DETERMINATION OF KINEMATIC CHARACTERISTICS OF THE COMPLEX MOTION OF A PARTICLE BY MATHCAD

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ABSTRACT: The current problem considers the complex motion of a point M on the surface of a circular disk such as the latter performs a plane motion. The disc rolls without sliding on a horizontal straight line. The problem is solved graphically for a given time interval. The part of the trajectory of the point M is shown. The values of the transfer, relative and absolute velocities are graphically shown. The same is done for the transfer, relative, Coriolis and absolute accelerations, as well as for their components. The above mentioned kinematic characteristics are determined for the same time interval. The problem is solved using the MathCAD application. In this way the complexity of the graphical solution by hand is overcome.

Keywords: material particle, motion, velocity, acceleration, MathCAD

ОПРЕДЕЛЯНЕ НА КИНЕМАТИЧНИТЕ ХАРАКТЕРИСТИКИ НА СЛОЖНОТО ДВИЖЕНИЕ НА ТОЧКА С MATHCAD

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РЕЗЮМЕ: Задачата разглежда сложното движение на точка М върху повърхността на кръгъл диск, като последният извършва равнинно движение. Дискът се търкаля без плъзгане по хоризонтална права линия. Задачата се решава графично за даден интервал от време. Показана е частта от траекторията на точка М. Стойностите на преносната, относителната и абсолютната скорости са представени графично. Същото се прави за преносното, относителното, кориолисовото и абсолютното ускорения, както и за техните компоненти. По-горе споменатите кинематични характеристики се определят за същия времеви интервал. Задачата се решава с помощта на приложението MathCAD. По този начин се преодолява сложността на графичното решение на ръка.

Ключови думи: материална точка, движение, скорост, ускорение, MathCAD

Introduction

It is well known that there are difficulties in the 'manual' determination of the 3D-temporal characteristics of a material particle performing complex motion for a certain time interval. They arise from the graphical nature of the solution itself, as well as from possible errors caused by the complexity of the resulting expressions due to the twofold differentiation. Similar results are published by different authors (Doev and Doronin, 2016; Bertyaev, 2005; Velichenko, 1988; Gurskyi, 2003; Horn and Johnson, 2012, Lucko and Kavalchuk, 2018; etc.). In the means of the above mentioned the present study improves and complements the solution of this problem.

Formulation of the problem

The material particle $M\left(x_{2M}(t),\ y_{2M}(t),\ z_{2M}\right)$ moves on the surface of a round disk "T" with a radius $R = 25\,cm$ according to the law:

$$\begin{cases} x_{2M}(t) = \rho[\theta(t)].\cos[\theta(t)]; \\ y_{2M}(t) = \rho[\theta(t)].\sin[\theta(t)]; \\ z_{2M} = 0; \end{cases}$$

where - $\rho[\theta(t)] = 7 + 1,7.\sin(2,5.\theta(t))^2, cm$;

$\theta(t) = [\pi.(1+\cos(\pi.t)], rad$.

The disk lies in the plane Ox_0y_0 and rolls on a horizontal line without slipping (Fig. 1). The position of the disk center - point $C\left(x_{OC}, y_{OC}, z_{OC}\right)$ is determined according to the law:

$$\begin{cases} x_{0C} = s(t) = R.\lambda, cm; \\ y_{0C} = 0; \\ z_{0C} = 0; \end{cases}$$

where - $\lambda = 0,5.t^2 + t, rad$.

In the plane of the disc lie the following four reference (coordinate) systems - $Ox_0y_0z_0$, Cxy, Cx_1y_1, Ax_2y_2:

1) The first of them $Ox_0y_0z_0$ is global - conditionally motionless;

