RELIABILITY FUNCTION IMPROVED FITTING USING GOODNESS-OF-FIT PROCEDURES

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ABSTRACT. Presented research paper analyze assessments of reliability function using goodness-of-fit procedures. The analysis is deepen following the modern increased software and computational abilities which allows using better and accelerated mathematical procedures. Particularly goodness-of-fit /GOF/ procedures are compared with median rank estimation /MRE/ and maximum likelihood estimation /MLE/ in resolving fitted values for Weibull models in reliability function assessment. This study shows diagram CDF – ECDF comparison, as well as the criteria comparison using Kolmogorov-Smirnov, Anderson-Darling and Crammer-Von Mises distances. Data sets from jaw crusher lining plates repair periods are used. There are developed hazard and survival functions for reliability assessment based on fitted models.

Keywords: goodness-of-fit; liner plate wear; jaw crusher; Weibull model fit; Weibull reliability assessment

ИЗВЕЖДАНЕ НА ФУНКЦИЯ НА НАДЕЖДНОСТТА ЗА ОБЛИЦОВКИТЕ НА ЧЕЛЮСТНА ТРОШАЧКА

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РЕЗЮМЕ. Представената статия разглежда методите за интерполиране и оценка на показателите на функциите на надеждността. Ресурсните показатели на ремонтните въздействия поради естеството им са времево зависими дискретни данни, които при интерполиране към непрекъсната функция биха дали различни показатели в зависимост от използваните методи. Сравнени са класически методи: метод на ранга на медианата, максимум на функцията на възможността с метод максимизиращ метод на подобрението (goodness-of-fit). Оценката на показателите на функциите на надеждността е проведено с методите: Колмогоров-Смирнов, Андерсон-Дарлинг, Крамер-фон Мизес. Разработен е алгоритъм за приемане на показателите на функциите и оценъчните показатели на разпределенията и описващите ги модели. Представени са таблични и графични резултати за визуализация и оценка на моделите, оценките и сравнителните им показатели.

Ключови думи: goodness-of-fit; облицовъчни плочи, износване, челюстна трошачка, Вейбул модели, надеждностни модели

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Introduction

Operational properties of machines and their parts subjected to unsteady loading and abrasive wear tend to show variability despite the fact of strictly determined calculation design procedures. Following chain structure levels: part - sub-assembly - assembly- machine, the complete machine also tends to have inconsistency in its operational properties, repair periods and maintenance works. All this is enveloped in reliability model and has to be used in product lifecycle Minin (2017) management regarding the machine operation parameters aggravate in wear increasing Hristova (2016). Reliability model structure could be based on Weibull function, thoroughly described and proven in experimental scientific studies (Weibull (1951), Stephens (1974)) regarding such kind of problems.

The examined object Minin (2017) is a jaw crusher operating as gold ore sizing crusher in Bulgarian mine. Working process inconsistency depends mostly on the variance of ore fragment size and then on rock strength and abrasive properties, which vary with mineral-composite structure.

Although finding the proper reliability model, based on Weibull function could be a problem itself, the suitable function interpolation is equally important issue. The aim of this research is to resolve that issue with the significance and the adequate values in Weibull function. The solution for estimated Weibull model values using median rank estimation /MRE/ and maximum likelihood estimation /MLE/ is expanded by goodness-of-fit tests (Delignete-Muller (2014), Murthy (2004), Krit (2014)), which assists the research engineer to choose between various valuated models.

Some improvements in MRE algorithm with precise Bernard interpolation formula (eq. 6) can be found but that improvement averages in 0.1 % which is not valuable subject. Finding improved mathematical method is done successfully when goodness-of-fit functions are implemented as an algorithms in fitting software procedures (Delignete-Muller (2014), Krit (2014), Krit (2016)). That gives improved abilities in estimation of adequate and confident values in chosen reliability function. Presented research is focused in reliability function estimation improving and its application in real repair and maintenance of jaw crusher lining plates.

