A DESIGN OF EXPERIMENTS APPROACH FOR ESTABLISHING KEY FACTORS DETERMINING THE PROFITABILITY OF ULTIMATE PIT LIMITS

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ABSTRACT. Pit limits are determined by numerous factors of different nature, including commodity prices, geologic uncertainty, pit geometric design features, operational costs, etc. On the one hand, a substantial amount of these factors is related to uncontrollable conditions, such as economic and financial trends on a global scale, which lead to varying commodity prices and operational costs. On the other hand, geological uncertainty, geotechnical and technological constraints are also of great importance when determining the ultimate pit limits. This article is based on using the Design of Experiments (DOE) paradigm to quantify the dependences between different factors and the net present value (NPV) of the ultimate pit contour. Different factors are represented as a set of input variables for a pit limit optimisation software (*MiningMath*) and the obtained net present value under different conditions is evaluated. Two regression models are built for the purpose of providing guidance for making robust design decisions. This approach provides a way of quantifying factor influences on the magnitude of the calculated NPV. In addition, a smaller set of design alternatives can be considered, which would accelerate the process of reaching an optimal ultimate pit design scenario, taking uncertainty into account.

Key words: open-pit design, geological block model, economic model, Design of Experiments, regression analysis

ИЗСЛЕДВАНЕ НА ОСНОВНИТЕ ФАКТОРИ, ОПРЕДЕЛЯЩИ РЕНТАБИЛНОСТТА НА КРАЙНИЯ КОНТУР НА ОТКРИТ РУДНИК, ЧРЕЗ ПЛАНИРАНЕ НА ЕКСПЕРИМЕНТА

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РЕЗЮМЕ. Границите на откриния рудник до голяма степен се определят от фактори с различно естество – цена на продукцията, геоложки риск, конструктивни елементи на минните изработки, оперативни разходи и др. Една част от тях се свързват с икономическите и финансовите тенденции в международен план, като това рефлектира върху цените на металите и върху разходите за добив и преработка на рудата. От друга страна ключово значение върху границите на откриния рудник има геоложката изученост на находището, както и геотехнически и технологични ограничения, определящи конструкцията на открития рудник. В настоящото изследване е използван подход, базиран на парадигмата на Планирането на експеримента с цел търсенето на зависимости между отделните фактори и получаваната нетна осъвременена стойност (NPV) за крайния контур на рудника. Отделните фактори са представени като множество от входящи променливи за оптимизационен софтуер (*MiningMath* и получената нетна осъвременена стойност се оценява при различни условия. Създадени са два регресионни модела, които имат за цел да послужат като насока при избора на проектен вариант. Този подход позволява количествено да се оцени влиянието на всеки фактор върху рентабилността на крайния контур, както и да се намали броят на разглежданите варианти. Приложението на подхода цели да ускори намирането на оптимален вариант за краен контур на открития рудник, отчитайки вероятностните уловия.

Ключови думи: проектиране на открити рудници, геоложки блоков модел, икономически модел, планиране на експеримента, регресионен анализ

Introduction

Mining operations are complex activities that require careful planning to ensure maximum profitability while at the same time taking into account multiple environmental and safety considerations. Among the critical decisions in open-pit mining is the determination of the ultimate pit contour, which directly impacts the economic viability of the project's economic viability (Koprev and Aleksandrova, 2022). Numerous factors influence the profitability of these contours, and understanding their interactions is vital for informed decision-making. These factors can be grouped into two major categories:

1) controllable
2) uncontrollable.

In terms of pit limits consideration, a number of factors can be considered uncontrollable, which have a great influence on the profitability of the mining operation. Hence, this requires a thorough investigation of the effects of such factors on the profitability of the mining operation with respect to the set of controllable factors. This paper presents a comprehensive investigation using a Design of Experiments (DOE) approach to identify and evaluate the key factors that significantly affect the profitability of ultimate pit contours. By applying statistical techniques to post-optimisation data of the net present value (NPV) for each ultimate pit scenario, this paper aims to provide valuable insights and practical guidelines for mining professionals to enhance their decision-making process and achieve optimal economic outcomes. For this purpose, modern-day mining is heavily dependent on the consideration of multiple ultimate pit scenarios. These scenarios serve as a basis for mining engineers and geologists and lead to making informed decisions on how to further optimise the mine’s design and operational planning with the gained insight. Hence, this can be considered a trial-and-error process, where each consecutive solution and considered scenario is used to refine the following one. Therefore, today’s mining requires the use of robust and fast-working algorithms, which provide mine planners with the tools to optimise the mining operation’s profit under the tough conditions of uncertainty and multiple safety, technological and environmental constraints.

Pit optimisation algorithms

The main purpose of each ultimate pit optimisation algorithm is to determine the optimal configuration of an open-pit mining operation, maximising the economic value of the utilised mining and processing technology, as well as the geometric features of the pit under the conditions of geological and economic uncertainty. Each algorithm is specifically designed to address these challenges, while considering various constraints and factors. They include the geological characteristics of the deposit, mining costs, commodity prices, processing costs, environmental regulations, safety considerations, and infrastructure limitations (Hustrulid et al., 2013). Therefore, the algorithm's primary goal is to determine the optimal pit shell, which is the outer boundary of the pit that encloses the economically viable material to be mined. This involves finding the optimal trade-off between the economic value of the extracted material, the associated mining costs, and different environmental and social aspects.

Generally speaking, ultimate pit optimisation algorithms utilise mathematical and optimisation techniques to solve complex problems involving multi-variable constraints and objectives. During the past 60 years, optimisation algorithms have been extensively researched to the point that they have displaced manual methods for an initial evaluation of the deposit and for mine planning (ibid.). Several of the more notable and flexible algorithms, which have shaped the understanding of how such algorithms perform in a real-life environment, include: the floating cone method, the Lerchs-Grossman algorithm, the Pseudoflow algorithm, and algorithms for solving Mixed-Integer Linear Programming (MILP) problems.

