ON THE DISTORTED CONTOUR OF AN ELLIPTICAL OPENING PASSING A ROCK MASS WITH A HORIZONTAL PLANE OF ISOTROPY

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ABSTRACT. The question of determining the displacements in transversal isotropic rock mass is discussed in the article. The plane of isotropy is horizontal. The specified class of tasks is solved by the complex variable theory. According to this method, it is necessary to solve two tasks for determining the displacements inthe environment under consideration. In the first task the rock is undistorted, but in the second task it is expressed after advancing the opening. The displacements in the first task are known and those inthe second task are expressed by two complex potential functions of stresses. These functions and the displacements themselves in involved form at points of the surrounding rock mass around the opening have been identified in a previous work by the author. Displacements' values on a contour of elliptical opening for a real transverse isotropic rock mass have been obtained. The deformed contour of the opening is depicted through them. The presented solution complements the idea of changing the type of contour.

Keywords: elliptical opening, transversal isotropic rock mass, displacements

ВЪРХУ ДЕФОРМИРАНИЯ КОНТУР НА ЕЛИПТИЧНА ИЗРАБОТКА, ПРЕМИНАВАЩА МАСИВ С ХОРИЗОНТАЛНА РАВНИНА НА ИЗОТРОПИЯ

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РЕЗЮМЕ. В статията се разглежда въпросът за определяне на преместванията в трансверзалноизотропен масив около елиптична изработка. Равнината на изотропия е хоризонтална. Указаният клас задачи се решава с теория на функцията на комплексна променлива. Според него е необходимо да се решат две задачи за определяне на преместванията в разглежданата среда. В първата задача скалата е ненарушена, а във втората задача тя се разглежда след прокопаване на отвора. Преместванията в първата задача са известни, а тези от втората задача се изразяват чрез де комплексни потенциални функции на напреженията. Тези функции и самите премествания в явен вид в точки от околната скална маса около отвора са определени в предишна работа на автора. За реален трансверзалноизотропен масив са получени стойностите на преместванията по контура на елиптична изработка. Чрез тях е изобразен деформирания контур на изработката. Представеното решение допълва представата за промяната на вида на контура.

Ключови думи: елиптична изработка, трансверзалноизотропен масив, премествания

Introduction

In order to examine the displacements of the rock mass in underground mining opening it is necessary to periodically determine the mutual position of individual points of its profile and compare with their original location in ground. Study and tracking of displacements is done by organising and conducting instrumental measurements over a fixed period of time according to the observed process of deformation. Special observing stations are used for this purpose. The geodesic and mine surveying methodshave an important role in the study (Tzonkov et al., 2018a; 2018b). Specific software packages, software products of a more general nature and separate programmes are used to process measurement data.

In addition to these methods, classical methods of the elasticity theory are known. In these methods the solution depends on an appropriate function of stress. This function in (Nikolaev et al., 2016) is presented in an infinite ascending order. The function in (Muskhelishvili, 1963) is a sum of two functions of a complex variable (England, 1971; Denis, 2011).These functions have been output for a single round hole passing through an isotropic rock mass (Sahoo et al., 2014; Savin, 1961; Minchev, 1960). This solution is also developed for an elliptical opening (Savin, 1961; Minchev, 1960a; 1960b) and for a square opening(Lei et al., 2001; Guangpu et al., 2015).

When the rock mass has the horizontal plane of isotropy and the opening is elliptic, the expressions of stresses are obtained in (Minchev, 1960b). In this work the expressions of the displacements aren't given. This made it necessary to write the analytical expressions of the complex potential functions (Trifonova-Genova, 2018a) and to determine the expressions of the displacements in involved form(Trifonova-Genova, 2018b). Thus,the deformable contour of the hole for real rock mass can be obtained and this is the purpose of the present paper.

Methods

Formulation of the problem

A horizontal opening in the form of an ellipse is drawn in the rock mass at a sufficiently large depth*H*.The beginning of the coordinate system is selected to bein the centre of the opening. The rock mass is transversally isotropic with a horizontal plane of isotropy. The vertical stress in undistorted rock mass$Q$ is obtained after multiplying the volumetric weight γ and the depth $H$of the opening (fig.1). The coefficient k_1 of the lateral pressure participates in the expression of horizontal stress in same rock mass.

