ANALYSIS OF PARAMETER DEPENDENCY IN MATHEMATICAL MODELING OF OPEN-PIT MINING

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ABSTRACT. Different processes in open-pit mining can be optimised by applying various mathematical models. Due to the large number of elements determining an open pit, and the complexities of the possible unknown links between these elements, it is difficult to estimate the dependency of the parameters in many processes. Such estimation analysis can be very useful in understanding the operational mechanisms of the major and minor factors to a given process, as well as to serve as a basis for optimisation.
This paper suggests a systematic analysis for determining the dependency of the parameters among some of the mathematical models most frequently used in open-pit mining

Key words: mathematical modelling, open-pit mining.

АНАЛИЗ НА ЗАВИСИМОСТТА НА ПАРАМЕТРИТЕ ПРИ МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ В ОТКРИТО РАЗРАБОТВАНЕ

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РЕЗЮМЕ. Различните процеси в откритиия добив могат да бъдат оптимизирани чрез прилагане на различни по точност и сложност математически модели. Поради големия брой на елементите, които са част от система открит рудник, и сложността на възможните неизвестни връзки между тези елементи, е трудно да се оцени зависимостта на параметрите в много процеси. Такива анализи за оценка могат да бъдат много полезни за разбиране на механизмите на действие на основните и второстепенни фактори за даден процес и да подобрят възможностите за оптимизация.
Тази статия предлага систематичен анализ за определяне на зависимостта на параметрите сред някои от най-използваните математически модели при открит добив.

Ключови думи: математическо моделиране, открито разработване

Introduction

The variety of problems referring to the open-pit mining determines a complex set of solutions, including those which are related to the mathematical optimisation methods. Bench height, pit slopes, boundaries of the different areas, processing sequence, truck scheduling problem, etc. are some of the main factors subject to different kinds of optimisation. On the other hand, each of the listed factors (F) depends on various numbers of parameters (f_j,) which are elements of the open mine system.

$$F_i = F_i(f_1, f_2, ..., f_m). \qquad (1)$$

where:
in F_i, i = 1, 2...,n are the numbers of the different factors and in f_j, $j = 1,2…,m$ are the numbers of the parameters.

For example, optimum bench height can be constructed if the maximum cutting height and the capacity of the loading machines, the capacity of the drilling machines, rock properties, geological characteristics, etc. are considered. In their work, Soltanmohammadi et al. (2010) have analysed bench height optimisation criteria divided in terms of two main aspects: economical (production scheduling, dilution, operating and capital costs) and technical (practicability, safety, and equipment).

It is known that the design of the pit slopes (slope angle) strongly depends on the factor of safety and the probability of failure. The economic impacts of potential slope failures have been suggested by Contreras (2015) as yet another factor that can be implemented to design pit slopes.

Many researchers have been focused to determine the optimum boundaries of the different areas in an open pit: from the graph theory proposed by Lerchs and Grossmann (1965), through the floating cone algorithm based on the heuristic theory (Berlanga et al., 1989), to the uncertainty representation method suggest by Beak et al. (2016) using the variation in mineral prices. All these different approaches show the parameters diversity in the open-pit mining.

The operational planning and the processing sequence mostly depend on the economic parameters; and the known optimisation methods for extraction sequence consider this fact. Indeed, one of the most used factors here is the Net Present Value (NPV) analysis, which is related to the estimation of cash inflows and outflows over the life of a project (Elevli, 1995). Using the dynamic cut-off grade implemented in binary integer programming, Moosavia and Gholamnejad (2016) show that NVP can be dynamically optimised during different stages of mining.

Controversially, the truck scheduling problem mostly involves the technical capacity of an open pit. This includes the number of available truck, loading and unloading capability, and dump selection. A common approach in optimising this problem is a mixed integer programming model considering transport revenue varied with different loading points (Chang et al., 2015).

Having summarised these problems and their possible solutions, we can determine the dependency of the different parameters to the unified open pit system.

Determination of the parameters

An open pit is a complex system with an enormous amount of elements and relationships among them, which can be considered as parameters for different processes. The description of all parameters is impossible, but determining those which are essential in the modelling of mining and processing sites is a necessity.

Different weight dependencies can be identified and quantified through the statistical analysis and the correlation among parameters and the measured results and specification.

The parameter dependency changes (PDC) often occur during development processes, especially due to unpredictable reasons, such as economic changes, customer requirements, weather changes, or other nonlinear factors. In this sense, parameter determination is somehow crucial for the implementation of the model optimisation. Nevertheless, most of the existing methods are not applicable when PDC are significant. The schematic view of decision making and models optimisation is shown in Figure 1.