The floating cone method

The floating cone method is a popular technique used in open-pit mining for designing and planning the layout of a new mine or the expansion of an existing one. It helps determine the optimal pit shell and the sequence of mining blocks to maximise the value of the mineral deposit while considering various economic and operational constraints.

The process starts with a geological resource model of the mineral deposit, which includes information on the ore body's shape, size, and grade spatial distribution. The next step is to create a pit shell, which represents the ultimate boundary of the open pit. The floating cone method assumes that the mining will start from the pit's highest point. At this stage, the "ultimate pit" includes all blocks with positive values above the cut-off grade. The floating cone is a theoretical slope that moves down from the highest point of the pit, progressively encompassing blocks with positive values as it descends. The blocks that fall within the floating cone are considered part of the mineable reserves, and those outside are left as waste (ibid.). The floating cone is iteratively adjusted to maximise the project's economic value, considering the value of the extracted ore, operating costs, and other financial factors. As the floating cone descends, it may encounter practical limitations, such as physical pit slopes or mining equipment restrictions. When these constraints are reached, "pushbacks" are designed. Pushbacks involve re-evaluating the pit design and creating a new shell around the existing one to include additional blocks that were previously excluded due to constraints. The process of adjusting the floating cone and adding pushbacks is repeated until no more profitable blocks can be included. The result is the final pit design, which includes the ultimate boundary and the sequence of pushbacks that maximise the profitability of the mining project. A notable drawback of the algorithm is that it is time-consuming, especially for deposits represented by a large number of blocks (Ares et al., 2022). Another crucial problem of the floating cone method is its inability to find an optimal solution when two overlapping pit contours are the optimal solution (Jodeiri et al., 2021). However, it was demonstrated that the problem can be overcome by more recent iterations of the method (Ares et al., 2022).

The Lerchs-Grossman algorithm

The Lerchs-Grossman (LG) algorithm is a widely used optimisation technique in open-pit mine design and planning. It was developed by H. Lerchs and I. F. Grossman in 1965 and has since become a fundamental tool for determining the optimal ultimate pit shell in a mineral deposit, considering economic and physical constraints (Lerchs & Grossman, 1965). However, the method became practical after its implementation by J. Whittle in the mid-1980s (Poniewierski, 2017). The algorithm seeks to maximise the economic value of the mining operation while accounting for factors, such as the geometry of the orebody, mining costs, processing costs, and commodity prices. It is named after its developers, and its main goal is to identify the ultimate pit that contains the highest-value blocks while considering the economic viability of the mining operation (Alford & Whittle, 1986).

The algorithm works by creating a series of nested pit shells. Each pit shell represents a hypothetical pit that includes all blocks with a grade above the cut-off grade. The algorithm uses a series of mathematical calculations to determine the optimal ultimate pit shell. It does this by finding the set of nested pit shells that maximises the NPV of the mining operation. In some cases, the resulting ultimate pit shell may not be practically achievable due to physical constraints or other factors. If necessary, additional design modifications, known as pushbacks, can be applied to the pit to make it more feasible for mining operations. The final output of the LG algorithm is the boundary of the ultimate pit, which represents the maximum economically viable extent of the open-pit mine.

The LG algorithm is highly regarded because it considers both economic and geological factors in determining the optimal pit design. It provides a systematic and efficient approach for mine planners to evaluate various scenarios and make informed decisions about the mine's layout, leading to better resource utilisation and financial outcomes (Nikolov et al., 2019; Svilenov and Nikolov, 2021). However, similar to the Floating cone method, a notable drawback of the LG algorithm is that it is time-consuming due to the number of computations required to navigate the search space. Nonetheless, it performs faster than the Floating cone method, deeming it the most preferable algorithm for several generations.

The Pseudoflow algorithm

The Maximum Flow Method as an alternative to generating an optimal pit contour has been proven to be a more efficient approach for finding the maximum closure, as direct searching for it has been deemed inefficient (Hochbaum & Chen, 2000; Hochbaum, 2001; Chandran & Hochbaum, 2009; Bai et al., 2017). A variant version of a graph, known as a flow graph or flow network, has been established as a more effective method for this purpose. To comprehend this concept more easily, a suitable analogy is that of a flow graph as a network of pipes designed for transporting water from one city to another. In this flow graph, two special nodes are present: the source node, where the flow initiates, and the sink node, where the flow concludes.

Each arc in the flow graph, analogous to a pipe, possesses a capacity property, allowing a flow to pass through, but only up to the capacity limit. It is essential to note that both the flow and capacity along an arc must be positive. Moreover, the nodes in the graph represent a junction of pipes, meaning that the flow into a node must be equal to the total flow out of the same node, adhering to the flow balance criteria. The primary objective in this network is to search for a flow distribution that yields the maximum total flows entering the sink node (or leaving the source node). This is commonly referred to as the maximum flow problem, which has been demonstrated to be equivalent to the maximum closure problem for obtaining an optimal pit solution (Bai et al., 2017). However, the relationship between water flow and mining concepts is not straightforward. To elucidate this association, ore is represented as water stored in a source city that must be efficiently transported to a destination city through a pipe network. In this network, the economic value of a block is not reflected on a node, but rather measured by the capacity of the pipe (arc) connecting it with the source or destination city. The three types of pipes identified are "waste-to-destination," "source-to-ore," and "block-to-block." When the maximum flow is achieved, it guarantees the optimal utilisation of ores to cover the costs of waste excavation (Bai et al., 2017).

The Pseudoflow algorithm has been proven to produce the same results as the LG algorithm and is considered to be one of the most efficient methods for solving the maximum flow problem (ibid.; Poniewierski, 2017). Its time complexity allows for larger models, containing millions of blocks to be solved in significantly less time, compared to the LG algorithms, which deems it preferable in the newer pit optimisation solvers.

