INFLUENCE OF THE TRANSITION SECTION OF A STEPPED SHAFT ON THE FREQUENCY OF NATURAL OSCILLATIONS

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ABSTRACT. The article examines the frequency of natural oscillations in a stepped shaft. Natural oscillations act on the shaft in the course of operation. An approximate method is used to determine them. According to this method, each section of the shaft is divided into segments. The shaft is presented as a beam on two supports, loaded with concentrated forces, the values of which are equal to the weights of the segments. The application points of the forces are located in the middle of the selected lengths of the segments into which the shaft is divided. The displacements at points of the simple beam are determined by the differential equation of the elastic line. Forces and displacements are involved in the expression of the frequency natural oscillations of the shaft. The method is applied on specific shaft with two sections. The stiffness and weights of each segment of the plots are determined. The support reactions, the bending moments, and the displacements at the application points of the forces are obtained. The proposed solution for determining the frequency of natural oscillations of the stepped shaft has been compared with the solution for a stepped shaft a transitional section.

Key words: stepped shaft, frequency of natural oscillations, approximate method, differential equation of the elastic line.

ВЛИЯНИЕ НА ПРЕХОДНИЯ УЧАСТЪК НА СТЪПАЛЕН ВАЛ ВЪРХУ ЧЕСТОТАТА НА СОБСТВЕНИТЕ ТРЕПТЕНИЯ

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РЕЗЮМЕ. В статията се изследва честотата на собствените трептения в стъпален вал. В процеса на работа върху вала действат собствени трептения. За тяхното определяне е приложен приблизителен метод. Според този метод всеки участък на вала се разделя на сегменти. Валът се представя като греда на две опори, натоварена със съсредоточени сили, чиито стойности са равни на теглата на сегментите. Приложните точки на силите се намират в средите на избраните дължини на сегментите, на които е разделен валът. Преместванията в точки от простата греда се определят чрез диференциално уравнение на еластичната линия. Силите и преместванията участват в израза за честотата на собствените трептения на вала. Методът е приложен върху конкретен стъпален вал с два участъка. Определени са коравините и теглата на всеки сегмент от участъците. Получени са опорните реакции, огъващите моменти и преместванията в приложните точки на силите. Предложеното решение за определяне на честотата на собствените трептения на стъпален вал е сравнено с решението на стъпален вал с преходен участък.

Ключови думи: стъпален вал, честота на собствените трептения, приблизителен метод, диференциално уравнение на еластичната линия.

Introduction

The frequency of natural oscillations in a stepped shaft is determined by an approximate method. Schematically, the shaft is presented as a beam on two supports, loaded with two types of forces. For each iteration, the support reactions, the bending moments, the displacements at the application points of the forces, and the natural frequency of the shaft are determined. This process continues until the accepted accuracy is reached. The method used is applied to a shaft with a transitional section in Trifonova-Genova et al. (2017), and the algorithm is published in Tonkova et al. (2018). Based on the applied method, the current radius and the weights of the segments from the transitional section of the shaft studied are obtained.

Purpose

The purpose of the study is to clarify the influence of the transition section of the stepped shaft on the frequency of natural oscillations.

Exposition

In order to achieve the objective set out in the article, the algorithm of the method used is applied on the stepped shaft without a transitional section. The end result is compared with that published by Tonkova and Trifonova-Genova (2018).

1. Staging the task

Figure 1 shows a stepped shaft with two sections having lengths l_1 and l_2. The total length of the shaft is l. The diameters in both sections are respectively D_1 and D_2.

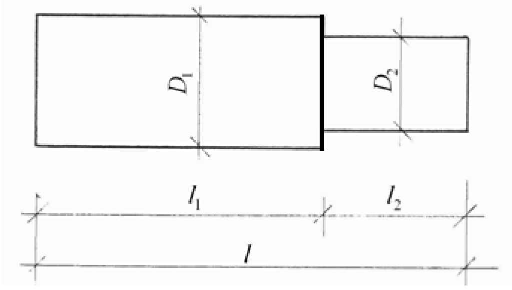

Fig.1. Scheme of the studied shaft

2. Method for determining internal moments

According to the approximate method described in Feodosev (1965), the research shaft is divided into segments. The length of each segment in the first section is Δx_1 , and the length of each segment in the second section is Δx_2 (Fig. 2). The computational scheme is a beam of two supports, loaded with forces.

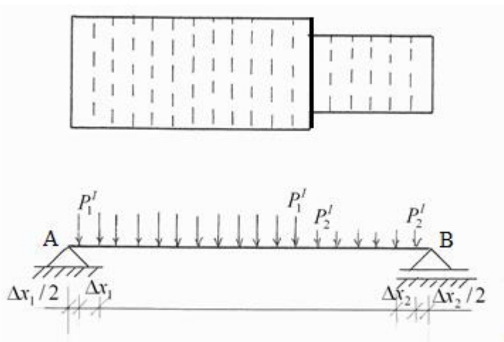

Fig.2. Computational scheme

2.1. Weights and inertia moments of the shaft segments.

The weights of segments and inertia moments to the axis y are determined by the expressions:

$$P_1^I = 0,25\pi D_1^2 \Delta x_1 \gamma \ ;$$
$$P_2^I = 0,25\pi D_2^2 \Delta x_2 \gamma \ ;$$
$$J_{yi} = \frac{\pi D_i^4}{64} \ ;$$
$$i = 1, 2 \ , \qquad (1)$$

where:

γ is the density of the material from which the shaft is made;

i - section number;

D_i - diameters (Fig.1).

