STRESS AND STRAIN STATE AROUND ELLIPTIC OPENING IN TRANSVERSALLY ISOTROPIC ROCK MASS – 2

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ABSTRACT: This article discusses the question of analytical determining the stresses and strains around opening. Its cross section is an ellipse. The rock mass has inclined plane of isotropy. This class of plane tasks is solved by complex function method. The behavior surrounding the opening is described by differential equation of function for stresses. This function is represented by two functions of complex variables. They took part in expressions of stresses and strains in the point of examined medium.

The expressions of complex functions in condition for plane strain are given on contour of the opening. The analytical expressions of stresses and strains are presented in points on contour of opening. These expressions are written in natural and orthogonal coordinates systems.

Keywords: elliptical opening, transversally isotropic rock mass, analytical solution, complex function method

НАПРЕГНАТО И ДЕФОРМИРАНО СЪСТОЯНИЕ ОКОЛО ЕЛИПТИЧНА ИЗРАБОТКА В ТРАНСВЕРЗАЛНО ИЗОТРОПЕН МАСИВ - 2

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РЕЗЮМЕ: Тази статия се разглежда въпросът за аналитично определяне на напреженията и деформациите около изработка. Нейното напречно сечение е елипса. Масивът притежава наклонена равнина на изотропия. Този вид равнинни задачи се решават с метода на комплексна функция. Околността около отвора се описва с диференциално уравнение на функция за напрежението. Тя е представена посредством две функции на комплексни променливи. Те участват в изразите за напреженията и деформациите в точка от разглежданата област.

По контура на отвора са дадени изразите за комплексните функции в условие на равнинна деформация. Представени са аналитичните изрази за напреженията и деформациите в точки от контура на отвора. Тези изрази са записани в естествена и декартова координатни системи.

Ключови думи: елиптична изработка, трансверзално изотропен масив, аналитично решение, метод на комплексна функция

Introduction

The behavior of the rocks is influenced by the mechanical properties when underground construction works are performed and materials are extracted. These properties describes by mechanical models. They idealize the rock mass. In each of the models only basic essential properties are reflected. Depending on the properties of the rock mass and the nature of the processes, different theories are applied. The theory of elasticity is used here. She applies the following models of rock mass (Bulichev, 1984): elastic, viscosoelastic, elastoplastic nonhomogeneous, viscosoplastic and elastoplastic model and etc.

The rock mass has natural empty space and fissure. It is considered as a system of macro blocks. The relations of the contact between them are destroyed. Then the rock mass is modulated as stochastic medium. For her two group models are typical (Vulkov, 2016): linear and nonlinear. The first group includes: parabolic (Trifonova, 2008; 2010), hyperbolic, elliptic and generalizing model. The second group encloses: nonlinear model of Litwiniszyn, nonlinear parabolic model (Vulkov, 1987-88), nonlinear hyperbolic and nonlinear generalizing model.

The refinement of these models and methods for determination of stresses and displacements around opening is the goal of researchers. The opening may be close to the earth's surface or at a considerable depth.

In this article elastic model of theory of elasticity is used. The stresses and the strains are related by linear dependence for transversally isotropic rock mass. The mechanical properties of medium are the same in the plane of isotropy and are different in direction, perpendicular to her.

Methods

Formulation of the problem

Horizontal opening in the shape of an ellipse is drawn at a great depth H (fig.1). The rock mass has inclined plane of isotropy. This plane forms an angle φ with the horizontal axis. The width of the opening is $2a$ and the height is: $2b$. The rock mass has volumetric weight: γ .

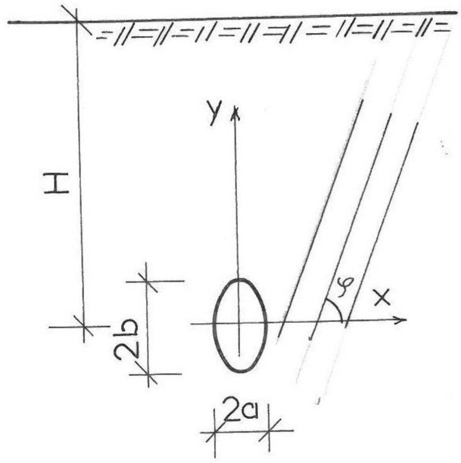

Fig.1. Horizontal elliptical opening in rock mass with inclined plane of isotropy