***MiningMath*'s MILP Branch and Cut algorithm**

Unlike current best practices, *MiningMath* handles all parameters simultaneously and generates multiple scenarios, enabling a more comprehensive solution space exploration. By doing so, *MiningMath* provides a more accurate representation of the mining operation, optimising the entire project instead of focusing on individual stages. *MiningMath* employs a flexible mining optimisation algorithm that combines Mixed Integer Linear Programming (MILP) with linearisation methods to address the non-linear aspects of the problem. It also uses its own Branch and Cut algorithm, which is fine-tuned to the specific optimisation problem and provides more efficiency than standard MILP optimisers (https://miningmath.com/docs/knowledgebase/).

The *MiningMath* and LG/Pseudoflow methods are both used for pit optimisation, but they differ in their approach to maximising project cash flow. One of the key features of *MiningMath* is its use of surface-based formulations, allowing the inclusion of geometric constraints and the provision of solutions that more closely resemble real mining operations, compared to the other algorithms (https://miningmath.com/docs/knowledgebase/). The user has the flexibility to include mining and bottom widths, maximum vertical advance rates, and other constraints that guide the geometry and direct the optimisation problem toward a more realistic and technologically feasible search space. This approach also eliminates geotechnical errors that can arise with block precedence methods, which may provide slopes that are steeper than what is desired. Furthermore, *MiningMath*'s model optimises all periods simultaneously, and hence, it has the potential to find higher NPVs than traditional procedures based on LG/Pseudoflow nested pits. Last but not least, the Branch and Cut method allows for accelerating the solving time.

Assumptions regarding pit limits optimisation

Established practices

The process of strategic mine planning and pit limits optimisation involves breaking down a large pit into smaller problems, called pushbacks, for efficient mine production scheduling. Typically, this process is divided into three stages: finding nested pits, defining pushbacks, and creating schedules. This division is made in order to obtain the best solution within a realistic time frame. Various techniques and algorithms can be used for each stage, but the ultimate goal is to identify the final pit limit that maximises cash flow and afterwards to determine the optimal extraction block sequence within this limit. However, this approach is not short on its drawbacks in terms of underestimating the NPV of the project, as already mentioned.

Conventional optimisation practices in mining problems assume that all inputs are fixed values and usually do not consider uncertainty associated with different parameters, such as orebody attributes and market uncertainties. This practice of assuming constant metal prices and block values calculated from limited data has major limitations that may lead to unrealistic assessments and decisions. However, as practical and efficient as this paradigm may be, it has some crucial limitations, which need to be acknowledged. The pushback definition process is usually performed manually, and automatic approaches focusing on NPV optimisation often overlook geometric requirements and resource constraints. To mitigate some of the factors that contribute to errors, uncertainties, and approximations in the pit optimisation process, certain practices have been introduced. Scenario strategies proposed by Hall (2014) and Whittle (2009) can be helpful in designing for risk minimisation. However, it is also important to check optimistic scenarios to determine infrastructure boundaries. Ultimately, the choice of pit optimisation method depends on the project's stage, objectives, and resources.

Stochastic approaches

Recent efforts have been made to include uncertainty in mine design by using a set of equally likely orebody models in combination with stochastic optimisation for the mine production scheduling problem (Dimitrakopolous et al., 2002, Rimélé et al., 2019, Jelvez et al, 2022). Stochastic integer programming (SIP) models have been developed to maximise net present value (NPV) and minimise deviations from production goals while streamlining the use of a stochastic stockpile. However, the SIP approach is computationally demanding, and researchers are currently addressing this issue through parallelisation.

Leite and Dimitrakopoulos (2009) have developed stochastic optimisation methods using a simulated annealing algorithm to optimise scheduling. Although computationally efficient, the approach requires a labour-intensive preparation of inputs.

Additionally, scheduling methods, such as genetic algorithms and dynamic programming, generate a limited variety of solutions due to the single pushback input. Although thousands of potential schedules can be generated with different methods, they all follow the same stepwise approach, which restricts the exploration of the solution space. Additionally, a notable drawback of heuristic and meta-heuristic approaches is that a global optimum solution is not guaranteed and hence this field of study of their implementation for pit optimisation can still be considered experimental. Nonetheless, they prove to be a promising approach to scheduling and potentially to pit limits optimisation due to the reduced calculation time required for converging to an acceptable solution.

Unresolved problems and possible solutions related to pit limits optimisation

In the mining industry, planning models often rely on shortcuts and approximations to accommodate the complexities and constraints of a project. This necessitates using powerful computational resources to find the optimal pit limit and mining sequence that delivers maximum project value. One common approximation is using blocks with vertical sides to represent a pit design with nonvertical sides, leading to errors in the tonnes and grade output. Hence, slope accuracy representation plays a crucial role in modelling, with the accuracy relying on the number of dependencies used to define the slope. Larger blocks may give less slope accuracy, while more, smaller blocks can allow for greater accuracy but will slow down computations. An average error of 1° is considered acceptable (Poniewierski, 2017). Additionally, the effect of minimum mining width on the bottom of a shell is often overlooked, even when the package used provides such a facility, which can change the value of the selected shell used for design. It is important to note that no algorithm can give a completely accurate solution to the pit optimisation problem, as all three algorithms rely on different assumptions making the problem solvable in a discrete space.

Despite the significant efforts put into developing advanced optimisation algorithms, there is often not enough emphasis on ensuring the accuracy and reliability of the data used in the optimisation problem and the proper utilisation of the results obtained. As with any optimisation method, the accuracy and reliability of the results heavily depend on the input parameters, such as metal prices, mining costs, and slope angles. Subjectivity in these parameters can lead to biased outcomes. A common way of performing pit optimisation is by assuming a single value for each input variable in each scenario. This may lead to certain biases in the model as influential factors usually vary over time. Although this can be avoided by dividing the life of mine into distinct periods, assuming different sets of values for the input variables is a labour and computationally intensive process and hence it is avoided in many cases. An alternative shortcut to this problem is by assuming factors, dependent on bench levels taking into account the variability of costs for a single scenario.

