology "St. Ivan Rilski", 1700 Sofia; assendragomirov59@gmail.com

ABSTRACT. In this paper a numerical solution is suggested by using the package MathCAD 15. The solution determines the rods' force reactions, which support the homogeneous plate. A homogeneous body, which is loaded unilaterally with wind pressure, is lying on this plate. Sometimes routine and labour-intensive calculations complicate the calculated process. The research shows that the difficulties are due to the complicated spatial configuration of the homogeneous body. They influence both the determination of the body's volume (impossible accurate dual integration of complicated functional dependence $z = f(x,y)$) and the determination of the surface (the "shadow" of the body over the plane Oxz) loaded with wind. The study demonstrates the application of the numerical methods in mechanics and the spatial visualisation of complicated functional dependencies.

Keywords: centre of gravity, 3D structure, surface plot, MathCAD

ОПРЕДЕЛЯНЕ НА ВЪНШНИТЕ РЕАКЦИИ НА 3-D СТРУКТУРА, ПОД ДЕЙСТВИЕТО НА ПРИЛОЖЕНИ ТОВАРИ СЪС СЛОЖНА КОНФИГУРИРАЦИЯ

Асен Стоянов

Минно-геоложки университет "Св. Иван Рилски", 1700 София

РЕЗЮМЕ. Статията предлага числено решение с MathCAD 15. Решението определя пръговите усилия в пръти, които подпират хомогенна плоча. Върху нея лежи хомогенно тяло, натоварено едностранно с вятър. Понякога рутинни и трудоемки пресмятания усложняват изчисленият процес. В показаното изследване, трудностите поизтичат от сложната пространствена конфигурация на хомогенното тяло. Те влияят както за определяне обема (невъзможно точено двойно интегриране на сложна функционална зависимост $z = f(x,y)$) на тялото, така и за определяне повърхнината („сянката" на тялото върху равнината Oxz) натоварена от вятър. Изследването демонстрира приложението на числените методи в механиката, а също и пространствената визуализация на сложни функционални зависимости.

Ключови думи: център на тежестта, 3D структура, повърхностен участък, MathCAD

Calculation of 3D structure through the package MathCAD

The three dimensional structure is shown in Fig.1. It is supported by six rods anchored with ball-and-socket connections at points A, B and C. The rods are loaded with the weights of homogeneous plate and body as well as the wind.

The load from a body is unevenly distributed over the surface of the plate according to the law $z = f(x,y)$ – see Figures 1. 2 and 3. The wind direction is the same as the axis y – see Figures 1 and 2.

The input data for the problem are as follows:

$a = 3{,}5m;\ b = 3m;\ c = 6m;\ w = 3kN/m^2;\ q = 4kN/m^3;$
$g = 6{,}8kN/m^3;\ h = 0{,}2m;$
$z = f(x,y) = y.\sin[(0{,}8.x)^2] + 2 + 0{,}5.y.\sin(2.y)$.

The following symbols are used:

- $a,\ b\ and\ h$ – Dimensions of the plate;
- $a,\ b\ and\ z = f(x,y)$ – Dimensions of the body – see Figures 1 and 2.

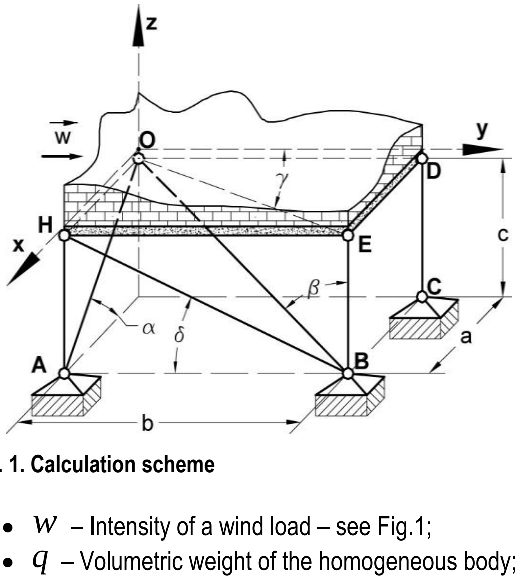

Fig. 1. Calculation scheme

- w – Intensity of a wind load – see Fig.1;
- q – Volumetric weight of the homogeneous body;

- g – Volumetric weight of the homogeneous plate;
- $\vec{e}_p$ – Unit vector in the "p" direction – see Fig.3.

