# CONTOUR STRESSES OF A CIRCULAR OPENING IN A TRANSVERSELY ISOTROPIC ROCK MASS

***Malina Ivanova***

*University of Mining and Geology “St. Ivan Rilski”, 1700 Sofia; E-mail: : malina_vatz @abv.bg*

**ABSTRACT:** A horizontal circular opening is excavated in a transversely isotropic rock mass. It possesses a plane of isotropy. The plane is inclined to the horizontal axis of the cross-section of the opening. The stresses at a point of the hole are determined by the complex potential theory. The behavior of the rock mass is described by a fourth-order partial differential equation. Its characteristic equation is of the fourth degree. An iterative method was used to determine its roots. These roots are complex numbers and are involved in the analytical expressions for the stresses along the hole contour.
The coefficients in the characteristic equation and its complex roots are determined for a real rock mass. The values of the stresses at points on the contour of the hole were calculated.

**Key words:** complex potential theory, transversely isotropic rock mass, stresses.

**НАПРЕЖЕНИЯ ПО КОНТУРА НА КРЪГОВА ИЗРАБОТКА В ТРАНСВЕРЗАЛНО-ИЗОТРОПЕН МАСИВ**

***Малина Иванова***
*Минно-геоложки университет „Св. Иван Рилски", 1700 София.*

**РЕЗЮМЕ:** Хоризонтална кръгова изработка е прокопана в трансверзално изотропен масив. Той притежава равнина на изотропия. Равнината е наклонена спрямо хоризонталната ос на напречното сечение на изработката. Напреженията в точка от отвора се определят с комплексна потенциална теория. Поведението на масива се описва с диференциално уравнение в частни производни от четвърти ред. Неговото характеристично уравнение е от четвърта степен. За определяне на неговите корени са прилага итеративен метод. Тези корени са комплексни числа и участват в аналитичните изрази за напреженията по контура на отвора.
За реален масив са определени коефициентите в характеристичното уравнение и комплексните му корени. Изчислени са стойностите на напреженията в точки от контура на отвора.

**Ключови думи:** комплексната потенциална теория, трансверзално изотропен масив, напрежения.

## Introduction

A large number of publications have been published on the issue of stress distribution around a circular horizontal long opening. In most of them, the researchers use a rock mass model in which the elastic properties are the same in the different directions. To solve this problem, a complex potential theory is described (Muskhelishvili, 1953; Savin, 1961; Lekhnitskii, 1963; Lu et al., 2007). But in nature, the case of a rock mass which possesses a plane with equivalent elastic properties in all directions is often encountered. Such a rock mass is transversely isotropic. When the plane is horizontal and vertical, the development of the analytical solution can be seen in (Obert et al., 1967; Hefny, 1999; Trifonova-Genova, 2018).

The subject of the present study is an underground linear opening crossing an array with an inclined plane of isotropy. For this purpose, a development of the complex potential theory is applied. With it, the analytical expressions for the stresses in points of the contour of a circular opening will be given.

## Methods

**1.Formulation of the problem**

The plane of isotropy in the transversely isotropic rock mass is inclined at an angle $\varphi$ (Fig.1). A horizontal circular opening has a radius $r_o$. The volumetric weight of the rock mass is $\gamma$ and the embedding depth of the opening is $H$ (Fig.2). The physical characteristics of the transversely isotropic rock mass are five: $E_1$, $E_2$ - Young’s modulus in the plane of isotropy and perpendicular to it, $\mu_1$, $\mu_2$ - Poison’s ratio in the plane of isotropy and perpendicular to it, and $G_2$ is the shear modulus for a plane perpendicular to the plane of isotropy. By means of these characteristics, the deformation coefficients are determined. They are involved in the relationships between stresses and strains. The stresses in an undisturbed rock mass are: $\sigma_z^o = Q = \gamma H$ $\sigma_x^o = \lambda_x Q$, and $\tau_{xz}^o = \lambda_\tau Q$. The expressions for the coefficients $\lambda_x$ and $\lambda_\tau$ are given in (Trifonova-Genova, 2019).