Another problem worth mentioning is the choice of using discounted or undiscounted cashflows to calculate a pit contour's profitability. The discounted ultimate pit is proven to be no larger than the undiscounted ultimate pit, according to Caccetta and Hill (2003), who stated that the discounted ultimate pit is a subset of or equal to the undiscounted ultimate pit. This theorem was validated through a case study, although no robust mathematical proof was used (Askari-Nasab et al., 2011). Nonetheless, the global validity of this theorem is a key issue, as optimisation of NPV instead of undiscounted profit is more idealised. On the one hand, introducing time as a factor influencing profitability is not an easy task, especially when considering cashflow modelling under uncertainty. Hence, the NPV approach is an easy-to-use alternative compared to using predictions of commodity prices, annual costs and inflation levels. On the other hand, the extraction of certain blocks, which are considered unfeasible during an optimisation stage, occurs during the later stages of the life of mine. Hence, the economic value for these blocks is not entirely accurate, considering the stochastic nature of the economic value due to the uncertainty of commodity price and ore grade estimations. Nonetheless, the calculation of the NPV is a more preferred parameter accounting for the project's cashflows, given that time is considered a factor in the decision-making process.

A DOE approach to the pit limits optimisation problem

The use of a Sensitivity analysis is a common way of interpreting the most influential factors on the NPV of a mining project at a design stage. However, the choice of a sampling strategy can be problematic when dealing with multiple factors and multiple factor levels which can influence the magnitude of the NPV. The most commonly used method for studying the influence of different factors on the magnitude of the NPV are first-degree models by applying the One-At-a-Time (OAT) approach (Nikolov et al., 2019; Chaves et al., 2020; Svilenov and Nikolov, 2021). However, a more efficient and more robust approach is required when dealing with multiple factors. The Design of Experiments (DOE) approach can be considered a complementary way to Sensitivity analysis, for several reasons:

1. Compound factor influences can be investigated in a controlled and systematic way in the input feature space. In addition, DOE allows studying the variation of multiple factors by efficiently gathering new data points, while at the same time, the required number of runs can be drastically reduced.
2. DOE can be used for testing the system's response by applying different inputs to optimise the system. Hence, this provides a systematic way of finding the best set of controllable factors under different external conditions.

3. The DOE approach provides a way for the investigation of the robustness of a system by exploring the variation of the response variable. Hence, this approach is suitable for the future implementation of optimisation algorithms with randomised components for solving the ultimate pit limits problem.

Fractional factorial design

Fractional factorial design is used in statistics and experimental studies to efficiently investigate the effects of multiple factors on a response variable. It is particularly useful when many factors are involved, and conducting a full factorial experiment (testing all possible combinations of factors) would be impractical or resource-demanding.

In a full factorial design, if there are a *k* number of factors, each with *n* levels, a total of n^k experimental runs should be conducted. In contrast, fractional factorial design reduces the number of experimental runs by running only a carefully selected subset of the full factorial combinations. The goal is to obtain enough information about the main effects and some interaction effects while minimising the number of experiments needed. The selection of the fractional factorial design is based on different design strategies, which ensure a balanced distribution of factor combinations. Fractional factorial designs are commonly denoted as 2^{k-p} designs, where *k* is the number of factors, and *p* is the number of factors that are fractionated (run at only a subset of levels). The division from the full factorial design is expressed as $1/2^p$ (Anthony, 2014). This can significantly reduce the workload and resources needed for experimentation.

While fractional factorial designs provide useful information about the main effects and certain interactions, they have their limitations. Some higher-order interactions may be confounded with main effects or lower-order interactions, making their interpretation challenging. As such, these designs are best suited for situations where the primary focus is on identifying significant main effects and the most important lower-order interactions. Other design methods are considered more appropriate for more complex studies.

Screening design

In Design of Experiments (DOE), a Screening design is a type of experimental approach used to quickly and effectively pinpoint the key factors that strongly influence the response variable of interest (ibid.). The primary goal of a screening design is to identify these influential factors while reducing the number of experimental trials, resources, and time required for the overall experimentation process. Screening designs are particularly valuable when dealing with numerous factors, making it impractical to investigate all possible combinations exhaustively. Instead, these designs involve selecting a smaller subset of experimental trials that represent a fraction of the total feasible combinations of factors. The chosen subset is thoughtfully selected to ensure that critical main effects and, if possible, some interactions between factors be accurately estimated.

The most commonly used Screening design is the Fractional factorial design. By employing mathematical algorithms, like orthogonal arrays or other statistical methods, the design ensures that the selected subset of runs provides enough information to estimate the main effects and, potentially, some two-factor interactions with precision. Upon conducting the experimental runs and collecting response data, statistical methods are applied to identify the significant factors that have a notable impact on the response variable. These influential factors can then be further investigated in subsequent experiments, such as optimisation or response surface designs, to refine the understanding of their effects and interactions.

DOE application in the current case study

For the purpose of this study, the use of the NPV was considered to be a better-suited optimisation criterion, as the aim of this paper is to rank key input variables, which represent factors, influencing the ultimate pit profitability by also introducing time as a factor. The purpose of this ranking is to provide the pit design team with quantifiable information for the set of input factors, which deserve further attention when developing different ultimate pit scenarios. In addition, each scenario assumes a single value for each factor variable. A total of 12 factors were considered for this analysis which range in their corresponding domains, depending on the set of technological solutions proposed for the exploitation of the deposit (Table 1).