The functional dependence $z = f(x, y)$ is illustrated graphically in Fig.2.

All reactions in the support rods are determined.

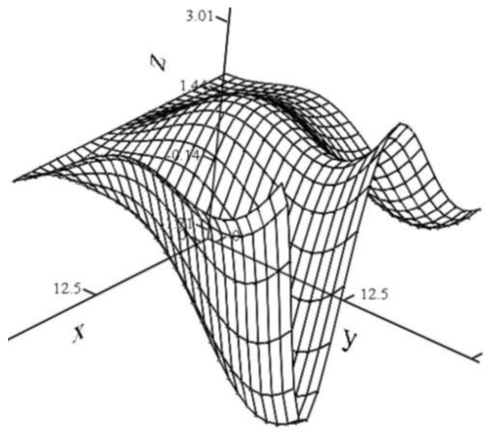

Fig. 2. 3G graphics of the function
$z = y.\sin[(0{,}8.x)^2] + 2 + 0{,}5.y.\sin(2.y)$

A similar problem is published in Doev and Doronin study (2016), but this article complements and expands the solution.

Algorithm of the solution:

1) *Data introduction. The weight $\vec{G}_{pl}$ and the centre of gravity C_{pl} of the plate are determined – see Figures 2 and 3.*
2) *Determination of the body's volume V_b – see Figures 1, 3 and 6;*
3) *Determination of the body's weight $\vec{G}_b$ – see Figures 2 and 3;*
4) *Determination of the body's centre of gravity $C_b\,(x_{cb}\,;\,y_{cb})$ – see Figures 2. and 3;*
5) *Graphic visualisation with MathCAD of the projection (or "shadow") of the body over the coordinate plane Oxz – see Fig.3;*
6) *Determination of the equivalent concentrated load $\vec{P}_w$ over the body, which is subjected to the distributed load from the wind, as well as the centre of gravity of the loaded area – see Figures 2 and 3;*
7) *Determination of the external reactions of the supported plate;*
8) *Checking the solution.*

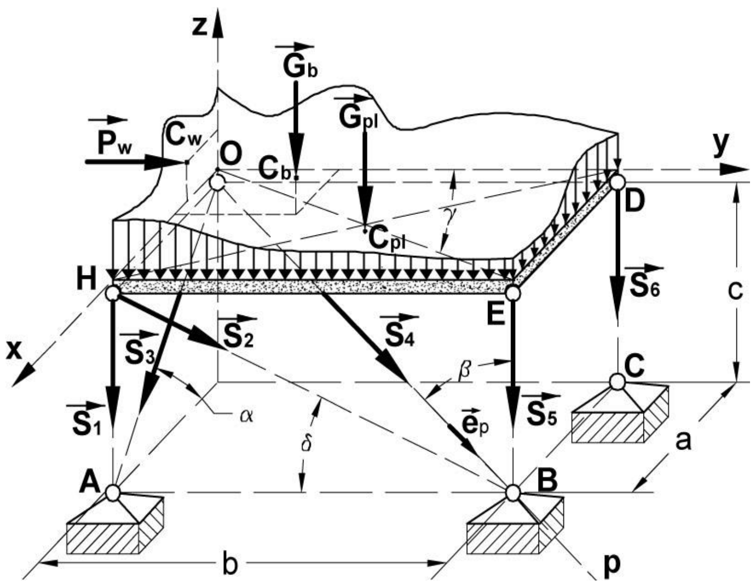

Fig. 3. External reactions of the supported plate loaded with concentrated forces which are equivalent to the distributed loads

The projection of the body (or "shadow") over the plane Oxz is graphically visualised in Fig.4.

The input data, boundaries of amendment, as well as the variable steps, are introduced in Fig.5.

On the same figure, the weights of the plate and body, as well as the gravity body centres and projections over the plane Oxz, are determined.

The determination of the external plate reactions, as well as their verifications, are shown in Fig.6.