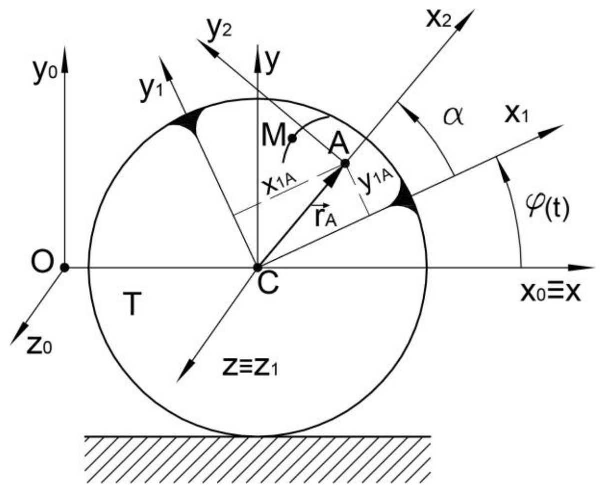

Fig. 1. Calculation scheme

2) The second $Cxyz$ is local and it is connected to the disk in a point C. It moves translationally. Its coordinate axes are parallel to the axes $Ox_0y_0z_0$;

3) The third coordinate system $Cx_1y_1z_1$ is a local and it is rigidly connected to the disk. It rotates around the axis Cz_1 according to the law $\varphi(t)=\dfrac{s(t)}{R}, rad$. The angle $\varphi(t)$ is accounted from an axis Cx;

4) The fourth reference system $Ax_2y_2z_2$ is a local with an arbitrary point $A\left(x_A=8cm, y_A=4cm, z_A=0\right)$. It is rotated around the axis Az_2 at an angle $\alpha=\dfrac{\pi}{5}, rad$ counted by an axis Cx_1 in counterclockwise direction.

In the global coordinate system it is necessary to determine the kinematical characteristics of the point M:

A) For the moment $t_1=2\,s$:

- The transfer, relative and absolute point velocity;
- The transfer, relative, Coriolis and absolute point acceleration.

B) The trajectory of the point for the time interval $[0,\ 6\,s]$;

C) For the same time interval, in a graphical form - the components and modules of:

- The transfer, relative and absolute velocity of the point;
- The transfer, relative, Coriolis and absolute acceleration of the point.

Solution of the problem by MathCAD package

The calculation and graphical visualization are performed by using the MathCAD package.

The angle α is accounted in a positive direction, i.e. in counterclockwise direction.

At the rolling of the disc the angle $\varphi(t)$ changes in the negative direction.

The analytical expressions for all velocities and accelerations are too long and they are not shown in Fig. 2.a. and Fig. 2.b.

All unknown quantities as well as their modules are determined for the time moment $t_1=2\,s$.

$$R := 25 \quad \lambda(t) := .5\cdot t^2 + t \qquad s(t) := R\cdot\lambda(t) \quad xOC(t) := s(t) \quad yOC(t) := 0$$

$$\varphi(t) := \frac{-s(t)}{R} \quad \alpha := \frac{\pi}{5} \quad t1 := 2 \qquad t2 := 6 \quad x1A := 8 \qquad y1A := 4 \quad rOC(t) := \begin{pmatrix} s(t) \\ 0 \end{pmatrix}$$

$$r1A := (x1A\ \ y1A)^T \quad \theta(t) := \pi\cdot(1+\cos(\pi\cdot t)) \quad \rho(t) := 7 + 1.7\cdot\sin(2.5\cdot\theta(t))^2$$

$$x2M(t) := \rho(t)\cdot\cos(\theta(t)) \quad y2M(t) := \rho(t)\cdot\sin(\theta(t)) \quad r2M(t) := (x2M(t)\ \ y2M(t))^T$$

$$Sz1(\alpha) := \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \quad r1M(t) := r1A + Sz1(\alpha)^T\cdot r2M(t) \qquad r1Mx(t) := r1M(t)_0$$

$$r1My(t) := r1M(t)_1 \quad r1M(t) := (r1Mx(t)\ \ r1My(t))^T \quad Sz\varphi(t) := \begin{pmatrix} \cos(\varphi(t)) & \sin(\varphi(t)) \\ -\sin(\varphi(t)) & \cos(\varphi(t)) \end{pmatrix}$$

$$rOM(t) := rOC(t) + Sz\varphi(t)^T\cdot r1M(t) \quad xOM(t) := rOM(t)_0 \quad yOM(t) := rOM(t)_1$$

$$rOM(t) := (xOM(t)\ \ yOM(t))^T \qquad v1M(t) := \frac{d}{dt}r1M(t) \quad vOC(t) := \frac{d}{dt}rOC(t)$$