Table 1. *Input parameters range, accounting for uncertainty*

Parameter	Notation	Min	Base	Max
Variation in Cu grade	A	-15%	0	+15%
Variation in Au grade	B	-15%	0	+15%
Cu Price, USD/t	C	6000	8000	10 000
Au Price, USD/g	D	35	55	75
Cu Extraction	E	0.80	0.85	0.90
Au Extraction	F	0.65	0.73	0.80
Bottom width, m	G	60	90	120
Mining width, m	H	90	120	150
Overall slope, °	I	35	40	45
Mining costs, USD	J	3.40	3.70	4.00
Processing costs, USD	K	10.63	12.50	14.38
Mine recovery, %	L	0.9	0.93	0.95

The discount rate is assumed to be 10% for all scenarios, which are represented as a set of equally probable economic block models. It should be noted that all 12 factors were considered to be independent of one another, as the inherent randomness of the noise in a mining operation results in the shift of each parameter in a random direction from the initially planned value. One can argue that certain dependences can exist, e.g. between processing costs and the extraction of Cu and Au; however, the implementation of such relationships would require a different sampling method, which is not in the scope of this case study. Additionally, when dealing with a deposit at a pre-feasibility stage, these relations cannot be described robustly. Nonetheless, it is worth investigating how different relationships between input parameters can influence the NPV distribution at a later stage of the life of mine, when actual field data is used for certain factors.

For this case study, the Marvin deposit (https://miningmath.com/docs/knowledgebase/formatting-data/datasets) was used for 2 major reasons:

1. The Marvin deposit’s block model is a well-known model, used as a “testing ground” for different illustrations of concepts and experimental studies in the field of open-pit

mining. Hence, results for this model are comparable, as it is a public dataset;
2. The deposit is fairly "light" with a total of 53 271 blocks, which makes it an easy-to-handle model from a computational point of view.

The software used for ultimate pit optimisation for this case study is *MiningMath (v2.3.52)* (https://miningmath.com/) due to its flexibility for applying different geometrical constraints and features regarding the open-pit design. Table 2 shows the constraints used for this case study, which aim to introduce an element of realism into the software's solution by using a ramp-up in the ore's annual output, as well as different periods in the life of mine in terms of waste output.

Table 2. *Material output constraints*

Period (years)	Ore volume constraint, Mt	Waste volume constraint, Mt
1 – 5	10	75
2 – 10	25	125
10 – 30	50	125
30 – end period	25	0

As shown, production periods are grouped in 5-year bins. Two experimental design alternatives were used for this case study – a Definitive Screening design, (25 runs) and a Fractional factorial design (128 runs). All 12 factors were introduced in both cases, so as to compare how different each estimation would be, given that a different number of runs is used for each experiment. Mining sites, which have to deal with block models consisting of millions of blocks, are limited in the number of considered scenarios, as each one can take up to several hours to find an optimal ultimate pit contour, depending on the complexity of the algorithm used. So far, such mining sites can rely solely on a limited number of scenarios, which leads to dealing with scenarios that may not be representative of the vast number of solutions in the feature space. However, this can be avoided by using the DOE approach. The software used for the DOE analysis is *Minitab*.

Results and discussion

A Resolution IV design was assumed for this case study. The design generators used include the following relations: H = ACDG; J = ABCD; K = BCFG; L = ABDEFG; M = CDEF. Hence, the number of runs for the Fractional Factorial design is 128 (2^{12-5}). The defining relations used are as follows: I = ACDGH = ABCDJ = BCFGK = ABDEFGL = CDEFM = BGHJ = ABDFHK = BCEFHL = AEFGHM = ADFGJK = CEFGJL = ABEFJM = ACDEKL = BDEGKM = ABCGLM = CFHJK = ADEFHJL = BCDEFGHJM = EGHKL = ABCEHKM = BDHLM = BEJKL = ACEGJKM = DGJLM = AFKLM = ABCDEGHJKL = DEHJKM = ACHJLM = CDFGHKLM = BCDFJKLM = ABFGHJKLM.

Fig. 2a and 2b show the distribution of the NPV, provided from each optimisation scenario with its corresponding set of input variables.

It can be observed that both NPV distributions resemble the Normal distribution, which can be attributed to its well-known property that it is the product of the influence of a number of independent random variables. This can be observed in the case of investigating the set of optimal solutions in a pre-planning stage or at a pre-feasibility stage, when the input data regarding the cost model of the pit optimisation problem is very limited.

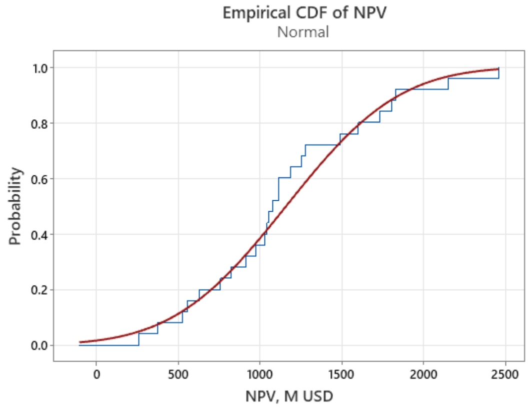

Fig. 2a. Empirical CDF vs Normal distribution CDF (Screening design)

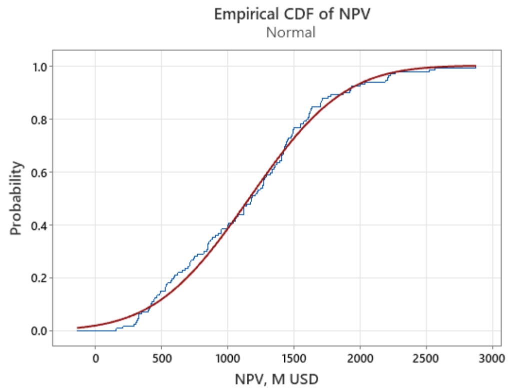

Fig. 2b. Empirical CDF vs Normal distribution CDF (Fractional Factorial design)

Indeed, there is no evidence that both distributions behave in a non-normal manner, as the results from the Kolmogorov-Smirnov test yield values above 0.05 ($p > 0.150$) for both the Screening and the Factorial design.