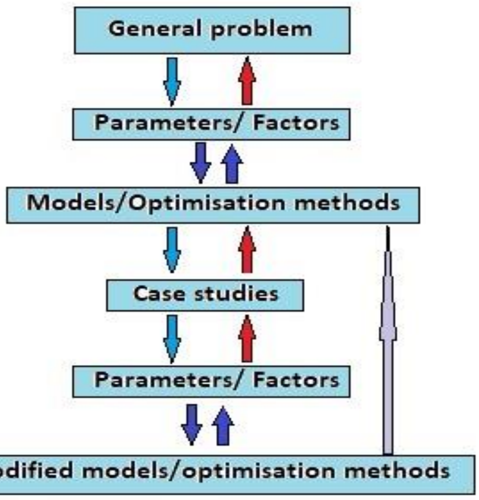

Fig. 1. Flow chart of the decision making and models optimisation

Here, the parameter dependency is summarised and how models can be optimised taking into account different case studies. The term “General problem” refers to all possible optimising problems in an open pit, such as pit design optimisation, open pit scheduling problem, uncertainty, risk, etc. Identifying the most important parameters and factors that affect every aspect concerning open-pit mining is a decision making process. Taking into account accuracy, flexibility, and applicability of the solution methods (for example a mathematical model), the numbers of the parameters and factors can vary significantly. Subsequently, another process decision could be taken, based on the model results. This is an example of how the reverse flow from a non-real object, such as a mathematical object (model or methods), can affect the ongoing process. In other words, solution methods by themselves can be considered as factors determining the general problem. With the development of the computer methods and software products (Mariko et al., 2018; Rimele et al., 2020; Rakhmangulov et al., 2021), the weight of the solution method as a factor in open-pit mining has become more significant. In this sense, equation (1) can be rewritten as follows:

$$F_i = F_i(f_1, ..., f_{m-n}, q_{m-n+1}, ..., q_{m-1}, s), \qquad (2)$$

where $f_1, f_2, ..., f_{m-n}$ are the parameters that characterise the system of the open pit, such as geophysical, geological, and technical. The parameters $q_{m-n+1}, q_{m-n+2}, ..., q_{m-1}$ refer to economic factors, customer requirements, weather changes, human factors, etc. The parameter *s* is related to the solution method. The total number of the parameters describing any given problem or situation is *m*.

Having the explicit type of a model that solves a given general problem in open-pit mining, it can be used to some extent in a specific situation, or the so called case studies. Every single case study is characterised by additional parameters different from those determining other cases. Actually, we can consider different case studies in terms of PDC. In this sense, equation (2) transforms into:

$$F_i = AF_i(f_1, ..., f_{m-n}, q_{m-n+1}, ..., q_{m-1}, s), \qquad (3)$$

where *A* is a weight matrix.

In general, this matrix is in the form:

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{pmatrix} \qquad (4)$$

Determining the coefficients depends on every single case study, and if the problem is similar to the general description, then A is close to the unit matrix.

The difference between the parameters used to describe the general problem and those which characterise a specific situation and unique approach is embedded in the weight matrix *A*. Thus, the modified models or optimisation methods with comparison to the “basic models” (the models that describe the general problem) use not different parameters but different weight coefficients. Moreover, in many cases the modified models improve the initial models.

Table 1. *Models used and parameters involved for different problems in open-pit mining*

Problem	Model	Parameters
Open-Pit Design Problem	Meagher et al., 2014	Numerical indicators – number of ore and waste blocks; Economic value of blocks;
Pit slopes (slope angle optimisation)	Contreras, 2015	Probability of slope failure; Volume of excavated mass; Economic impact of slope failure;
Optimum boundaries	Beak et al., 2016	Numerical indicators – number of ore and waste blocks; Mass of ore block (tonne), the grade and recovery of ore;
Operational planning and the processing sequence	Mousavi et al., 2016	Numerical indicators – number of blocks; Technical parameters - extraction capacity, effectiveness;
Truck scheduling problem	Chang et al., 2015	Numerical indicators – loading and dumping points, number of trucks; Time variable - loading and unloading times per truck, moving times for an empty and full-loaded trucks;

In Table 1, some examples of the models used in solving the general problems in open-pit mining are shown, and in the third column, the most important parameters are listed.

Method

From a statistical perspective, correlations among measured quantities of the problem including final results, such as productivity, cost price, duration of the process, etc., and different parameters including optimisation models used can be analysed to estimate the degree of influence.

One possible method in determining the correlation between measured quantities and different parameters is the calculation of the Spearman rank correlation coefficient (ρ) (Myers et al., 2006; Zhang et al., 2020). This coefficient can be determined by the following equation:

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}, \tag{5}$$

where:
d_i is the difference between every two pairs;
n is the number of observations.

Solving equation (5) for the existing pairs of the measured quantities and parameters will give the heating correlation matrix (Fig. 2):

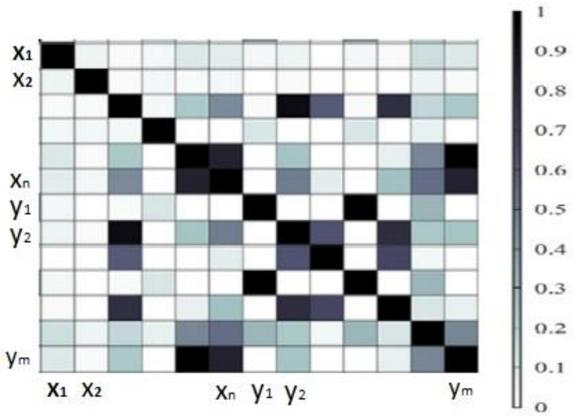

Fig. 2. Heating correlation matrix (Zhang et al., 2020)

In the heating matrix, x_i (i=1,…,n) and y_j (j=1,…,m) are respectively the measured quantities of the problem (results) and parameters involved in the process. The method allows for possible dependency between different parameters to be presented.

Discussion and results

In this work, one of the possible methods for determining the relationships between parameters and different processes in open-pit mining is presented. Having the heating correlation matrix, different analysis and conclusions about the possible solution can be made. These include even the used mathematical optimisation methods, as a component of the parameters variety set. Such kind of analysis can be very useful in long term scheduling problems.

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