REDUCING THE ELASTIC TORSIONAL MOMENT OF A VIBRATING SHAFT

Gergana Tonkova, Stefan Pulev

University of Mining and Geology “St. Ivan Rilski”, 1700 Sofia; E-mail: gtonkova@mgu.bg, spulev@mgu.bg

ABSTRACT. This work presents the mechano-mathematical modelling of the torsional vibrations of a shaft with two rotating masses with harmonic kinematic disturbance. A model with two degrees of freedom is used. The differential equations describing the vibrations are derived with methods from analytical mechanics. Values are obtained for the periodically changing elastic torsional moments of the sections of the shaft. The possibilities for reducing the elastic torsional moment are studied. A numerical example is provided.

Key words: torsional vibrations, shaft, kinematic disturbance, elastic torsional moment.

НАМАЛЯВАНЕ НА НАТОВАРВАНЕТО ПРИ УСУКВАЩИ ТРЕПТЕНИЯ НА ВАЛ

Гергана Тонкова, Стефан Пулев

Минно-геоложки университет „Св. Иван Рилски“, 1700 София

РЕЗЮМЕ. Разглежда се механо-математичен модел на усукващите трептения на вал с две ротиращи маси при наличие на хармонично кинематично смущение. Моделът е с две степени на свобода. Диференциалните уравнения на трептенията са решени с методите на аналитичната механика. Изведени са изразите за периодично променящите се еластични моменти в участъците на вала. Изследват се възможностите за минимизиране на техните стойности. Проведен е числен експеримент.

Ключови думи: усукващи трептения, вал, кинематично смущение, еластичен усукващ момент.

Introduction

When constructing shafts, strength and deformation calculation is performed, from where we move to their geometric sizing, taking into account the structural and technological features, such as quality, mutual location and features of the constituent surfaces, steps, transitions, channels, holes, etc. All these factors affect the internal stresses, and thereupon they also influence the geometric dimensions of the shaft. Strength calculation includes design calculation and verification, for which it is necessary to know the values of the torsional moments in the sections of the shaft. The data are published by Tonkov et al. (2020).

Purpose

The purpose of this study is to clarify the possibilities for reducing torsional moment and tangential stress during oscillations. Reducing the periodically changing dynamic load allows reduction of the resistance moment and selection of a smaller diameter. This ensures greater reliability and workability of the shaft.

Exposition

Fig. 1 shows the dynamic model for examining torsional oscillations.

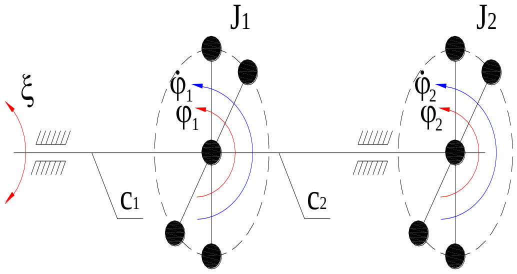

Fig. 1. Mechano-mathematical model of the mechanical system

A shaft with two sections bearing two rotating tables is considered. Their inertia moments are denoted by J_1 and J_2. For generalised coordinates, the spins of the disks φ_1 and φ_2 are chosen relative to the position of the shaft at which it is not twisted. The corresponding generalised speeds of the spins are $\dot{\varphi}_1$ and $\dot{\varphi}_2$. The coefficients of elasticity in both sections are c_1 and c_2. The data are published by Pulev and Tonkova (2017).

The source of the oscillations is the periodically changing kinematic disturbance $\xi = \lambda.\sin\omega t$, where λ and ω are its amplitude and frequency.

After the application of the Lagrange equations of second genus, published by Pisarev et al. (1975) and Babakov (2004), the differential equations for motion of the considered mechanical system are obtained:

$$\begin{vmatrix} J_1\ddot{\varphi}_1 + (c_1 + c_2)\varphi_1 - c_2\varphi_2 = c_1\lambda\sin\omega t \\ J_2\ddot{\varphi}_2 - c_2\varphi_1 + c_2\varphi_2 = 0 \end{vmatrix}, \qquad (1)$$

and the obtained frequency equation has the type:

$$\Delta(k^2) = \begin{vmatrix} c_1 + c_2 - k^2 J_1 & -c_2 \\ -c_2 & c_2 - k^2 J_2 \end{vmatrix} = 0. \qquad (2)$$

The determination of the first and second own frequencies of the mechanical system with two degrees of freedom is given from the roots k_1^2 and k_2^2:

$$k_{1,2}^2 = \frac{1}{2}\left[\frac{c_2}{J_2} + \frac{c_1 + c_2}{J_1} \pm \sqrt{\left(\frac{c_2}{J_2} + \frac{c_1 + c_2}{J_1}\right)^2 - 4\frac{c_1 c_2}{J_1 J_2}}\right] \qquad (3)$$