Theory

Weibull probability distribution (density) function /pdf/ is defined as three parameter function but in reliability modeling the location parameter $\gamma \in (-\infty,\infty)$ is often set to 0 (γ=0), so the distribution is presented in following form:

$$f\left(\tau|\gamma=0\right)=\frac{\beta}{\eta}\cdot\left(\frac{\tau}{\eta}\right)^{(\beta-1)}\cdot\exp\left[-\left(\frac{\tau}{\eta}\right)^{\beta}\right] \quad (1)$$

and cumulative density function /CDF/ is:

$$F\left(t|\gamma=0\right)=\int_{-\infty}^{t}f\left(\tau\right)d\tau=1-\exp\left[-\left(\frac{\tau}{\eta}\right)^{\beta}\right] \quad (2)$$

Other important functional features are:

- survival function S(t) defined as admission life period to exceed some time interval P(T > t), as follows:

$$S\left(t\right)=1-P\left(T\le t\right)=1-F\left(t\right)=exp\left[-\left(t/\eta\right)^{\beta}\right] \quad (3)$$

- hazard function is defined by:

$$h(t)=\frac{f\left(t\right)}{S\left(t\right)}=\frac{f\left(t\right)}{1-F\left(t\right)}=\frac{\beta}{\eta^{\beta}}\cdot t^{(\beta-1)} \quad (4)$$

- mean time to failure /MTTF/ or the expected time to failure as an mathematical expectation:

$$MTTF=E=\eta\cdot\mathbf{\Gamma}\left(\frac{1}{\beta}+1\right) \quad (5)$$

Computational application of MRE method with precise approximation Bernard algorithm formula for median rank is:

$$F_T\left(t_i\right)\cong Z_i\cong\frac{i-0.3175}{N+0.365} \quad (6)$$

, where:
N, number - total number of data;
i - data point ascending rank;

After double logarithm of upper formulas, with some idealizations it can be achieved:

$$\ln\left\{\ln\left[F_T\left(\ln t_i\right)\right]\right\}\cong y=\beta\cdot x-\beta\cdot\ln\left(\eta\right)=A_1\cdot x+A_0 \quad (7)$$

, placed in linear interpolation, for example in spreadsheet, so the MRE non-iterative algorithm in calculation of Weibull function coefficients uses:

$$\left|\begin{array}{l}\beta=A_1\\ \beta\cdot\ln\left(\eta\right)=A_0\end{array}\right.\Rightarrow\left|\begin{array}{l}\beta=A_1\\ \eta=\exp\left(-\frac{A_0}{A_1}\right)\end{array}\right. \quad (8)$$

The MLE method is accepted by Luciano [Luceno (2006)] as asymptotically optimal for large number of continuous distribution functions. Although Luceno (2006) and Krit (2014), Krit (2016) both based on their researches share the idea to use goodness-of-fit test as an algorithm in parameters estimation procedure. Initially the idea concerns any functions with large change in some intervals or points, which lead to large change in likelihood estimator and its impossibility of computing.

Goodness-of-fit estimation is based on algorithms that are previously developed as statistical methods proving the H0 hypothesis. Usually H0 hypothesis is about that two-samples of realizations are drawn from same distribution. One-sample gof tests H0 hypothesis respects the realizations and theoretical function to have the same distribution. There are large amount of hypothesis testing functions but in previous researches (Luceno (2006), Delignete-Muller (2014), Krit (2016), Lazov (2014)) Kolmogorov-Smirnov and Anderson-Darling tests are recommended.

General form of Kolmogorov-Smirnov test is evaluated as:

$$KS_{nm}=\sup_t\left|F_n\left(t\right)-G_m\left(t\right)\right| \text{ or } KS_n=\sup_t\left|F_n\left(t\right)-\hat{F}\left(t\right)\right| \quad (9)$$