Table 3 shows the descriptive statistics for the estimated NPV in both design methods.

Table 3. *Descriptive statistics for obtained NPV results, M USD*

Descriptive statistics	Screening design	Factorial design Resolution: IV Fraction: 1/32
Runs	25	128
Mean	1158.90	1161.07
Median	1073.50	1179.00
Std. deviation	544.65	560.15
Min	255.9	155.9
Max	2460.1	2873.97
Skewness	0.58	0.39
Kurtosis	0.14	-0.10

Results obtained by both methods are similar to one another in terms of mean, median, and standard deviation values. Both distributions tend to be positively skewed, however, the skewness seems to decrease with a higher number of observations. The kurtosis in both cases is relatively low, which further implies that both distributions resemble the shape of the Normal distribution.

These results alone can be considered a good argument that both methods yield a similar set of NPV values. However, this resemblance should be studied further by applying a suitable regression model in both cases. Results for both models are shown in Table 4.

Table 4. *Screening design model and Factorial regression model*

Parameters	**Screening design**	**Factorial design Resolution: IV Fraction: 1/32**
Centre points	1	0
R^2	0.9681	0.9902
R^2 (adjusted)	0.9361	0.9886
R^2 (prediction)	0.8614	0.9865

The regression model formula obtained from the Screening design is as follows:

$$\begin{aligned} NPV = {} & 34502 + 1153\,A + 849\,B + \\ & + 0.1605\,C + 17.04\,D - 98552\,E + \\ & + 1399\,F + 50.0\,G + 22.50\,H - \\ & - 131.0\,I - 34.2\,J + 58709\,E^2 - 0.2179\,H^2 \end{aligned} \tag{1}$$

It could be observed that certain variables are of second order, which would mean that the relationship between the factors and the NPV is non-linear, although it exhibits a predominantly linear behaviour in some sections of the feature space.

The formula for the regression model obtained from the Factorial design is as follows:

$$\begin{aligned} NPV = {} & 705 - 2232\,A - 1127\,B - \\ & - 0.2520\,C - 0.66\,D - 2280\,E - \\ & - 182\,F - 2.027\,H + 8.63\,I - \\ & - 13.5\,J - 31.44\,K + 1711\,L + \\ & + 0.1902\,A\,C + 2165\,A\,E + 17.74\,B\,D + \\ & + 1481\,B\,F + 0.2563\,C\,E + \\ & + 0.001344\,C\,I - 0.01331\,C\,J \end{aligned} \tag{2}$$

Once more, certain compound factor effects are also present in the Factorial design model. However, this model serves only as an indicator of the compound effects which have the highest influence on the estimated NPV. Higher-order interactions can indeed exist, however, the modelling accuracy is satisfactory, which does not necessarily imply that further investigations are required.

It should be pointed out that for both equations, the Variation of Cu estimation and the Variation of Au estimation assumed values of 0.85 and 1.15 for their lower and upper bounds. These are the fractions used for multiplication with the ore grades from the block model.

In order to verify both models, the residuals in both cases were investigated in terms of their randomness, independence, and homoscedasticity. As seen below, the residuals in both cases follow a near-normal distribution, judging by both histograms (Fig. 3a and 3b).

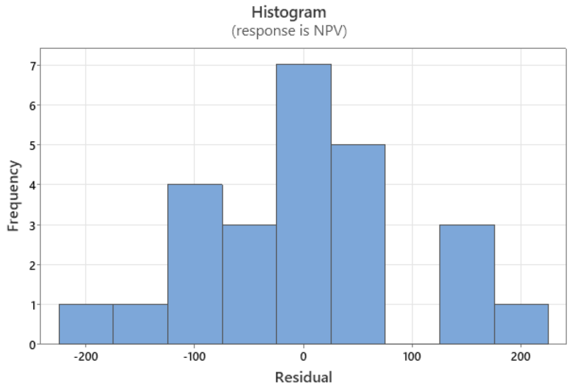

Fig. 3a. Residuals histogram (Screening design)

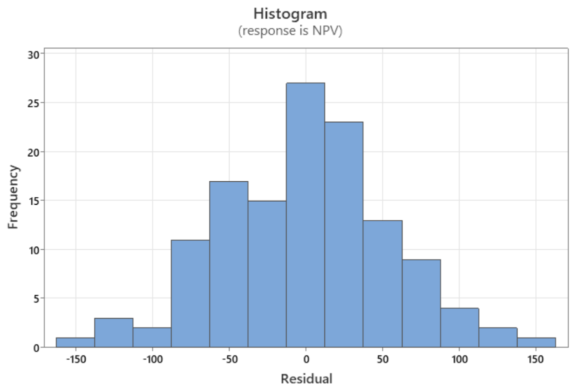

Fig. 3b. Residuals histogram (Fractional Factorial design)

This can be further supported by the graph for the empirical data and their corresponding values from the normal distribution (Fig. 4a and 4b).