In engineering practice, the purely forced oscillations of the shaft-masses system are of interest. They arise as a result of the periodic kinematic disturbance. The purely forced oscillations are expressed by the private solution of the equation system (1) as follows:

$$\varphi_1 = D_1 \sin\omega t\ ,\ \ \varphi_2 = D_2 \sin\omega t\ , \qquad (4)$$

where D_1 and D_2 are the unknown amplitudes and the frequency ω coincides with that of the disturbance. After replacement of (4) in the system of differential equations (1) and equating the coefficients in front of $\sin\omega t$ on both sides of the equations, the system of algebraic equations is obtained:

$$\begin{vmatrix} (c_1 + c_2 - J_1\omega^2).D_1 - c_2 D_2 = c_1\lambda \\ -c_2 D_1 + (c_2 - J_2\omega^2)D_2 = 0. \end{vmatrix} \qquad (5)$$

After solving equations (5), the amplitudes of the forced torsional vibrations of the shaft-mass system are obtained, as well as the frequency equation of the considered mechanical system:

$$D_1 = \frac{\Delta_1}{\Delta},\ D_2 = \frac{\Delta_2}{\Delta}, \qquad (6)$$

where:

$$\Delta = J_1 J_2\omega^4 - \left[J_1 c_2 + J_2(c_1 + c_2)\right]\omega^2 + c_1 c_2 = \\ = J_1 J_2(\omega^2 - k_1^2)(\omega^2 - k_2^2) \qquad (7)$$

$$\Delta_1 = \begin{vmatrix} c_1\lambda & -c_2 \\ 0 & c_2 - J_2\omega^2 \end{vmatrix} = c_1\lambda(c_2 - J_2\omega^2) \qquad (8)$$

$$\Delta_2 = \begin{vmatrix} c_1 + c_2 - J_1\omega^2 & c_1\lambda \\ -c_2 & 0 \end{vmatrix} = c_1 c_2\lambda. \qquad (9)$$

After replacing the expressions (6) in the private decision (4), the law on purely forced oscillations has the appearance:

$$\varphi_1 = \frac{c_1\lambda(c_2 - J_2\omega^2)}{J_1 J_2(\omega^2 - k_1^2)(\omega^2 - k_2^2)}\sin\omega t; \\ \varphi_2 = \frac{c_1 c_2\lambda}{J_1 J_2(\omega^2 - k_1^2)(\omega^2 - k_2^2)}\sin\omega t. \qquad (10)$$

With the help of the derived law of forced oscillations (10), the dynamic moments loading the two sections of the shaft are determined. They are given with the expressions:

$$M_1 = c_1(\varphi_1 - \xi) \text{ and } M_2 = c_2(\varphi_2 - \varphi_1)\ . \qquad (11)$$

In order to achieve the objective set out in this article, the approach to minimising the dynamic moment in the first section of the considered mechanical system is applied, since it has the greatest impact on the emerging torsional stresses (Fig. 1).

After substitution of (10) in (11) and converting, the following is obtained:

$$M_1 = \frac{c_1\lambda\omega^2(J_1 c_2 + J_2 c_2 - J_1 J_2\omega^2)}{J_1 J_2(\omega^2 - k_1^2)(\omega^2 - k_2^2)}\sin\omega t \qquad (12)$$

It is possible that the expression in brackets from the numerator of (12) become zero:

$$J_1 c_2 + J_2 c_2 - J_1 J_2\omega^2 = 0\ , \qquad (13)$$

whereby the following is obtained:

$$\frac{J_1 J_2}{J_1 + J_2} = \frac{c_2}{\omega^2} \qquad (14)$$

If the laying is made:

$$J^* = \frac{J_1 J_2}{J_1 + J_2}\ , \qquad (15)$$

then the expression $M_1 = 0$ is equivalent to the condition:

$$J^* = \frac{c_2}{\omega^2} \qquad (16)$$

Based on expression (16) obtained above, J^* can be called the equivalent moment of inertia of the shaft. In particular, when the two moments of inertia are equal, the following is obtained:

$$J_1 = J_2 = 2J* = \frac{2c_2}{\omega^2} \qquad (17)$$

Numerical experiment

In order to illustrate and analyse the above dependencies, a numerical experiment has been conducted. The kinematic scheme of the four-bar linkage mechanism is presented in Fig. 2. The data are published by Tonkova (2017).