$$\omega e(t) := \frac{d}{dt}(-\varphi(t)) \quad vOM(t) := \frac{d}{dt}rOM(t) \quad v1Mx(t) := v1M(t)_0 \qquad v1My(t) := v1M(t)_1$$

$$v1M(t) := (v1Mx(t)\ \ v1My(t))^T \quad K := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad vOMex(t) := vOMe(t)_0 \quad vOMey(t) := vOMe(t)_1$$

$$vOMe(t) := vOC(t) + \omega e(t)\cdot K\cdot Sz\varphi(t)^T\cdot r1M(t) \qquad vOMe(t) := (vOMex(t)\ \ vOMey(t))^T$$

$$vOMr(t) := Sz\varphi(t)^T\cdot v1M(t) \quad vOMrx(t) := vOMr(t)_0 \quad vOMry(t) := vOMr(t)_1$$

$$vOMr(t) := (vOMrx(t)\ \ vOMry(t))^T \quad vOMe(t) := vOC(t) + \frac{d}{dt}Sz\varphi(t)\cdot r1M(t) + Sz\varphi(t)^T\cdot v1M(t)$$

$$vOM(t) := vOMe(t) + vOMr(t) \qquad vOMx(t) := vOM(t)_0 \qquad vOMy(t) := vOM(t)_1$$

Fig. 2.a. Analytical expressions for determining the kinematics characteristics of point M

$$\mathrm{r2M(t1)} = \begin{pmatrix} 7 \\ -1.714\times 10^{-15} \end{pmatrix} \quad |\mathrm{r2M(t1)}| = 7 \quad \mathrm{rOM(t1)} = \begin{pmatrix} 84.928 \\ 5.036 \end{pmatrix} \quad |\mathrm{rOM(t1)}| = 85.077$$

$$\mathrm{r1M(t1)} = \begin{pmatrix} 13.663 \\ 8.114 \end{pmatrix} \quad |\mathrm{r1M(t1)}| = 15.891 \quad \varphi(\mathrm{t1}) = -4 \quad \omega e(\mathrm{t1}) = 3 \quad \mathrm{vOMe(t1)} = \begin{pmatrix} 59.891 \\ -45.216 \end{pmatrix}$$

$$|\mathrm{vOMe(t1)}| = 75.043 \quad \mathrm{vOMr(t1)} = \begin{pmatrix} -3.859\times 10^{-15} \\ -1.648\times 10^{-14} \end{pmatrix} \quad |\mathrm{vOMr(t1)}| = 1.692\times 10^{-14}$$

$$\mathrm{vOM(t1)} = \begin{pmatrix} 59.891 \\ -45.216 \end{pmatrix} \qquad |\mathrm{vOM(t1)}| = 75.043$$

$$\varepsilon(t) := \frac{d}{dt}\omega e(t) \qquad \mathrm{aOC}(t) := \frac{d}{dt}\mathrm{vOC}(t) \qquad \mathrm{aOMe}(t) := \mathrm{aOC}(t) + \left[\varepsilon(t)\cdot K + (\omega e(t)\cdot K)^2\right]\cdot \mathrm{rOM}(t)$$

$$\mathrm{aOMex}(t) := \mathrm{aOMe}(t)_0 \quad \mathrm{aOMey}(t) := \mathrm{aOMe}(t)_1 \quad \mathrm{aOMe}(t) := (\mathrm{aOMex}(t) \quad \mathrm{aOMey}(t))^T$$

$$\mathrm{a1M}(t) := \frac{d}{dt}\mathrm{v1M}(t) \quad \mathrm{aOMr}(t) := \mathrm{Sz}\varphi(t)^T\cdot \mathrm{a1M}(t) \quad \mathrm{aOMrx}(t) := \mathrm{aOMr}(t)_0 \quad \mathrm{aOMry}(t) := \mathrm{aOMr}(t)_1$$

$$\mathrm{aOMr}(t) := \left(\mathrm{aOMr}(t)_0 \quad \mathrm{aOMr}(t)_1\right)^T \quad \mathrm{aOMC}(t) := 2\cdot\omega e(t)\cdot K\cdot \mathrm{Sz}\varphi(t)^T\cdot \mathrm{v1M}(t) \quad \mathrm{aOMCx}(t) := \mathrm{aOMC}(t)_0$$