, where:
KS_{nm} - distance (statistic) value;
$F_n(t)$ - first CDF function with n - sample number;
$G_m(t)$- second CDF function with m - sample number;
$\hat{F}\left(t\right)$ - estimated (theoretical) CDF. It was proven that the probability P of distance KSn to be smaller than chosen parameter lean to Kolmogorov distribution, as follows:

$$\lim_{n\to\infty}P\left(\sqrt{n}\cdot KS_n\le x\right)=\frac{\sqrt{2\cdot\pi}}{x}\sum_{j=1}^{\infty}\exp\left(\frac{-\left(2\cdot j-1\right)^2\cdot\pi^2}{8\cdot x^2}\right) \quad (10)$$

, so the formal transformations have to be taken in attention:

$$P\left(KS_n>\delta_{n,1-\alpha}\right)=\alpha\Rightarrow P\left(KS_n<\delta_{n,\alpha}\right)=1-\alpha \quad (11)$$

, where:
α - significance level or -
CI = 1 - *α* - confidence interval.

Statistic value (KS_n) appraise the distance between CDF and ECDF (respectively theoretical function and observed) at the significance level. The p-value in that statistic (KSn) represents probability in finding the calculated distance value between CDF and ECDF. So the smallest distance (KSn) with high p-value means the smoothest and the most precise fit between observations and theoretical function.

It is known that Kolmogorov-Smirnov test Krit (2016), Delignete-Muller (2014), Luceno (2006) sensitivity to deviations from a cumulative distribution function /CDF/ is not variable independent which leads to more sensitivity of test around the median and less sensitivity at the extreme ends (tails) of the distribution, where cumulative distribution function get near 0 or 1. Anderson-Darling test remove these disadvantages with weighted statistic, but it is in integral form which is computational expensive. Real data application for Anderson-Darling is usually in the form of:

$$AD_n^2=n\cdot\int_{-\infty}^{\infty}\frac{\left[F_n\left(x\right)-\hat{F}_0\left(x\right)\right]^2}{\hat{F}_0\left(x\right)\cdot\left[1-\hat{F}_0\left(x\right)\right]}d\hat{F}_0\left(x\right) \quad (12)$$

, where: $F_n(x)$ - is empirical distribution function /ECDF;
$\hat{F}_0\left(x\right)$ - theoretical (estimated) distribution function /CDF;

The other test statistic placed upwards is Cramer - von Mises usually in the form of:

$$CM_n=n\cdot\int_{-\infty}^{\infty}\left[F_n\left(x\right)-\hat{F}_0\left(x\right)\right]^2d\hat{F}\left(x\right) \quad (13)$$

, using same assignments as in the upper formulas, using same assignments as in the upper formulas.

Data and results

Here are used Kolmogorov-Smirnov and Anderson-Darling procedures in comparison to maximum likelihood estimation (mle) and median regression estimation (MRE). Checking procedure is done in one-sample Kolmogorov-Smirnov test and Crammer-Von Mises test conducted in different computational procedures.

There are used three data samples marked with D1 to D3 with different sample size and representing different maintenance periods for the considered parts. Data in D1 presents changing period for lining plate for movable jaw with 38 samples, D2 is for stationary jaw lining with 87 samples. Dataset D3 is chosen from data set D2 after 38-th row, so it has 50 samples. Dataset D3 is chosen from same samples in order to test computational procedures.