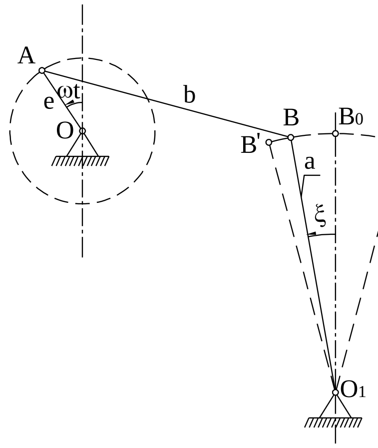

Fig. 2. Kinematic scheme of the four-bar linkage mechanism

The parameters of the oscillating system have the following values:

$$b = 135\ \ mm$$
$$a = 121\ \ mm$$
$$e = 29{,}5\ \ mm$$
$$\lambda = \frac{e}{a} = 0{,}244 \qquad (18)$$
$$c_1 = 5117\ \ Nm$$
$$c_2 = 22594\ \ Nm$$
$$\omega = 70{,}5\ \ s^{-1}$$

For the numerical values given above, and in accordance with (16), the following value for the equivalent moment of inertia is obtained:

$$J^* = 4{,}546\,kg.m^2 \qquad (19)$$

In particular, according to (17), the following result is obtained for the two inertia moments:

$$J_1 = J_2 = 9{,}092\,kg.m^2 \qquad (20)$$

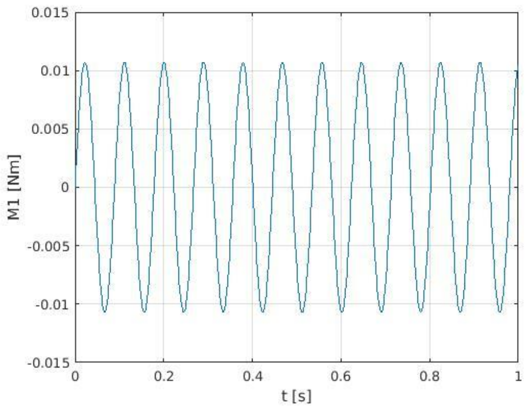

Fig. 3. Amendment of the elastic torsional moment in the first section of the shaft

At these values of the parameters, with the help of the program MATLAB, the graphs for the change of the elastic torsional moments M_1 and M_2 are obtained, which are presented in Figures 3 and 4. From Fig. 3, very small amplitude of the torsional moment can be reported in the first section of the shaft $\max|M_1| = 0{,}011\ N.m$. The maximum value of the torsional moment in the second section of the shaft according to Fig. 4, is $\max|M_2| = 11000\ N.m$. These results of the experiment confirm the reliability of the resulting equations (13) - (17) and prove that the purpose of the study has been achieved.

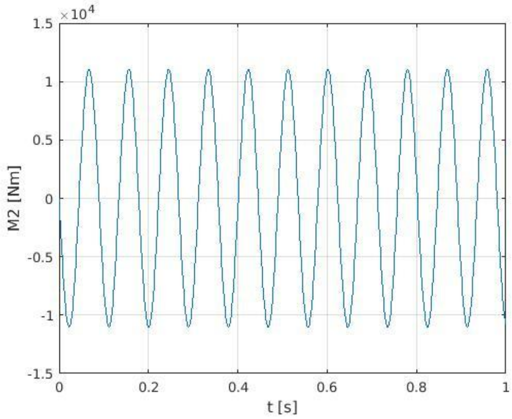

Fig. 4. Amendment of the elastic torsional moment in the second section of the shaft

Conclusion

The torsional oscillations of the investigated shaft, occurring in the process of work, create periodically changing torsional moment and tension, which leads to fatigue of the material.

The analytically derived equations (13) – (17) provide a relationship between the parameters, which provides virtually zero torque value in one of the shaft sections. If these dependencies are observed in the process of constructing, voltage minimisation could be achieved despite the intense vibrations.

The presented method for reducing the load in torsional vibrations can increase the working capacity and life of the shaft. In this way, an increase in the reliability of the machine, part of which is the investigated shaft, is achieved. This is especially important in responsible cases, in precise technologies and heavily loaded machines, such as in mine mechanisation.

References

Babakov, I. M. 2004. Teoria kolebaniy. Drofa, Moskva, 591str. (in Russian).

Pisarev A., Ts. Paraskov, S. Bachvarov. 1975. Kurs po Teoretichna mehanika - chast II. Tehnika, Sofia, 506 str. (in Bulgarian).

Pulev S., G. Tonkova. 2017. Torsional vibrations of shaft in kinematic disturbed by a four-bar linkage mechanism, *Journal of Industrial Technologies*, IV (1), 75-79 (in Bulgarian with English abstract).

Tonkov G., D. Ralev, A. Hinkov. 2020. Posobie za proektirane na mashinni elementi. Propeler, Sofia, 152 str. (in Bulgarian).

Tonkova G. 2017. Izsledvane na vliyanieto na ogavashti i usukvashti treptenia na valove, izpolzvani v tezhkata minna mehanizatsia. Doktorska disertatsia, MGU “Sv. Ivan Rilski“. Sofia, 142 str. (in Bulgarian).