$$\mathrm{aOMCy}(t) := \mathrm{aOMC}(t)_1 \quad \mathrm{aOMC}(t) := (\mathrm{aOMCx}(t) \quad \mathrm{aOMCy}(t))^T \quad \mathrm{aOM}(t) := \mathrm{aOMe}(t) + \mathrm{aOMr}(t) + \mathrm{aOMC}(t)$$

$$\mathrm{aOMx}(t) := \mathrm{aOM}(t)_0 \qquad \mathrm{aOMy}(t) := \mathrm{aOM}(t)_1 \qquad \mathrm{aOM}(t) := (\mathrm{aOMx}(t) \quad \mathrm{aOMy}(t))^T$$

$$\varepsilon(\mathrm{t1}) = 1$$

$$\mathrm{aOMe(t1)} = \begin{pmatrix} -744.389 \\ 39.601 \end{pmatrix} \quad \mathrm{aOMr(t1)} = \begin{pmatrix} 49.5 \\ 211.324 \end{pmatrix} \quad \mathrm{aOMC(t1)} = \begin{pmatrix} 9.885\times 10^{-14} \\ -2.315\times 10^{-14} \end{pmatrix} \quad \mathrm{aOM(t1)} = \begin{pmatrix} -694.889 \\ 250.925 \end{pmatrix}$$

$$|\mathrm{aOMe(t1)}| = 745.442 \quad |\mathrm{aOMr(t1)}| = 217.044 \quad |\mathrm{aOMC(t1)}| = 1.015\times 10^{-13} \quad |\mathrm{aOM(t1)}| = 738.806$$

Fig. 2.b. Certain values for the positions and velocities of point M as well as analytical expressions for its accelerations and their values for the moment $t_1 = 2\,s$

The position of a particle M for the moment $t_1 = 2\,s$ in the space of the local coordinate system $Ax_2y_2z_2$ is shown in Fig. 3.a. The trajectory of the point for its relative motion is also described.

The same figure shows also the position of the point M for the moment $t_1 = 2\,s$ on the trajectory described by it in the space of the global coordinate system $Ox_0y_0z_0$.

The dependences between the projections of velocities and accelerations on the global axes and time are graphically presented on Fig. 3.a., 3.b. and 3.c. In addition, on the same figures are shown also the changes of their respective modules.

The presented graphic visualization refers only to the specified time period $[0,\ 6\,s]$.

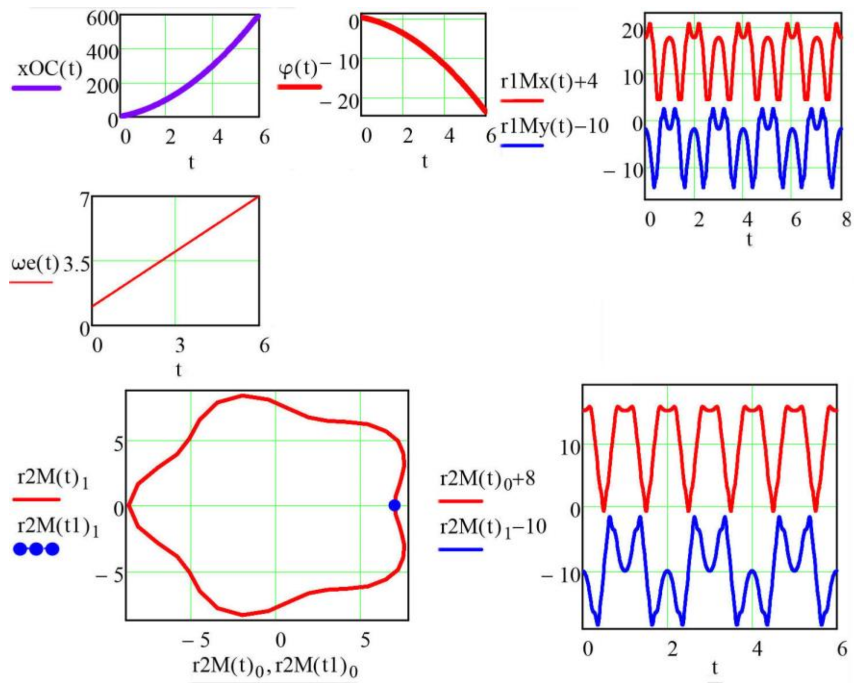

Fig. 3.a. Graphs expressing the dependence of the kinematics characteristics of the point M on time $t\,(s)$