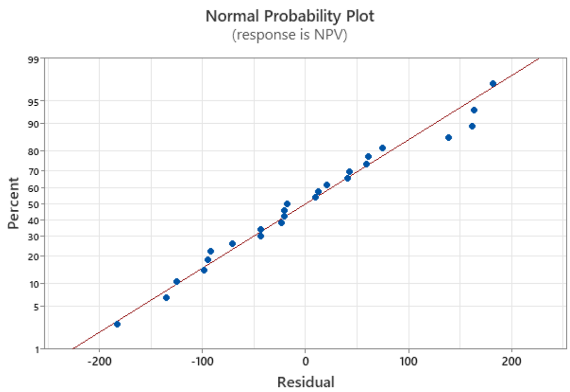

Fig. 4a. Regression model residuals normality resemblance (Screening design)

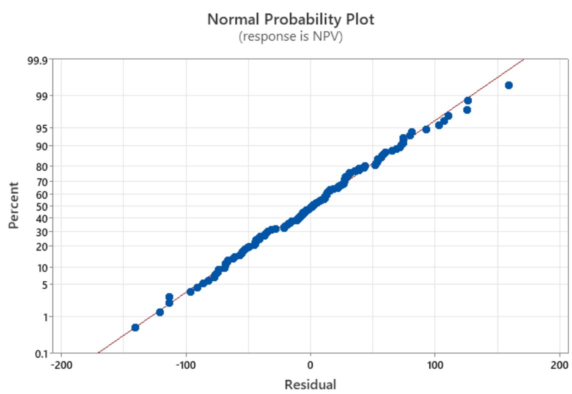

Fig. 4b. Regression model residuals normality resemblance (Fractional Factorial design)

Once more, the Kolmogorov-Smirnov test was used, which yielded p > 0.150 for both design methods. Hence, there is no evidence to believe that both distributions are different from the Normal distribution. In terms of their independence, in both cases, the residuals do not display evidence of autocorrelation, as well as heteroscedasticity (Fig 5a and 5b).

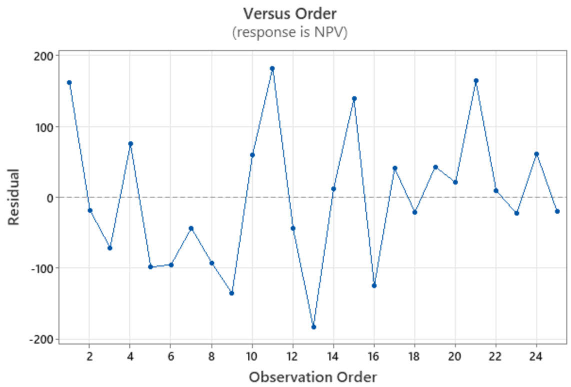

Fig. 5a. Variation and randomness of regression model residuals (Screening design)

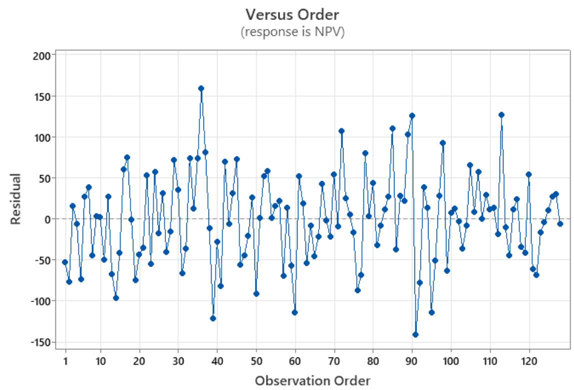

Fig. 5b. Variation and randomness of regression model residuals (Fractional Factorial design)

There is a residual with a higher value, as can be observed in Fig 5b; however, this should not be surprising as the NPV values can vary significantly with a small change in a certain factor. Additionally, big changes in the input parameters may lead to smaller changes in the NPV. This is especially true when a certain constraint is relaxed for the optimisation problem.

Indeed, both models can be considered valid, and their superb accuracy evidently shows that the NPV response variable is predominantly linearly dependent on the assumed factor levels. Although there is still evidence of the variability of the estimated NPV by the software, on a bigger scale the NPV response can be approximated by a non-linear model with low-level interactions between the major factors. However, a certain level of the NPV's variation cannot be explained by this approach. This can be attributed to the inherent problem with models, which rely on the NPV as an optimisation criterion – the mining of certain blocks is postponed due to their infeasibility at the initial stage of their evaluation. Additionally, the relaxation of some constraints can also lead to non-linear behaviour. Nonetheless, the actual relationship between the 12 input parameters with the NPV can be estimated with satisfying accuracy when considering combinations of best and worst-case scenarios. Therefore, the purpose of both DOE regression models is not to eliminate the further need to use the optimiser, but rather to quantify the relative importance of different factors, denoted by the model's input variables. Furthermore, this can provide a rational way of identifying which controllable factors deserve attention in terms of applying changes in the considered mining method, pit design geometry, or processing technology. Additionally, this would also provide a way of estimating which uncontrollable factors require further investigation in order to reduce the variability of NPV's distribution (geological surveying, financial modelling, etc.). Last but not least, the estimated regression models can also serve as a basis for an optimisation problem for finding the best search space for exploring different sets of controllable parameters in a certain scenario. In any case, the obtained results should be validated by solving the optimisation problem via the pit optimiser with different sets of input variables, as the absolute error of the regression model can reach over 150 M USD. Nevertheless, this approach can significantly accelerate the process of refining the ultimate pit design, as well as the choice of mining method and processing technology.

Fig. 6a and 6b show the standardised importance of the considered 12 factors with the addition of second-order and compound factor influences.