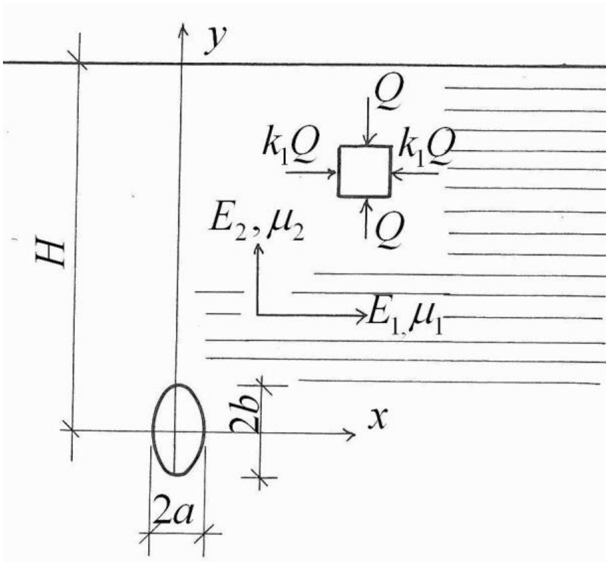

Fig. 1. Horizontal elliptic opening in rock mass with a horizontal plane of isotropy

Expressions of the displacements

To solve the problem the plane elastic complex variable function methodis used.The displacements area sum of the displacements in undistorted rock mass and the displacements as a result of work progress (Trifonova-Genova, 2018a; Minchev,1960a):

$$u_x = u_{xo} + 2\,\mathrm{Re}\left[p_1\Phi_1(z_1) + p_2\Phi_2(z_2)\right];$$
$$u_y = u_{yo} + 2\,\mathrm{Re}\left[q_1\Phi_1(z_1) + q_2\Phi_2(z_2)\right], \qquad (1)$$

where

$$z_l = x + s_l\, y\ ;\ \ l = 1{,}2\,.$$

Here x and y are coordinates of a point of rock mass, u_x is the displacement of the direction x, u_y is the displacement of the direction y, s_1 and s_2 are the roots of characteristic equation of the field (Minchev, 1960a).

The components of the displacements in undistorted field which take part in equation (1) are:

$$u_{xo} = \{k_1C_{11} + C_{12}\}Q\,x\ ; \qquad (2)$$
$$u_{yo} = (k_1C_{12} + C_{22})Q\,y\ .$$

Here u_{xo} is the displacement of the direction x, u_{yo} is the displacement of the direction y.

In expression (2) C_{jl} arethe reduced coefficients of the deformation, whose expressions can be seen in (Trifonova-Genova, 2012; Minchev, 1960a). They are expressed by the physical and the mechanical characteristics of the rock mass. These characteristics are two types: in the direction parallel to the plane of isotropy and in the direction perpendicular to the plane. The first type includes the Young's modulus E_1 and the Poisson's ratio: μ_1. In the second type, the Young's modulus: E_2, the Poisson's ratio μ_2 and the shear modulus G_2 participated.

The two functions Φ_j of equation (1) are complex potential functions. They are analytical in the region S occupied by the elastic material (Fig.1). To transform this region into an outer region of a unit circle the transformation given in (Minchev, 1960a) is used. The limit values of complex potential functions and their expressions in the outer region of the unit circle ς are determined. Then they are replaced in the expressions of the displacements. The latter are a function of the polar angle θ:

$$u_x = 2u_x^{'}\cos\theta;\ u_y = \left(u_{yo}^{'} + 2u_y^{'}\right)\sin\theta\,, \qquad (3)$$

where

$$u_x^{'} = \mathrm{Re}\,\overline{\Phi}_{11} + \mathrm{Re}\,\overline{\Phi}_{21} + \mathrm{Re}\,\overline{\Phi}_{12} + \mathrm{Re}\,\overline{\Phi}_{22};$$
$$u_y^{'} = \mathrm{Im}\,\overline{\Phi}_{32} + \mathrm{Im}\,\overline{\Phi}_{42} - \mathrm{Im}\,\overline{\Phi}_{31} - \mathrm{Im}\,\overline{\Phi}_{41}; \qquad (4)$$
$$u_{yo}^{'} = (k_1C_{12} + C_{22})Q\,b\,.$$