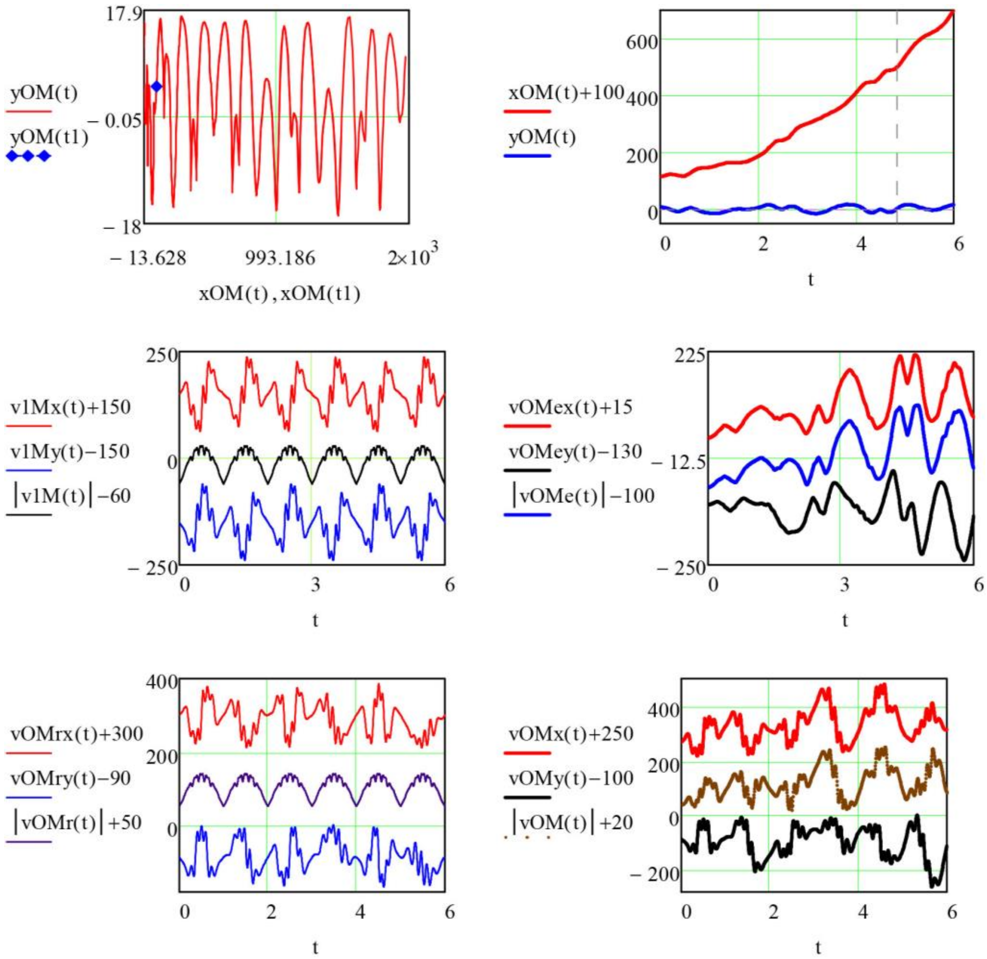

Fig. 3.b. Graphs expressing the dependence of the kinematics characteristics of the point M on time $t\,(s)$

