ON THE DETERMINATION OF THE FUNCTION FOR STRESS IN TRANSVERSALLY ISOTROPIC ROCK MASS WITH ELLIPTIC OPENING

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ABSTRACT: The opening with form of ellipse is drawn in transversally isotropic rock mass. He owns the plane of isotropy, which is inclined. Stresses in the point of the medium surrounding the opening are determined with complex function method. The conduit of rock mass in condition of plane strain is described with partial differential equation. The main integral of this equation is function for stress. She depends by roots of characteristic equation. This equation is of fourth degree. To determine the roots of equation are applied iterative method.
The characteristic equation and his complex roots are obtained for real rock mass. The roots take part in expressions of function for stress and for stresses in point on opening.

Keywords: elliptical opening, transversally isotropic rock mass, analytical solution, complex function method

ВЪРХУ ОПРЕДЕЛЯНЕ НА ФУНКЦИЯТА ЗА НАПРЕЖЕНИЕТО В ТРАНСВЕРЗАЛНО ИЗОТРОПЕН МАСИВ С ЕЛИПТИЧЕН ОТВОР

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РЕЗЮМЕ: Изработка с форма на елипса е прокопана в трансверзално изотропен масив. Той притежава равнина на изотропия, която е наклонена. Напреженията в точка около отвора се определят с метода на комплексна функция. Поведението на масива в условие на равнинна деформация се описва с частно диференциално уравнение. Общият интеграл на това уравнение е функция за напрежението. Тя зависи от корените на характеристичното уравнение. Последното е от четвърта степен, За определяне на корените на уравнението се прилага итеративен метод.
За реален масив са определени характеристичното уравнение и комплексните му корени. Последните участват в изрази за функция за напрежението и за напреженията в точки от отвора.

Ключови думи: елиптична изработка, трансверзално изотропен масив, аналитично решение, метод на комплексна функция.

Introduction

The distribution of stresses in rock mass around the opening is problem, solved by analytical, numerical, geological and surveying methods. Between analytical methods, the most applying is complex variable method. The rock mass is modulated by isotropic medium. The relation between the stresses and strains are linear.

In this research the transversally isotropic rock mass is examined. For her it will be applied the algorithm for determining the function for stresses.

Methods

Formulation of the problem

The elliptic opening is drawn in rock mass with the plane of isotropy. She is inclined by angle φ to horizontal axis (fig.1). The stresses, the strains and the displacements are expressed by the function for stresses: $F(x, y)$. She depends of the coordinates x and y of the point of the medium around the opening. This function fulfills the partial differential equation:

$$D_1 D_2 D_3 D_4 (F) = 0 \qquad (1)$$

and the boundary conditions of the contour of the opening .

The symbol in (1) D_k (к=1, 2, 3, 4) notes with the operator

$$D_k = \frac{\partial}{\partial y} - s_k \frac{\partial}{\partial x}, \qquad (2)$$

Here s_k are the roots of characteristic equation:

$$b_{11}s^4 - 2b_{15}s^3 + b_{17}s^2 - 2b_{35}s + b_{33} = 0, \qquad (3)$$

where

$$b_{17} = (2b_{13} + b_{55}).$$

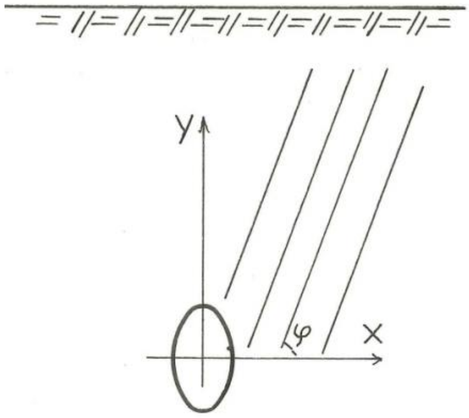

Fig.1. Horizontal elliptic opening in rock mass with inclined plane of isotropy

The coefficients in this equation can be seen in (Ivanova et al., 2018; Trifonova-Genova, 2019). The equation (3) can be written as:

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0, \qquad (4)$$

where

$$A_3 = \frac{2b_{13}}{b_{11}}, \quad A_2 = \frac{2b_{13} + b_{55}}{b_{11}}, \quad A_1 = \frac{-2b_{35}}{b_{11}},$$
$$A_0 = \frac{b_{33}}{b_{11}}.$$