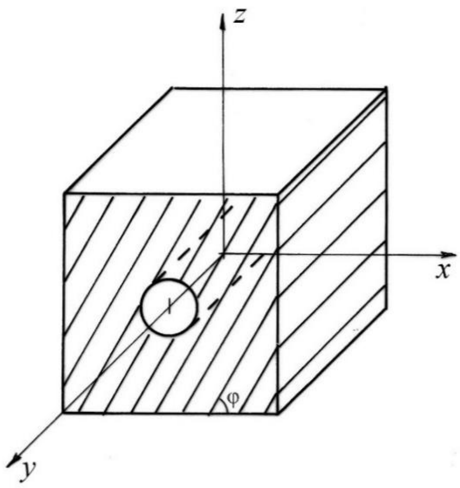


**Fig.1. Rock mass with inclined plane of isotropy**

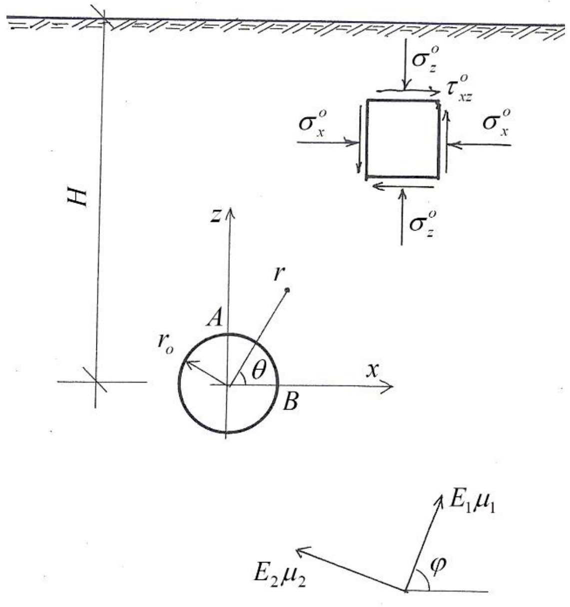


**Fig. 2. Horizontal circular opening in a transversely isotropic rock mass**

## 2. Stress

The behavior of the rock mass is described by a 4th-order partial differential equation of the stress function. A detailed view of this function can be found in the specialised literature given above. This function expresses the stresses at a point around the hole with polar coordinates ( $r\theta$ ). The general integral of a partial differential equation depends on the roots of a characteristic equation:

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0\,, \qquad (1)$$

Expressions for the coefficients in this equation can be seen in (Ivanova et al., 2018). These coefficients depend on both the physical characteristics of the rock mass and the slope of the plane of isotropy.

The roots of equation (1) are complex numbers:

$$s_1 = \alpha_1 + \beta_1 i\,;\; s_2 = \alpha_2 + \beta_2 i\,;$$
$$s_3 = \alpha_1 - \beta_1 i\,;\; s_4 = \alpha_2 - \beta_2 i\,. \qquad (2)$$

Here, $i$ is an imaginary unit.

The components of the stresses in the rock mass in a Cartesian coordinate system can be seen in (Muskhelishvili, 1953; Savin, 1961; Lekhnitskii, 1963). In a polar coordinate system, these components are obtained in (Minchev, 1980). There are no radial or tangential stresses along the opening contour. For a circular opening, the normal tangential stresses have the form:

$$\sigma_\theta = \sigma_\theta^o + \sigma_\theta^{oo}\,, \qquad (3),$$

where:

$$\sigma_\theta^o = \sigma_x^o \sin^2\theta + \sigma_z^o \cos^2\theta - \tau_{xz}^o \sin 2\theta\,;$$
$$\sigma_\theta^{oo} = 2\,\mathrm{Re}\left[d_1^2 \Phi_1'(z_1) + d_2^2 \Phi_2'(z_2)\right];$$
$$d_1 = \cos\theta + s_1 \sin\theta\,;\; d_2 = \cos\theta + s_2 \sin\theta\,.$$

The variables $z_k = x + s_k y$ for $k = 1{,}2$ are complex generalised because $s_k$ is a complex number and x and y are Cartesian coordinates of a point. In expression (3), "Re" denotes the real part of the complex expression.

At $\theta = 0, \pi$, the expressions for the stresses in an undisturbed array and the stresses after excavation of the opening from (3) are:

$$\sigma_\theta^o = -Q\,;\sigma_\theta^{oo} = 2\,\mathrm{Re}\left[s_1^2 \Phi_1'(z_1) + s_2^2 \Phi_2'(z_2)\right] \text{ (4).}$$

At $\theta = \pi/2, 3\pi/2$, the two stresses from (3) have the form:

$$\sigma_\theta^o = -\lambda_x Q\,;\sigma_\theta^{oo} = 2\,\mathrm{Re}\left[\Phi_1'(z_1) + \Phi_2'(z_2)\right] \qquad (5).$$