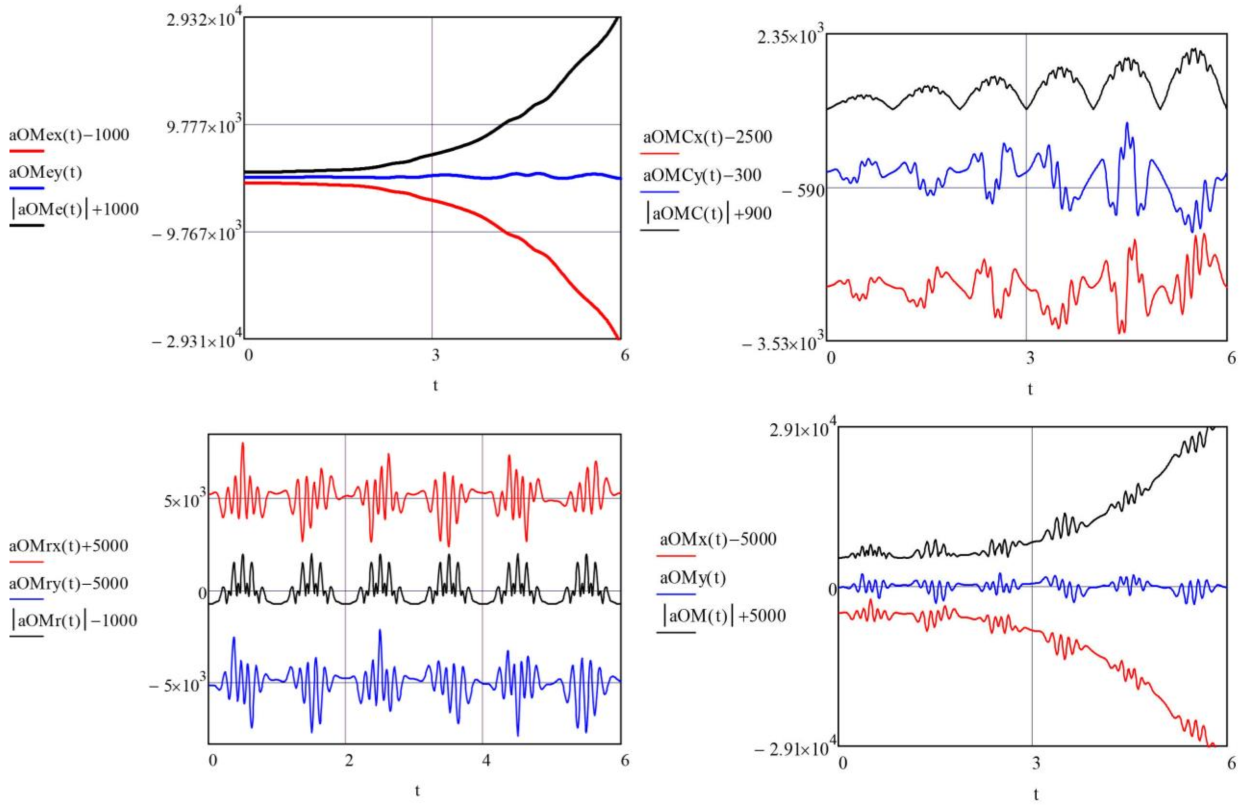

Fig. 3.c. Graphs expressing the dependence of the kinematics characteristics of the point M on time $t\,(s)$

Conclusion

The conventional solution of the problem, solved in the current paper, is related to some difficulties.

These difficulties are easily overcome by using of the modern mathematical packages in dependences of the aim of the study. In the considered case, the program MathCAD package can be used to check the problem solved by the traditional way.

The automated solution and the graphical editor of MathCAD provide control and optimization on the obtained results.

Some adjustments in the initial data are possible in order to achieve certain values of specific kinematic quantities.

The contemporary teaching of all fields of mechanics at the universities is related to using the most advanced mathematical packages such as MatLab, MathCAD, Mathematica, MuPAD and others (Doev and Doronin, 2016; Bertyaev, 2005; Stoyanov, 2016; Ivanov, 2017; Stoyanov and Javorova, 2012; Pavlov et al., 2013; Lekova et al., 2013, Stoyanov, 2017; Pavlov and Evlogiev, 2015; Ivanov, 2018; Aleksandrov et al., 2012, Ivanov, 2019, Pavlov, 2018).

The presented study demonstrates a fast solution and excellent graphical visualization as can be seen for example from the last two figures of the paper. By this way the main advantages of the MathCAD package are confirmed.

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