2.2. Reactions of supports. To determine the reactions of supports, the shaft studied is presented as a simple beam on two supports A and B (Fig. 2). According to the method used, iterations are made. In the first iteration, which is denoted by I , the beam is loaded by the weights of the segments (1). With the methods of statics presented by Valkov (2014), as well as by Valkov et al. (2013), the reactions in the supports are calculated:

$$A = \frac{1}{l}\left[P_1^m \sum_{i=1}^{n_1} a_{i1} + \sum_{i=1}^{n_2} P_2^m a_{i2} \right];$$
$$m = I, II, III \ldots$$
$$B = \frac{1}{l}\left[P_1^m \sum_{i=1}^{n_1} a_{i3} + \sum_{i=1}^{n_2} P_2^m a_{i4} \right], \qquad (2)$$

where m is the iteration number. The arms of the forces P_1^I and P_2^I (Fig. 3) are defined by the expressions:

$$a_{i1} = \frac{\Delta x_1}{2} + \Delta x_1 (i-1); \quad 0 \le i \le n_1 ;$$
$$a_{i2} = \frac{\Delta x_2}{2} + \Delta x_2 (i-1); \quad 0 \le i \le n_1 ; \qquad (3)$$
$$a_{i3} = \frac{\Delta x_1}{2} + \Delta x_2 (i-1); \quad 0 \le i \le n_2 ;$$
$$a_{i4} = \frac{\Delta x_2}{2} + \Delta x_2 (i-1); \quad 0 \le i \le n_2 ,$$
$$n_1 = l_1 / \Delta x_1 , \ n_2 = l_2 / \Delta x_2 ,$$

where:

n_1 and n_2 are the number of segments in the first and second sections of the shaft;

i is an integer.

2.3. Bending moments.

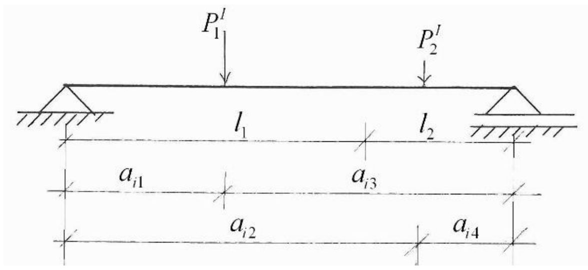

Fig.3. Computational scheme with partial load

The moments in the application points of the forces (Fig. 3) are determined by the methods of resistance of the materials (Kisyov, 1978; Valkov, 2011) and have the appearance:

$$M_i^I = -Ab_{1i} + P_1^I \Delta x_1 S_1 ; \quad 0 \le i \le n_1 ;$$
$$M_i^I = -Bb_{2i} + P_2^I \Delta x_2 S_1 ; \quad 0 \le i \le n_2 , \qquad (4)$$

where:

b_{1i}, b_{2i} are the shoulders of the reactions of supports A и B ;

$\Delta x_1 S_1$, $\Delta x_2 S_1$ are shoulders of forces.

These values are determined by the expressions:

$S_1 = \sum_{j=1}^{i-1} (i-j)$;

$j = 1, 2$;

$k = 1, 2$;

$$b_{jk} = 0,5\Delta x_k + (i-1)\Delta x_k \ . \quad (5)$$

3. Natural oscillations

Natural oscillations occur in the process of shaft operation. The differential equation of elastic line is composed. The displacements at the applicable points of forces are calculated by numerical integration. Two mathematical works of forces and displacements are drawn up and the frequency of natural oscillations for the first iteration is calculated (Kisyov, 1978).

The emerging inertial forces that load the shaft are taken into account for the following iterations (Fig. 2):

The steps in the first iteration are repeated and the frequency of natural oscillations is determined. The iterations are terminated when two similar values for the frequency of natural oscillations are obtained in two consecutive iterations.

4. Numerical example

In order to illustrate and analyse the dependencies mentioned above, a numerical experiment has been conducted. The stepped shaft examined (Fig. 1) is of accepted dimensions and lengths of the segments. Those are given in Table 1.

Table 1. *Shaft dimensions, cm*

Section	l_i	D_i	Δx_i
1	12	5.6	1.0
2	6	4.0	1.0

Young's modulus is $E = 1{,}98.10^4 \kappa g / cm^2$, and the density of the material from which the shaft is made is $\gamma = 7{,}8.10^{-3} kg / cm^3$. The stiffness and weights of the segments are given in Table 2.