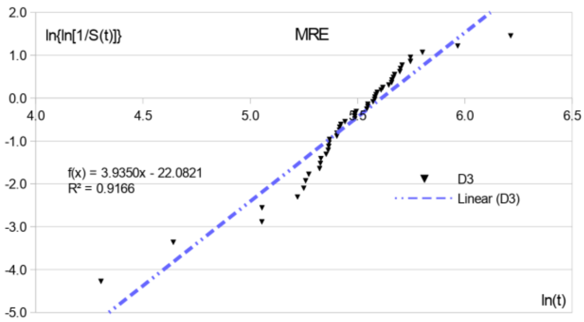

Fig.1. MRE regression for D1 with linear interpolation of MRE

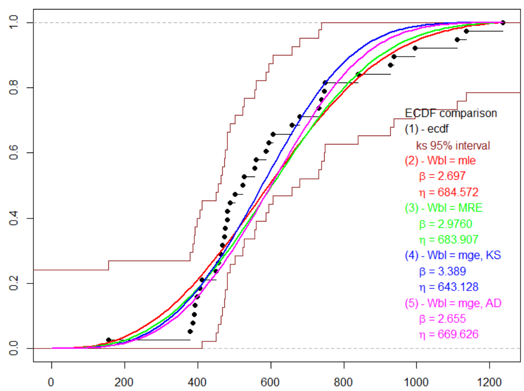

Fig.2. Comparison for algorithms fit with ECDF for D1

			KS	AD
	MRE	mle	mge	mge
β	2.9760	2.6965	3.3891	2.6545
η	683.907	684.572	643.128	669.626
R^2	0.8595			
ks.test				
KSn(D)	0.1570	0.1571	0.1269	0.1715
$KS_{p\text{-value}}$	0.276	0.275	0.532	0.191
cvm.test				
CMn (ω^2)	0.254	0.228	0.184	0.196
$CM_{p\text{-value}}$	0.184	0.220	0.302	0.275

Fig.3. Algorithms fit numerical values comparison for D1 (table view)

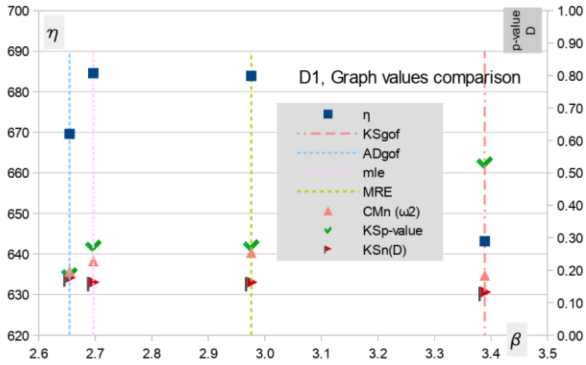

Fig.4. Algorithms fit values comparison for D1 (chart view)

Interpolation of distribution parameters with different algorithms is shown on fig. 1 and fig. 3 as it's noted. This particular data set fits quite well in Weibull model despite estimation method used. Kolmogorov-Smirnov and Cramer-Von Mises test are used after the fitting procedure within different computational function. This is shown in comparison table 1 and fig. 2. On figures 2, 5, and 8:

- flags - Kolmogorov-Smirnov estimated distance KS_n;
- boxes - "η" values;
- check tick - p-value as and;
- triangle - estimated CM_n distance. Which representation aims easy and distinctive comparison between estimation models.

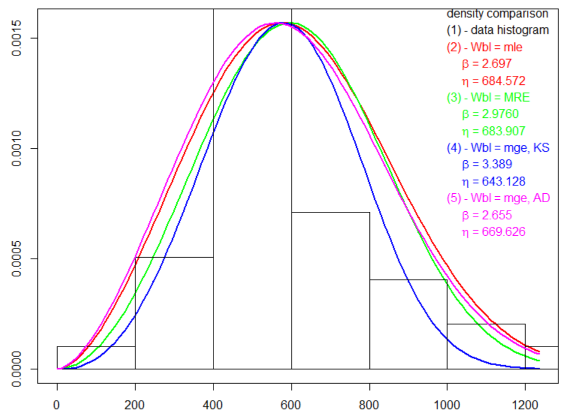

Fig.5. Density chart comparison for different interpolation algorithms - dataset D1

The same procedure is used, so following are the D2 processing results:

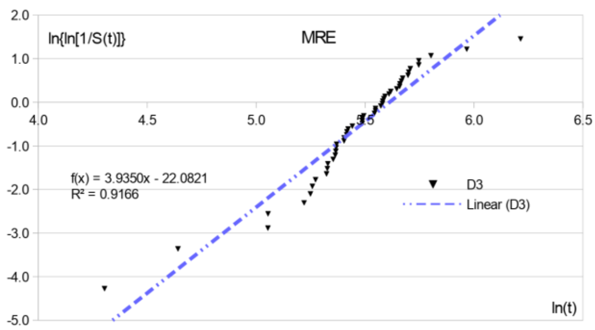

Fig.6. MRE regression for D2 with linear interpolation of MRE

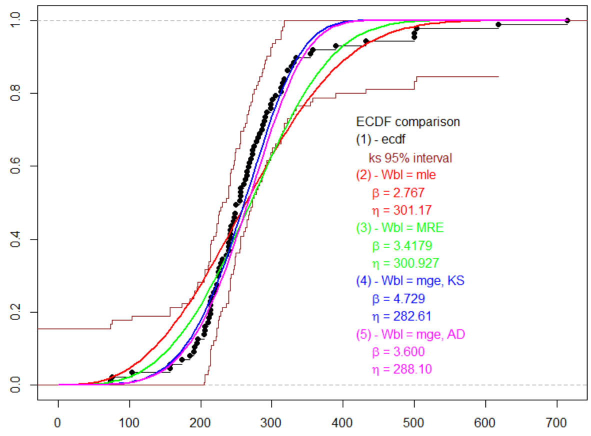

Fig.7. Comparison for algorithms fit with ECDF for D2

			KS	AD
	MRE	mle	mge	mge
β	3.4179	2.7671	4.7286	3.6000
η	300.927	301.169	282.609	288.098
R^2	0.8809			
	ks.test			
KSn(D)	0.1511	0.1654	0.0704	0.1282
$KS_{p\text{-}value}$	0.038	0.017	0.781	0.115
	cvm.test			
CMn (ω^2)	0.678	0.877	0.134	0.368
$CM_{p\text{-}value}$	0.014	0.005	0.442	0.087

Fig.8. Algorithms fit numerical values comparison for D2 (table view)

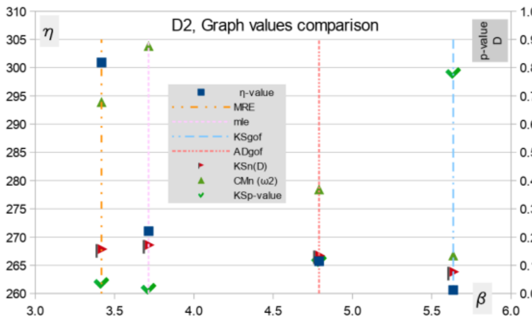

Fig.9. Algorithms fit values comparison for D2 (chart view)

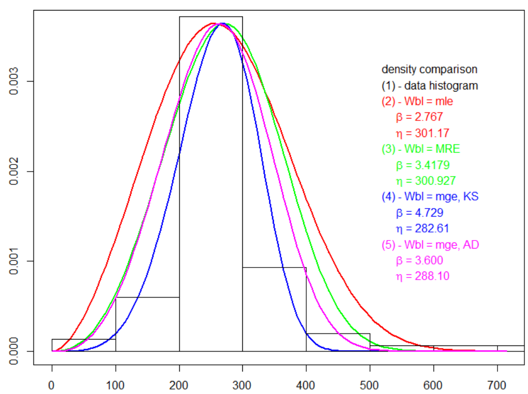

Fig.10. Density chart comparison for different interpolation algorithms - dataset D2

The improved procedure using goodness-of-fit procedure shows better results with both datasets, so it is supposed to be tested with some testing pieces, so following are the D3 processing results:

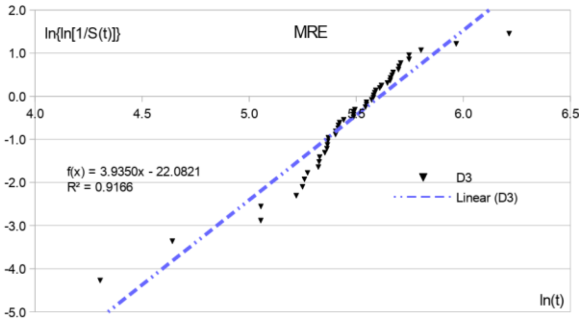

Fig.11. MRE regression for D3 with linear interpolation of MRE

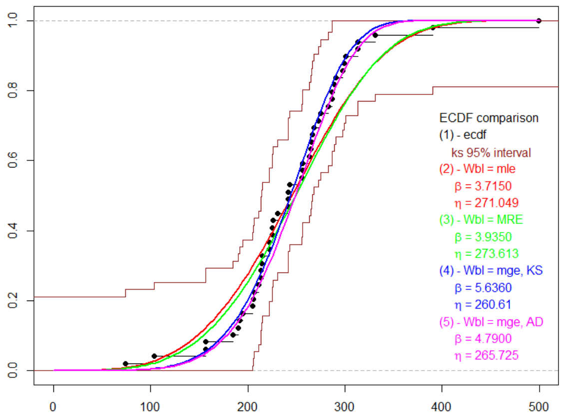

Fig.12. Comparison for algorithms fit with ECDF for D3

			KS	AD
	MRE	mle	mge	mge
β	3.9350	3.7150	5.6360	4.7900
η	273.613	271.049	260.614	265.725
R^2	0.9166			
	ks.test			
KSn(D)	0.338	0.135	0.065	0.088
$KS_{p\text{-}value}$	0.131	0.304	0.978	0.816
	cvm.test			
CMn (ω^2)	0.209	0.230	0.049	0.058
$CM_{p\text{-}value}$	0.251	0.216	0.887	0.830

Fig.13. Algorithms fit numerical values comparison for D3 (table view)

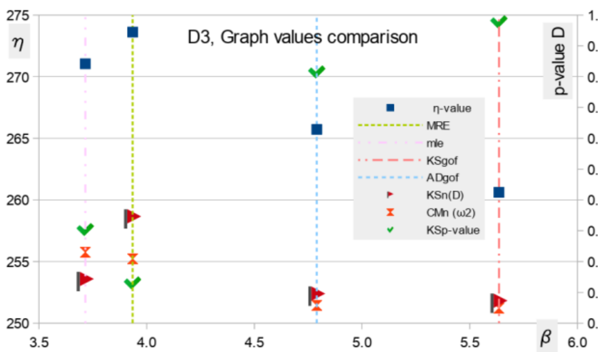

Fig.14. Algorithms fit values comparison for D3 (chart view)

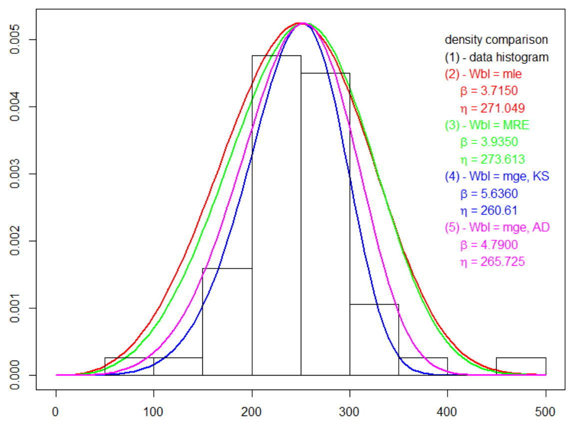

Fig.15. Density chart comparison for different interpolation algorithms - dataset D3

The acceptance of these models is controversial according to the graphical instruments. The better verification is comparison between numerical values from goodness-of-fit tests. Results from test verification are placed bellow interpolation values as shown in tables 1-3. Base researchers Stephens (1974) and O'Connor (2012) prefer Kolmogorov-Smirnov with critical asymptotical value calculated as:

$$KS_n < \frac{d_\alpha}{n^{1/2} + 0.12 + 0.11 \cdot n^{-1/2}} \qquad (14)$$

, where
d_α - tabulated constant for Kolmogorov-Smirnov distance Stephens (1974) at significance level α = 5%, so $d_{0.05}$ = 1.358, or for n > 40 as:

$$KS_n < d_\alpha \cdot n^{-1/2} = 1.358 \cdot n^{-1/2} \qquad (15)$$

For Cramer-Von Mises test critical values [Stephens (1974)] are asymptotically calculated as:

$$\left(CM_n - 0.4 \cdot n^{-1} + 0.6 \cdot n^{-2}\right) \cdot \left(1.0 + 1.0 \cdot n^{-1}\right) < CM_\alpha \qquad (16)$$