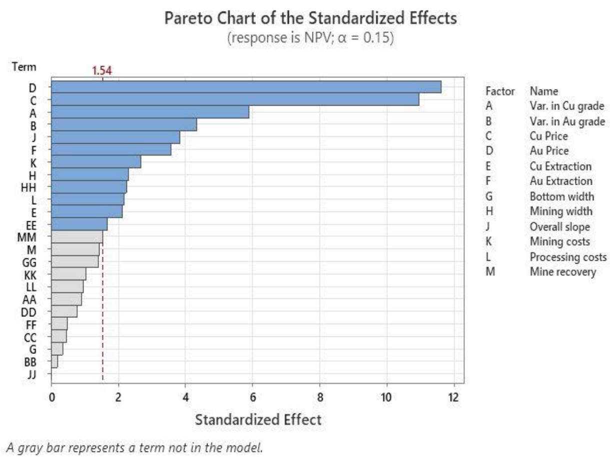

Fig. 6a. Pareto chart of standardized effects (Screening design)

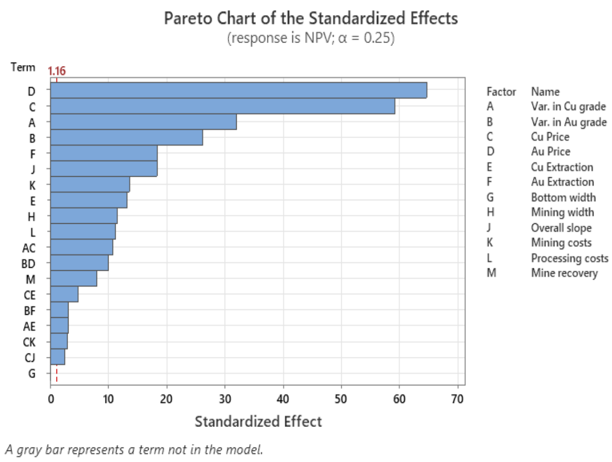

Fig. 6b. Pareto chart of standardized effects (Fractional Factorial design)

A comparison of both results can be seen in Table 5, which represents each factor's rank in terms of its influence on the NPV result from the optimiser.

Table 5. *Factor ranks for Screening and Fractional Factorial design*

Factor rank	Screening design	Factorial design Resolution: IV Fraction: 1/32
1	Au Price*	Au Price*
2	Cu Price*	Cu Price*
3	Var. in Cu grade*	Var. in Cu grade*
4	Var. in Au grade*	Var. in Au grade*
5	Overall slope*	Au extraction*
6	Au extraction*	Overall slope*
7	Mining costs*	Mining costs*
8	Mining width*	Cu extraction*
9	Processing costs*	Mining width*
10	Cu extraction*	Processing costs*
11	Ore losses	Ore losses*
12	Bottom width	Bottom width

** Statistically significant factors*

As can be observed, both design methods provide similar results regarding factor rankings, which additionally correspond to intuitions from practical experience.

Indeed, commodity prices are the most influential, as copper and gold prices are volatile and can vary in very wide ranges, especially over longer periods of time. Additionally, the accuracy of the Cu and Au grades in the geological block model can significantly influence the profitability of the mining operations. Both underestimating or overestimating ore grades can lead to significant changes in the estimated NPV value. Moreover, dilution from different mining technologies is also a factor worth further attention. The extraction of both commodities, yielded by the choice of mineral processing method, are also important parameters which deserve worth investigating. Additionally, the overall slope is also a crucial factor, which requires a detailed cost-benefit analysis in terms of finding a rational set of design parameters under geotechnical uncertainty and safety requirements. Mining costs and mining width correspond to the choice of mining method, as well as equipment choice, which can be considered a different optimisation problem on its own. Ore losses in the studied domain do not seem to affect significantly the NPV’s magnitude and therefore can be considered non-essential factors. Last but not least, it should be pointed out that factor rankings may vary for different site conditions and hence the point of the DOE approach is to estimate them in a robust way in the specifics of each mining site.

Conclusion

A major conclusion, which can be drawn for the magnitude of the ultimate pit NPV is that it is heavily dependent on the input parameters, as the MILP model applies a Branch and Cut algorithm for disregarding parts of the deposit that are deemed infeasible or suboptimal at a certain stage of the life of mine. Additionally, changes in the input parameters can lead to changes in the pit development schedule or the relaxation of certain constraints, which can further influence the NPV value of an evaluated scenario.

Regardless of its limitations in capturing the details of the NPV’s volatility, the DOE approach can be used as a guideline for estimating the key factors which influence the NPV results for the ultimate pit optimisation problem. Furthermore, the high modelling accuracy provides a way of improving the exploration of the feature space for more feasible design alternatives. Although the dependence between the input variables and the NPV value is non-linear in the hyperspace domain of the input parameters, applying a combination of best-case and worst-case scenarios or best-case, worst-case and realistic scenarios for the set of input parameters can provide a very good understanding of how the profitability of different scenarios is influenced by each factor. As a superior way, compared to the conventional One-At-a-Time approach, the DOE approach applies statistical power in order to regard each factor as significant or insignificant. Additionally, it also provides a way of quantifying the relationship between different factors and their compound effect on the magnitude of the NPV. Last but not least, a Screening design could be a reasonable solution in cases where block models of a substantial size are investigated with optimisation algorithms with higher time complexity (e.g. the Lerchs-Grossman algorithm), as 25 scenarios provide similar results, compared to a Fractional Factorial design with 128 runs. Results in this case study prove to be similar in every way (NPV distribution, factor ranks), which implies that both approaches can be utilised, depending on the scale and complexity of the deposit’s block model. Nonetheless, a design with 128 runs is preferable, as the variation of the model’s residuals is lower. Additionally, the estimate of the NPV’s variance is more accurate in the design with 128 runs.

For future work, this approach would require further verification in different geological conditions, as well as for more complex block models and more complex optimisation scenarios including ore blending constraints, the addition of dilution, varying hauling costs and commodity prices in time. Additionally, as an improvement over the current approach, more or fewer scenarios could be considered with different sampling strategies, with the addition of correlating factors, as this could lead to a more realistic understanding of the NPV’s volatility in the input parameter’s hyperspace. Furthermore, data mining (more specifically machine learning) could also prove to be a viable approach to this problem, given that a proper sampling strategy is used. Regardless of which approach is considered for practical use, the parallelisation for the ultimate pit scenario calculations is also a problem worth studying for the sake of reducing computational time.

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