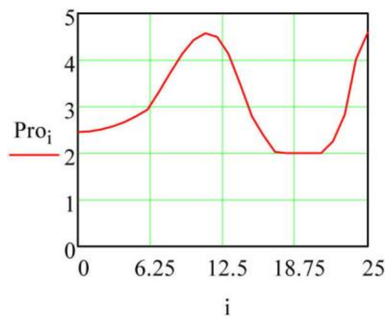

Fig. 4. Projection of the body over the plane Oxz

$$a := 3.5 \quad b := 3 \quad c := 6 \quad w := 3 \quad q := 4 \quad g := 6.8 \quad h := 0.2$$

$$f(x,y) := y \cdot \sin\left[(0.8 \cdot x)^2\right] + 2 + 0.5 \cdot y \cdot \sin(2 \cdot y) \quad Gpl := g \cdot a \cdot b \cdot h \quad N := 25$$

$$dx := \frac{a}{N} \quad dy := \frac{b}{N} \quad i := 0..N \quad j := 0..N \quad x_i := dx \cdot i$$

$$y_j := dy \cdot j \quad M_{i,j} := f(x_i, y_j) \quad Vb := \int_{y_0}^{y_N} \left(\int_{x_0}^{x_N} f(x,y)\, dx \right) dy$$

$$Gb := Vb \cdot q \quad Vb = 21.808 \quad Gb = 87.23 \quad Gpl = 14.28$$

$$xcb := \frac{\int_{y_0}^{y_N} \left(\int_{x_0}^{x_N} f(x,y) \cdot x\, dx \right) dy}{Vb} \quad ycb := \frac{\int_{y_0}^{y_N} \left(\int_{x_0}^{x_N} f(x,y) \cdot y\, dx \right) dy}{Vb}$$

$$xcb = 1.632 \quad ycb = 1.38 \quad i1 := 1..N$$

$$Pro_i := \max\left[\left(M^T\right)^{\langle i \rangle}\right] \quad F_{i1} := Pro_{i1} \cdot \left(x_{i1} - x_{i1-1}\right) \quad Pw := w \cdot \left(\sum_{i1=1}^{N} F_{i1} \right)$$

$$xcw := \frac{\sum_{i1=1}^{N} \left[F_{i1} \cdot \frac{\left(x_{i1} + x_{i1-1}\right)}{2} \right]}{\sum_{i1=1}^{N} F_{i1}} \quad zcw := \frac{\sum_{i1=1}^{N} \left(F_{i1} \cdot \frac{Pro_{i1}}{2} \right)}{\sum_{i1=1}^{N} F_{i1}}$$

$$zcw = 1.674 \quad xcw = 1.736$$

$$s\alpha := \frac{c}{\sqrt{a^2 + c^2}} \quad c\alpha := \frac{a}{\sqrt{a^2 + c^2}} \quad s\beta := \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + c^2}}$$

$$c\beta := \frac{c}{\sqrt{a^2 + b^2 + c^2}} \quad s\gamma := \frac{a}{\sqrt{a^2 + b^2}} \quad c\gamma := \frac{b}{\sqrt{a^2 + b^2}}$$

$$s\delta := \frac{c}{\sqrt{b^2 + c^2}} \quad c\delta := \frac{b}{\sqrt{b^2 + c^2}}$$

$$A := \begin{pmatrix} 0 & 0 & c\alpha & s\beta \cdot s\gamma & 0 & 0 \\ 0 & c\delta & 0 & s\beta \cdot c\gamma & 0 & 0 \\ -1 & -s\delta & -s\alpha & -c\beta & -1 & -1 \\ 0 & 0 & 0 & 0 & -b & -b \\ a & a \cdot s\delta & 0 & 0 & a & 0 \\ 0 & c\delta \cdot a & 0 & 0 & 0 & 0 \end{pmatrix} \quad |A| = -3.283$$

Fig. 5. Determination of the body gravity centre and the wind loaded area centre as well as the analytical expressions of trigonometric functions and matrix A

$$B := \begin{bmatrix} 0 \\ -Pw \\ Gb + Gpl \\ Gpl \cdot .5 \cdot b + Gb \cdot ycb + Pw \cdot zcw \\ -(.5 \cdot a \cdot Gpl + Gb \cdot xcb) \\ -Pw \cdot xcw \end{bmatrix} \quad S := A^{-1} \cdot B \quad S = \begin{pmatrix} -37.06 \\ -35.9 \\ 37.776 \\ -41.149 \\ -7.12 \\ -86.282 \end{pmatrix}$$

$$S1 := \begin{pmatrix} 0 \\ 0 \\ 37.06 \end{pmatrix} \quad S2 := \begin{pmatrix} 0 \\ -35.9 \cdot c\delta \\ 35.9 \cdot s\delta \end{pmatrix} \quad S3 := \begin{pmatrix} 37.776 \cdot c\alpha \\ 0 \\ -37.776 \cdot s\alpha \end{pmatrix}$$