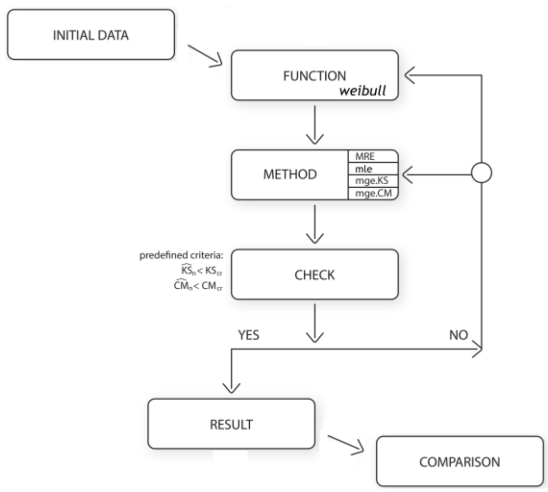

Fig.16. Function interpolation acceptance block scheme

Conclusions

Initial data are being interpolated using chosen function (Weibull) and method (MRE, mle, mge.KS, mge.CM). Approved result (YES flow) comes after examination check following assigned criteria between initial data and fitted function. Approved results are being presented graphically and are used to draw the other reliability functions, for example: hazard and survival function. If the result is not approved (NO flow), new function or method iteration are conducted. The iteration process continues until the result match the assigned criteria.

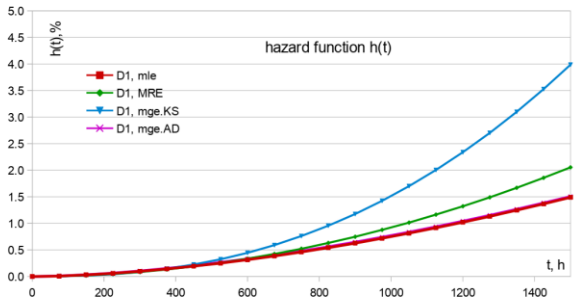

Fig.17. Hazard h(t) function based on estimated and accepted Weibull models for D1

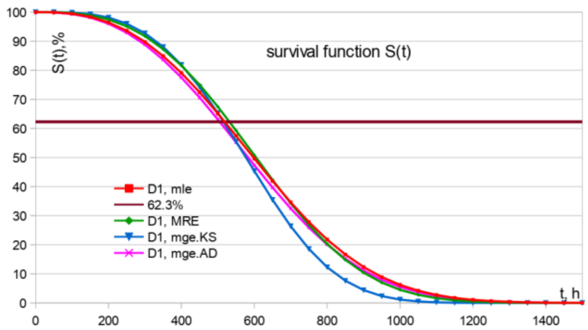

Fig.18. Survival S(t) function based on estimated and accepted Weibull models for D1

The result for hazard and survival functions shown in charts 17-18 match the assigned criteria for data set D1. Analogically for data set D2: in charts 19-20 are shown results for hazard and survival functions passing assigned criteria; and for data set D3: in charts 21-22 are shown results for hazard and survival functions passing assigned criteria.

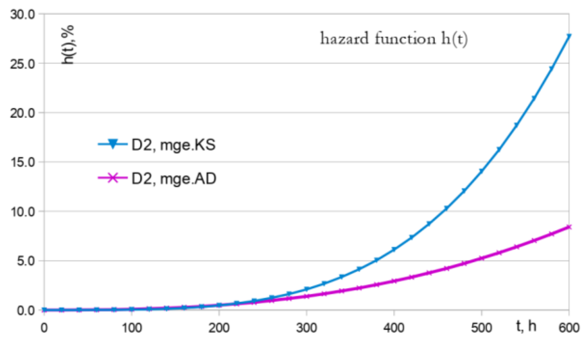

Fig.19. Hazard h(t) function based on estimated and accepted Weibull models for D3