The expressions of $\overline{\Phi}_{jl}$ ($j = 1 \div 4;\ l = 1{,}2$) are given in (Trifonova-Genova, 2018b). The horizontal displacements in the undistorted rock are small and therefore not included in (3).

Numerical example

An opening in the shape of an ellipse hasa width of $2b = 3m$ and a height of: $2a = 4.8m$. It draws at a depth of: $H = 300\,m$. The volumetric weight of the material of the rock is: $\gamma = 0.28.10^{-2}\,MN/m^3$. The vertical stress in situ is: $Q = 15MPa$.

The physical and the mechanical characteristics of the rock mass can be seenin Table 1.

The formulas of the coefficients of deformation can be seen in (Trifonova-Genova, 2018a; Minchev, 1960a). They participate in reduced coefficients C_{jl} of deformation as well as in the coefficients of the lateral pressure (Trifonova-Genova, 2018a). Their values are given in Table 2.

Table 1. *The physical and mechanical characteristics of the rock mass*

E_1	μ_1	E_2	μ_2	G_2
10^3		10^3		10^3
MPa		MPa		MPa
14.5	0.105	41.5	0.3	8.24

Table 2. *Reduced coefficients of deformation* C_{jl}

C_{11}	C_{12}	C_{22}	C_{66}	k_1
10^{-5}	10^{-5}	10^{-5}	10^{-5}	
cm^2/kg	cm^2/kg	cm^2/kg	cm^2/kg	
6.7	-0.94	2.2	12.15	0.34

These coefficients participate in the characteristic equation of the rock mass. The roots of this equation are imaginary numbers: $s_1 = 0.885i$ and $s_2 = 1.9708i$. Here i is an imaginary unit.

The coefficients: $\overline{\Phi}_{jl}$, related to the determination of the complex variable function, can be seen in (Trifonova-Genova, 2018a; 2018b).In the appendix tothe first article the expression of real and imaginary parts of the coefficients, used in equation (1), is given. The values for a concrete rock mass are given in Table 3.

Table 3. *Coefficients* p_j *and* q_j

j	$\operatorname{Re} p_j$	$\operatorname{Im} q_j$
	10^{-5}	10^{-5}
1	-6.193	-3.317
2	-26.963	-2.969

The second appendix in the same article describes the expressions of real and imaginary parts of the coefficients: g_{jl} and G_{jl}. They have thefollow values:

$$\operatorname{Re}[g_{jl}] = \begin{bmatrix} 0.115 & 6.848 \\ 1.886 & -0.416 \\ 0.416 & 0.177 \\ 6.848 & -0.540 \\ 1.962 & 1.421 \\ 10.789 & 1.421 \\ 0.115 & -0.177 \\ 1.886 & 0.540 \end{bmatrix};$$

$$\operatorname{Im}[g_{jl}] = 0; \quad \operatorname{Im}[G_{jl}] = 0; \quad \operatorname{Im}[G_{(j+2)l}] = 0;$$

$$\operatorname{Re}[G_{jl}] = \begin{bmatrix} -0.709 & -184.638 \\ -11.678 & 11.212 \\ -2.576 & -4.759 \\ -42.412 & 14.562 \\ -12.148 & -38.325 \\ -66.824 & -38.324 \\ -0.709 & 4.759 \\ -11.678 & 14.562 \end{bmatrix}.$$

The real and imaginary parts of the coefficients D_j and $\overline{\Phi}_{jl}$ are calculated by an expression, given in the second article. The numerical values of these coefficients can be seen in Table 4.