$$S4 := \begin{pmatrix} -41.149 \cdot s\gamma \cdot s\beta \\ -41.149 \cdot c\gamma \cdot s\beta \\ 41.149 \cdot c\beta \end{pmatrix} \quad S5 := \begin{pmatrix} 0 \\ 0 \\ 7.12 \end{pmatrix} \quad S6 := \begin{pmatrix} 0 \\ 0 \\ 86.282 \end{pmatrix}$$

$$Gpl := \begin{pmatrix} 0 \\ 0 \\ -14.28 \end{pmatrix} \quad Gb := \begin{pmatrix} 0 \\ 0 \\ -148.291 \end{pmatrix} \quad Pw := \begin{pmatrix} 0 \\ 32.37 \\ 0 \end{pmatrix}$$

$$ep := \begin{pmatrix} s\gamma \cdot s\beta \\ c\gamma \cdot s\beta \\ -c\beta \end{pmatrix} \quad ep = \begin{pmatrix} 0.463 \\ 0.396 \\ -0.793 \end{pmatrix}$$

$$ep \cdot (S1 + S2 + S3 + S4 + S5 + S6 + Gpl + Gb + Pw) = -1.153 \times 10^{-3}$$

Fig. 6. Determining the external reactions and their verification

Conclusion

The contemporary teaching of mechanics in the universities is related to using the most advanced mathematical packages such as MatLab, MathCAD, MuPAD and others, (Bertyaev, 2005; Doev et al., 2016; Ivanov, 2015; Stoyanov, 2016). In this case, the presented study is a continuation of the ideas published in the works (Ivanov et al., 2017; Stoyanov, 2017), where all calculations are made by using mathematical packages.

The traditional solution of the problem, solved in this paper, is related to some difficulties. The volume of the body V_b cannot be defined directly. In this case, it is necessary a function of two variables to be numerically integrated. But this action is associated with a large volume of calculations when it is executed by hand. The problems associated with determining the body shadow area on the plane Oxz are similar.

These difficulties are easily overcome by using the mathematical package MathCAD. In this case, the programme MathCAD can be used to check the problem solved in the traditional way.

We have a similar problem in the determination of the stability of excavators during their operation when a large piece of ore is loaded and the centre of gravity of the bucket has to be found (Minin, 2013).

The paper demonstrates a fast solution and excellent graphical visualisation - see (Bertyaev, 2005; Doev et al., 2016; Ivanov, 2017; Stoyanov, 2017).

References

Bertyaev, V. 2005, *Teoreticheskaya mehanika na baze MathCAD praktikum.* BHV-Peterburg, Sankt-Peterburg, 739 p. (in Russian)

Doev, V. S., F. A. Doronin, 2016. *Sbornik zadaniy po teoreticheskoy mehanike na baze MathCAD*, Lany, Sankt-Peterburg, Moscow, Krasnodar, 585 (in Russian).

Ivanov, A. I. 2015. Svobodni nezatihvashti prostranstveni trepteniya na nesimetrichno tvardo tyalo vurhu elastichna osnova. – *Sbornik nauchni trudove na RU Angel Kunchev, 54,* 2, 38–42 (in Bulgarian with English abstract).

Ivanov, A. I., P. C. Petkov. 2017. Izsledvane prinudenite prostranstveni trepteniya na mashinen agregat “Elektrodvigatel-kompresor”. – *Mehanika, Transport, Komunikacii, 15, 3, №1484*, VI-7 – VI-12 (in Bulgarian with English abstract).

Minin, I. 2013. Metodika za opredelyane koeficienta na ustoychivost na hidravlichnite ednokofovi bageri po vreme na rabota, – *Godishnik na MGU ”St. Ivan Rilski”, 56*, 3, 16–20 (in Bulgarian with English abstract).

Stoyanov, A. 2016. *Matrichni operacii s MathCAD v teoretichnata mehanika Statika*, Kineziologiya BG, Sofia, 165 p. (in Bulgarian)

Stoyanov, A. 2017. Survey of the relationship between the efforts in the bars and geometrical parameter on tree-dimensional truss. – *XVII International scientific conference VSU’2017 8-9 June*, Sofia, 103–111 (in Bulgarian with English abstract).