Stresses in natural coordinate system

The stresses are the sum of stresses in indestructible rock mass and the supplemental stresses that take into account the influence of the opening:

$$\sigma_n = \sigma_n^o + \sigma_n^{'};\ \sigma_t = \sigma_t^o + \sigma_t^{'};\ \tau_{nt} = \tau_{nt}^o + \tau_{nt}^{'}. \quad (1)$$

The stresses in indestructible rock mass are given in (Trifonova-Genova, 2019). The supplemental stresses have the form:

$$\begin{aligned}
\sigma_n^{'} &= \frac{2}{c_o}\mathrm{Re}\left[\Phi_1^{'}(z_1)c_1^2 + \Phi_2^{'}(z_2)c_2^2\right];\\
\sigma_t^{'} &= \frac{2}{c_o}\mathrm{Re}\left[\Phi_1^{'}(z_1)c_3^2 + \Phi_2^{'}(z_2)c_4^2\right]; \qquad (2)\\
\tau_{nt}^{'} &= \frac{2}{c_o}\mathrm{Re}\left[\Phi_1^{'}(z_1)c_1c_3 + \Phi_2^{'}(z_2)c_2c_4\right].
\end{aligned}$$

where

$$c_o = a\sin^2 t + b\cos^2 t;\ c_1 = s_1 a\sin t + b\cos t;$$
$$c_2 = s_2 a\sin t + b\cos t;\ c_3 = a\sin t - s_1 b\cos t;$$
$$c_4 = a\sin t - s_2 b\cos t.$$

In these expressions s_1 and s_2 are roots of the characteristic equation given in (Ivanova et al., 2018):

$$s_j = \alpha_j + i\beta_j. \quad (3)$$

Here i is imaginary unit and j=1,2.

The parameter t in the trigonometric dependencies in (2) is related to the polar angle θ by means of the relation (Trifonova-Genova, 2019) (fig. 2):

$$tgt = \frac{a}{b}tg\theta. \quad (4)$$

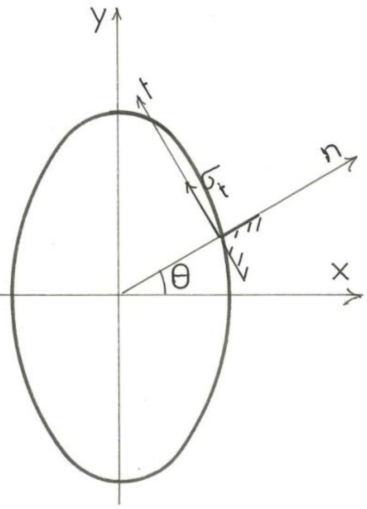

Fig.2. Contour stresses in natural coordinate system

The area occupied by the rock mass around the elliptical opening is transformed into the outer region of a unit circle. The functions of the complex variables from (1) along the contour of this circle have the form:

$$\Phi_1(z_1) = \frac{A_1}{e^{it}};\ \Phi_2(z_2) = \frac{B_1}{e^{it}}. \quad (5)$$

According to the method, described in (Minchev, 1960), the coefficients in (5) are:

$$A_1 = A_{11} + iA_{12};\ B_1 = B_{11} + iB_{12}, \quad (6)$$

where

$$\begin{aligned}
A_{11} &= A_{01}a(\lambda_\tau - \lambda_y s_2);\ A_{12} = A_{01}b(\lambda_x - \lambda_\tau s_2);\\
B_{11} &= A_{01}a(\lambda_y s_1 - \lambda_\tau);\ B_{12} = A_{01}b(\lambda_\tau s_1 - \lambda_x);\\
A_{01} &= 0{,}5Q(s_2 - s_1)^{-1};\ Q = \gamma H.
\end{aligned}$$

The expressions for the coefficients $\lambda_x, \lambda_y, \lambda_\tau$ are given in (Trifonova-Genova, 2019).