Table 2. *Stiffness and forces of the shaft segments*

Section	Points	EJ_y	P_i^I
		10^{+6}	10^{-1}
		kgcm^2	kg
1	1-12	95.536	1.921
2	13-18	24.869	0.980

The defined reactions of supports of the considered beam (Fig.2) are given in table 3, second column. In the third column the reactions of supports of a shaft with transitional section are obtained, published by Tonkova and Trifonova-Genova (2018). A comparison of these values has been made. In the fourth column, the difference between these values is given as percentage.

Table 3. *Reactions of supports, kg*

$i \rightarrow$	1	2	Differences in %
A_i^I	1.634	1.653	1.149
B_i^I	1.259	1.295	2.780

The values of the bending moments in the forces application points for the first iteration are presented in Table 4.

Table 4. *Bending moments, kgcm*

Point	M_i^I	Point	M_i^I
A	0	10	6.88
1	0.816	11	6.59
2	2.26	12	6.11
3	3.51	13	5.45
4	4.57	14	4.68
5	5.43	15	3.82
6	6.10	16	2.85
7	6.59	17	1.79
8	6.87	18	0.63
9	6.97	B	0

Table 5 provides the results of the first iteration. The value of $\omega^I = 18{,}715.10^3 s^{-1}$ is obtained for the frequency of natural oscillations.

Table 5. *Results of the first iteration*

	x_i	w_i^I	$P_i^I w_i^I$	$P_i^I \left(w_i^I\right)^2$
		10^{-6}	10^{-7}	10^{-13}
	cm	cm	kgcm	kgcm^2
A	0	0	0	0
1	0.5	0.29	0.056	0.016
2	1.5	1.40	0.165	0.142
3	2.5	0.86	0.269	0.376
4	3.5	1.89	0.363	0.686
5	4.5	2.32	0.445	1.059
6	5.5	2.68	0.515	1.380
7	6.5	2.98	0.572	1.706
8	7.5	3.20	0.615	1.967
9	8.5	3.36	0.655	2.199
10	9.5	3.34	0.641	2.143
11	10.5	3.45	0.663	2.286
12	11.5	3.40	0.653	2.221
13	12.5	3.13	0.307	0.960
14	13.5	2.67	0.262	0.699
15	14.5	2.05	0.200	0.411
16	15.5	1.32	0.129	0.171
17	16.5	0.52	0.051	0.026
18	17.5	0.30	0.029	0.009
B	18	0	0	0
Σ			6.590	18.457

In the second iteration, the shaft is loaded with the inertia forces P_i^{II}. The first iteration steps are repeated and the results are set out in Table 6. The value of $\omega^{II} = 18{,}176.10^3 s^{-1}$ is obtained for the frequency of natural oscillations of the second iteration; the value differs from the value in the first iteration by 2.88%. Since the resulting difference is permissible by engineering practice, no more iterations have been made.

Table 6. *Results of a second iteration*

	x_i	w_i^{II}	P_i^{II}	$P_i^{II} w_i^{II}$	$P_i^{II} \left(w_i^{II}\right)^2$
		10^{-6}		10^{-7}	10^{-13}
	cm	cm	kg	kgcm	kgcm^2
A	0	0	-	0	0
1	0.5	0.28	0.020	0.006	0.002
2	1.5	0.84	0.059	0.049	0.042
3	2.5	1.36	0.095	0.129	0.175
4	3.5	1.84	0.127	0.234	0.430
5	4.5	2.26	0.157	0.355	0.802
6	5.5	2.63	0.182	0.479	1.259
7	6.5	2.93	0.202	0.592	1.734
8	7.5	3.16	0.217	0.686	2.167
9	8.5	3.31	0.228	0.755	2.498
10	9.5	3.39	0.233	0.790	2.678
11	10.5	3.40	0.234	0.796	2.705
12	11.5	3.34	0.230	0.768	2.566
13	12.5	3.07	0.108	0.332	1.018
14	13.5	2.61	0.092	0.240	0.627
15	14.5	2.00	0.071	0.142	0.284
16	15.5	1.29	0.046	0.059	0.077
17	16.5	0.51	0.018	0.009	0.005
18	17.5	0.29	0.00	0.003	0.003
B	18.0	0	-	0	0
∑				6.422	19.070

Analysis of the result obtained

When comparing the frequency of natural oscillations of the shaft studied (Fig. 1) with the frequency of the natural oscillations of the shaft with a transition section studied by Tonkova and Trifonova-Genova (2018), a difference of 2.81% was obtained.

5. Key findings

Based on the approximate method applied in this article, a numerical example has been solved. The obtained value of the frequency of natural oscillations of a stepped shaft is compared with the value for a shaft with a transition section. A difference has been identified indicating that the impact of the transitional section is small, but it must be taken into account if these sections are more than one. In this case, it is necessary to apply an algorithm for calculating the shaft diameter, stiffness and weight of the segments from the transitional section.

Conclusion

The approach applied to determine the frequency of the natural oscillations of a stepped shaft is of a particular importance for a load retaining the straight-line shaft axis.

The numerical results of this work give an approximate solution when the shaft has two sections. This decision is shorter than the shaft decision with one transitional section.

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