The derivatives of the functions from (4) and (5) have the form (Minchev, 1968):

$$\Phi_1'(z_1) = \frac{Q s_5 g_3}{g_1}\,;\; \Phi_2'(z_2) = \frac{Q s_5 g_4}{g_2} \qquad (6),$$

where:

$$s_5 = (s_1 - s_2)^{-1}\,;\; g_3 = (i + s_2\lambda)\,;\; g_4 = -(i + s_1\lambda)\,;$$
$$\theta = 0, \pi\,; \qquad g_1 = -(1 - s_1 i) + (1 + s_1 i)\,;$$
$$g_2 = -(1 - s_2 i) + (1 + s_2 i)\,;$$
$$\theta = \pi/2, 3\pi/2\,; \qquad g_1 = (1 - s_1 i) + (1 + s_1 i)\,;$$
$$g_2 = (1 - s_2 i) + (1 + s_2 i)\,.$$

Substituting (6) into (4) and (3) and revealing the real part of this complex expression yields the expression for the normal tangential stresses at point A in algebraic form:

$$\sigma_{\theta,A} = -Q\left[\lambda_x(1 - \frac{-\beta_3\sigma_{\theta,5} + \alpha_3\sigma_{\theta,6}}{\sigma_{\theta,7}}) + \frac{\sigma_{\theta,5}}{\sigma_{\theta,7}}\right] \quad (7),$$

where:

$$\beta_3 = \beta_1 + \beta_2;\ \alpha_3 = \alpha_1 + \alpha_2;\ \sigma_{\theta,5} = (\alpha_1\alpha_2 - \beta_1\beta_2);$$
$$\sigma_{\theta,6} = (\alpha_1\beta_2 + \alpha_2\beta_1); \sigma_{\theta,7} = \sigma_{\theta,5}^2 + \sigma_{\theta,6}^2 .$$

The expressions from (6) are substituted into (5) and (3) and the following expression is obtained for the normal tangential stresses at point B:

$$\sigma_{\theta,B} = -Q\lambda_1 - Q[\beta_3 + \lambda_x(1 - \sigma_{\theta.5})] \quad (8),$$

where:

$$\lambda_1 = 1 + \lambda_x .$$

**3.Numerical example**

A circular opening is driven to a depth of $H = 200m$ (Fig.1). The rock mass has a plane of isotropy inclined at an angle $\varphi = 70°$ . For it, Young's modulus and Poisson's ratio are: $E_1 = 14.5.10^3 N/m^2$ , $\mu_1 = 0.105$ . In a direction perpendicular to this plane, the parameters are: $E_2 = 41.5.10^3 N/m^2$ , $\mu_2 = 0.3$ . The shear modulus in the same direction is $G_2 = 8.24.10^3 N/m^2$ . The volumetric weight of the rock mass is $\gamma = 0.025 MN/m^3$ . The vertical stress in the undisturbed rock mass is $\sigma_z^o = 5MPa$ .

Using the characteristics of the rock mass, the deformation coefficients were determined first, and then the coefficients from equation (1). The values of the latter are given in the following table (Vucheva et al., 2020):

Table 1. *Coefficients in characteristic equation*

| $A_0$ | $A_1$ 3 | 0 | $A_3$ |
|---|---|---|---|
| 2.1718 | -0.9512 | 0.3631 | -0.8810 |

An iterative method was applied to solve equation (1) and the following roots were obtained:
$s_1 = -0.6143 + i0.9979$ , $s_2 = 1.0548 + i0.73$ , $s_3 = -0.6143 - i0.9979$ , $s_4 = 1.0548 - i0.73$ .
The parameters in expressions (2) are: $\alpha_1 = -0.6143$ , $\beta_1 = 0.9979$ , $\alpha_2 = 1.0548$ , $\beta_2 = 0.73$ . They are involved in the expressions for the stresses at points A and B .

By means of the deformation coefficients for the environment, the coefficient was calculated $\lambda_x = 0.4196$ . Then, the normal tangential stresses are determined from equations (7) and (8). They are referred to the vertical stress in an undisturbed array $\sigma_z^o$ . The obtained results for both points are placed in Table 2.

Table 2. *Normal tangential stresses at two points*

| $\sigma_\theta^A / \sigma_z^o$ | $\sigma_\theta^B / \sigma_z^o$ |
|---|---|
| 0.68 | -4.14 |

It can be seen in the table that the values of the stresses along the horizontal axis are greater in absolute value compared to the same along the vertical axis for the given array. This ratio is 6.09.

**4. Major conclusions**

The article describes an algorithm for determining the stresses at points of a rock mass around a circular horizontal opening. The rock mass has an inclined plane of isotropy.

The analytical expressions for the stresses at points of the opening contour are obtained. The result is implemented for a real rock mass.

## Conclusion

The article considers a task from the mechanics of underground facilities. The boundary conditions of the problem are constant. The complex potential theory is used to determine the stresses.

The presented solution is an addition to the existing analytical methods by which the stresses along the contour of a circular hole are determined. This solution can be extended to other types of opening.

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