Expressions (6) are replaced by (5). The first derived of functions (5) are replaced by (1). The contour stresses are:

$$\begin{aligned}
\sigma_t &= \sigma_t^o + \frac{2}{c_o}\mathrm{Re}\left[\frac{A_1 i}{e^{it}c_5}c_3^2 + \frac{B_1 i}{e^{it}c_6}c_4^2\right];\\
\sigma_n &= 0;\ \tau_{nt} = 0, \qquad (7)
\end{aligned}$$

where

$$c_5 = a\sin t + s_1 b\cos t;\ \ c_6 = a\sin t + s_2 b\cos t.$$

Stresses in orthogonal coordinate system

Formulas are used to transform stresses from natural to orthogonal coordinates given in (Trifonova-Genova, 2019). The stresses in orthogonal coordinate system on the contour of the opening are (fig.3):

$$\sigma_x = \frac{\Delta_1^{'}}{\Delta}\sigma_t\,;\sigma_y = \frac{\Delta_2^{'}}{\Delta}\sigma_t\,;\tau_{xy} = \frac{\Delta_3^{'}}{\Delta}\sigma_t\,, \qquad (8)$$

where

$$\Delta^{'} = \frac{1}{c^3}\left[\Delta_4^{'}c_7 + \sin^2(2t)a^2b^2\Delta_5^{'}\right];$$

$$\Delta_1^{'} = \frac{a^2}{c^3}\left[0{,}5b^2\sin^2(2t) - \Delta_4^{'}\sin^2 t\right];$$

$$\Delta_2^{'} = \frac{b^2}{c^3}\left[0{,}5a^2\sin^2(2t) + \Delta_4^{'}\cos^2 t\right];$$

$$\Delta_3^{'} = -\frac{ab}{c^3}\sin(2t)\Delta_5^{'}\,;\Delta_4^{'} = b^2\cos^2 t - a^2\sin^2 t\,;$$

$$\Delta_5^{'} = b^2\cos^2 t + a^2\sin^2 t\,;$$

$$c_7 = b^4\cos^4 t - a^4\sin^4 t\,;c = b^2\cos^2 t + a^2\sin^2 t\,.$$

In expressions for stresses in the top expressions participate the tangential normal stresses of (7).

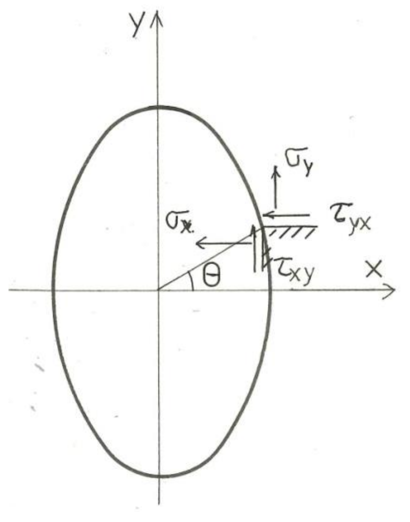

Fig.3. Contour stresses in orthogonal coordinate system

Strains

The strains on contour of the opening in natural coordinate system are:

$$\varepsilon_n = b_{13}\sigma_t\,;\ \varepsilon_t = b_{33}\sigma_t\ ;\ \gamma_{nt} = b_{35}\sigma_t\,. \qquad (9)$$

The stresses on contour of the opening in orthogonal coordinate system are:

$$\varepsilon_x = \frac{\sigma_t}{\Delta^{'}}\left(b_{11}\Delta_1^{'} + b_{13}\Delta_2^{'} + b_{15}\Delta_3^{'}\right);$$

$$\varepsilon_y = \frac{\sigma_t}{\Delta^{'}}\left(b_{13}\Delta_1^{'} + b_{33}\Delta_2^{'} + b_{35}\Delta_3^{'}\right); \qquad (10)$$

$$\gamma_{xy} = \frac{\sigma_t}{\Delta^{'}}\left(b_{15}\Delta_1^{'} + b_{35}\Delta_2^{'} + b_{55}\Delta_3^{'}\right).$$

The expressions of the strains of (9) and (10) involve the tangential normal stress. The coefficients with small letters in these expressions are given in (Ivanova et al., 2018), and those with capital letters are presented in (8).

Conclusion

The analytical expressions for stresses and strains, given in this paper, are applied to elliptical opening. It is drawn in a transversally isotropic rock mass. These expressions will write to detail. That will permit the numerical determination of stresses and strains.

These results should correspond to the strains observed in the rock mass. Theoretical studies (formulations) can be compared with other methods used to track strains. Modern photogrammetric methods are suitable for such purposes. In abroad and in Bulgaria there are developments related to the creation of numerical models in underground mines, using digital photogrammetry (Gospodinova, 2019; Gospodinova et al., 2018). These methods are not vector but tensor descriptions of geometric changes that give a better idea of the rock movements and the potential danger to the integrity of the rock. It may be interesting in the future to conduct a study related to the application of digital photogrammetry to track the strains, after then compare the results with the analytically determined stresses and strains along the contour of the opening.

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