Summary models have to be accepted at particular data set admit, with calculated value and author's preference for Kolmogorov-Smirnov method implementation of maximum goodness-of-fit estimation, so for D1 calculated $MTTF_{1\_KS}$ =

577.7 h, and hazard function deviations is at interval of 0.5%. For D2 calculated $MTTF_{2\_KS}$ = 240.9 h, witch show hazard function deviations in interval of 0.1%. For D3 calculated $MTTF_{2\_KS}$ = 258.6 h, hazard function deviations in interval of 0.3%. It is quite preferable to fit Weibull model using procedures of maximum goodness-of-fit with Kolmogorov-Smirnov or Anderson-Darling procedures. Related hazard function h(t) curves and survival function curves S(t) could be used in repair cycle analysis and reliability analysis for that particular machine.

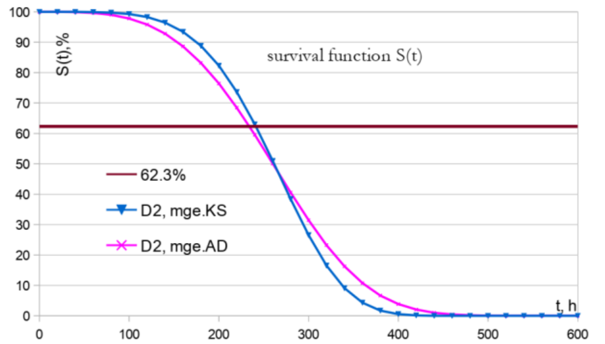

Fig.20. Survival S(t) function based on estimated and accepted Weibull models for D1

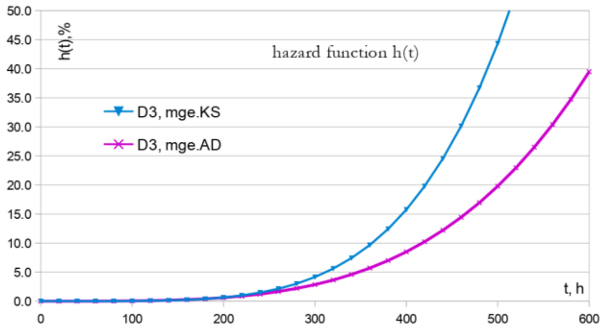

Fig.21. Hazard h(t) function based on estimated and accepted Weibull models for D3

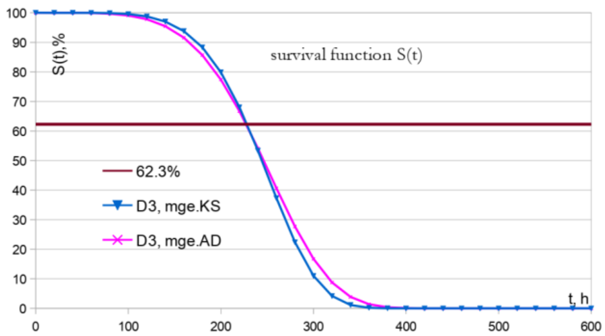

Fig.22. Survival S(t) function based on estimated and accepted Weibull models for D3

The particular fit procedure about distribution laws is in constant development, same the software instruments that achieve it. Many commercial software packages are widely used. In this research in development of program calculation as well as computational algorithms and procedures and graph presenting plots is used the iterative package for RStudio under GNU license in comparison with non iterative calculation made in spreadsheet.

Presented Weibull model fit is well fit to give an account of wearing process of jaw crusher liners with shown values in parameters interpolation. General view shows that using maximum goodness-of-fit procedures gave narrow density distribution graph and narrow interval in ECDF comparison so it have to be proffered in more precise interpolation of reliability function analysis.

The reliability assessment for hazard and survival functions through goodness-of-fit procedures gave very smooth and close neighbour graphs as it is shown on fig. 17 till 21 (KS and AD charts) which is main achievement and condition to recommend GOF procedure despite the fact it is computationally expensive and quite complicated in understanding. It is clearly recommended in examination of reliability procedures for stochastically phenomena in mechanical wearing for mining and processing machines.

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