By the algorithm given in (Ivanova et al., 2018), equation (4) is replaced with multiplication of two polynomials of the second degree:

$$\left(s^2 + e_{11}s + e_{10}\right)\left(s^2 + e_{21}s + e_{20}\right) = 0, \qquad (5)$$

where

$$e_{11} = B_1 + C_1, \quad e_{10} = B_0 + C_0, \quad e_{21} = B_1 - C_1,$$
$$e_{20} = B_0 - C_0.$$

The expressions above are given in recommended source. The roots of (5) are:

$$s_j = \alpha_j + i\beta_j, \qquad (6)$$

where i is an imaginary unit and j=1,2.

Numerical example

The rock mass has the plane of isotropy, inclined by angle $\varphi = 70°$ (fig. 1). For this plane Young's modulus is $E_1 = 14,5.10^3 N/m^2$ and Poisson's ration is $\mu_1 = 0,105$. In the direction perpendicular to this plane the parameters are: $E_2 = 41,5.10^3 N/m^2$, $\mu_2 = 0,3$. In the same direction the shear modulus is $G_2 = 8,24.10^3 N/m^2$. When calculating the coefficients of polynomials (5), an accuracy equal to: $\varepsilon_o = 1\%$ is assumed.

The elements of matrix A are calculated using formulas presented in (Trifonova-Genova, 2019):

$$[A] = 10^{-5}\begin{bmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{bmatrix},$$

where

$$a'_{11} = \begin{bmatrix} 3,8193 & -1,9117 & -2,6838 \\ -1,9117 & 6,8966 & -0,8814 \\ -2,6838 & -0,8814 & 7,2566 \end{bmatrix};$$
$$a'_{12} = \begin{bmatrix} 0 & 1,9117 & 0 \\ 0 & 0,8645 & 0 \\ 0 & 0,6748 & 0 \end{bmatrix};$$
$$a'_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 1,2094 & 0,8645 & 1,6748 \\ 0 & 0 & 0 \end{bmatrix};$$
$$a'_{22} = \begin{bmatrix} 14,9207 & 0 & 0,8811 \\ 0 & 10,0405 & 0 \\ 0,8811 & 0 & 12,8206 \end{bmatrix}.$$

The coefficients of equation (3) are obtained by formulas given in above literary source:

$$b_{11} = 3,2894\,.10^{-5}; \quad b_{12} = -4,3688\,.10^{-5};$$
$$b_{22} = 7,144\,.10^{-5}; \quad b_{26} = 1,5644.10^{-5};$$
$$b_{66} = 9,9321.10^{-5}.$$

The coefficients in equation (4) are presented in the following table:

Table 1. *Coefficients in characteristic equation*

A_0	A_1	A_3	A_3
2,1718	-0,9512	0,3631	-0,8810

By the first two step of algorithm, described in (Trifonova-Genova, 2018) are obtained: $D_I = -0,08455$, $D_{II} = 1,47371$.

The third step includes the subroutine for calculating: B_0. She compounded of two parts. Since the coefficient D_I is smaller than the coefficient: D_{II}, the required coefficient B_0

is determined by the second part. Its value is final after the second iteration.

The modulus of C_0 and C_1 and their multiplication are obtained according to fourth step of algorithm. The multiplication is negative. For that the coefficient C_0 has plus sign and C_1 - minus sign.

The coefficients in two polynomials of equation (5) are calculated following fifth step. The results of last three steps are given in following table:

Table 2. *Coefficients in equation (5)*

B_0	B_1	C_0	C_1
1,4774	-0,4405	-0,1043	1,6690
e_{10}	e_{11}	e_{20}	e_{21}
1,3731	1,2285	1,5817	-2,1095

The determination of the roots of characteristic equation (5) is the last step of algorithm. These roots are:

$$s_1 = -0{,}6143 + i0{,}9979,\ s_2 = 1{,}0548 + i0{,}73,$$
$$s_3 = -0{,}6143 - i0{,}9979,\ s_4 = 1{,}0548 - i0{,}73.$$

Conclusion

The obtained roots participated in the function for stresses $F(x, y)$ of (1). This function is compounded by two functions of complex variables (Trifonova-Genova, 2018). By them the stresses on contour of elliptic opening are obtained. When the stress values are small, the stability of the opening is guaranteed.

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