Table 4. *Coefficients* D_j

j	D_j	j	D_j
	10^{-5}		10^{-5}
1	2.576	5	1.379
2	42.412	6	22.713
3	-38.325	7	-4.220
4	-38.325	8	-4.220

The numerical values of the complex variable functions $\overline{\Phi}_{jl}$ ($j = 1,2$; $l = 1,2$) are:

$$\operatorname{Im}\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} = 0; \quad \operatorname{Re}\begin{bmatrix} \Phi_{31} & \Phi_{32} \\ \Phi_{41} & \Phi_{42} \end{bmatrix} = 0;$$

$$\operatorname{Re}[\Phi_{jl}] = 10^{-5}\begin{bmatrix} -14.414 & -12.824 \\ -266.007 & -30.538 \end{bmatrix};$$

$$\operatorname{Im}[\Phi_{(j+2)l}] = 10^{-5}\begin{bmatrix} 5.457 & -6.868 \\ -29.289 & -3.362 \end{bmatrix}.$$

These functions are involved in expressions for constants: $u_x^{'} = -647.564.10^{-5}$, $u_y^{'} = 27.204.10^{-5}$ and $u_{yo}^{'} = 67.593.10^{-5}$ of equation (4). The components of displacements by contour of the opening are obtained by (3). Their values in 7 points have reference to the vertical stress in undistorted rock mass and the results are given in Table 5.

The part of the final contour of a cross section for a shallow elliptic hole (Fig. 2) is drawn using the values in the table. This part is symmetricaltothe vertical and horizontal axis of the hole. The final contour is given in a continuous line, but the initial contour is in interrupted line.

Table 5. *Results*

point	θ	$\frac{u_x}{Q}$	$\frac{u_y}{Q}$
		10^{-5}	10^{-5}
		$\left[\frac{m^3}{kN}\right]$	$\left[\frac{m^3}{kN}\right]$
1	0	-43.17	0
2	15	-41.70	1.64
3	30	-37.39	3.16
4	45	-30.52	4.47
5	60	-21.59	5.47
6	75	-11.17	6.10
7	90	0	6.32

As can be seen in the figure, the contour is moved. The highest point rises, the low point goes down and the horizontal points move inwards in the hole. Thus, the contour of elliptical workmanship takes a curvilinear look.

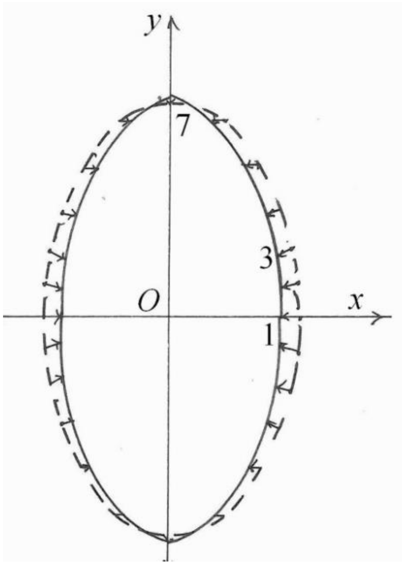

Fig. 2. Initial and final contours of the tunnel cross section for a shallow elliptic tunnel

Conclusion

The components of the displacements after advancing the opening are expressed by two complex potential functions of stresses. These potential functions are expressed by long formulas. This requires the use of modern means for calculating them (computer, Excel, etc.).These means are affordable and have a low price.

The algorithm implemented in the numerical example can be used in future work to determine the displacements in rock mass with an inclined isotropic plane.

The task under consideration can be solved by the numerical method of finite elements. The area around the hole occupied by a continuous elastic and homogeneous environment divides into triangular elements. The nodes of the elements are recorded equations of mechanics of the continuous environments. These equations are differential. Their numeric solution gives us the displacements in the nodes. This is doneby software packages or a separate programme. Such a programmeis given in (Mihaylov and al., 2013), applied in the solving of plane and elastoplastic tasks. They measure the body's stability with the failure criteria of Hock-Brown, Mohr Coulomb andthe modified stability criterion.

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