STRESSES AROUND A CIRCULAR OPENING EXCAVATED IN LAYERED AND SLIDING ROCK MASS

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ABSTRACT: The article focuses on the issue of determining the stresses in a steeply layered rock mass around a circular opening. The rock mass consists of homogeneous isotropic layers. The boundary between them is steeply inclined to the horizon. The influence of stresses due to the drawing of the opening extends to a square area. The specified class of problems is solved by the method of complex variable functions and the mechanics of layered media. This problem has been solved in previous works of the authors'.

In this paper, it is proposed to take into account the behaviour of the rocks around the hole over time. Here, the hereditary theory of elasticity, or more precisely the method of variable modules, is used. The rock mass consists of two layers. The expressions for the mechanical constants and the expressions for the stresses in each layer after a certain time are obtained.

The results are applied to a real rock mass. Mechanical constants are calculated at two points in time. Two diagrams of the normal tangential stresses for points from the contour of the hole are given. The first diagram shows the stresses at the initial point of time. The stresses after a certain time are shown in the second diagram.

Key words: method of complex variable functions, mechanics of layered media, method of variable modules.

НАПРЕЖЕНИЯ ОКОЛО КРЪГОВА ИЗРАБОТКА, ПРОКАРАНА В НАПЛАСТЕН И ПЪЛЗЯЩ МАСИВ

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РЕЗЮМЕ: В статията се разглежда въпросът за определяне на напреженията в стръмно напластен масив около кръгова изработка. Масивът се състои от хомогенни изотропни пласта. Границата между тях е стръмно наклонена спрямо хоризонта. Влиянието на напреженията, дължащи се на прокарването на изработката се простира в квадратна област. Указаният клас задачи се решава с методите на функции на комплексна променлива и на механика на среди с пластове. Тази задача е решена в предишни работи на авторите.

В тази работа се предлага да се отчете поведението на скалите около отвора във времето. Тук се използва наследствената теория на еластичността или по-точно метода на променливите модули. Масивът се състои от два пласта. Получени са изразите за механичните константи и изразите за напреженията във всеки пласт след определено време.

Резултатите са приложени за реален масив. Изчислени са механичните константи в два момента от време. Дадени са две диаграми на нормалното тангенциално напрежение за точки от контура на изработката. На първата диаграма са представени напреженията в начален момент от време. Напреженията след определено време са изобразени на втората диаграма.

Ключови думи: теория на функции на комплексна променлива, механика на напластените среди, метод на променливи модули.

Introduction

Analytical methods exist for determining the stresses around a circular mine opening. In most studies, researchers consider the rocks to be isotropic. The solution of the problem is complicated when taking into account the presence of different layers and their deformation over time. There are developments for an array that consists of parallel and homogeneous layers. Their thickness is commensurate with the opening of the working. The theory of layered media is applied to this class of problem (Trifonova-Genova, 2012). This theory has been extended to the thicker layers (Trifonova-Genova, 2018). This change in stresses over time is investigated by the linear theory of hereditary elasticity (Amisin et al., 1974). Part of this theory is the method of variable modules. It is applied to study the stresses around an opening passing through thin and parallel layers (Trifonova-Genova, 2012).

The aim of the present work is to extend this solution to an array consisting of thicker layers. The latter are isotropic and homogeneous. The opening is circular.

Methods

1. Formulation of the problem

An environment consisting of two isotropic steep layers is considered. The boundary between them is inclined at an angle α to the horizon (fig.1). The influence of stresses extends to a square area. The coordinate system is polar (r, θ). The considered area is loaded with vertical (Q_1, Q_2) and

($\lambda_1 Q_1$, $\lambda_2 Q_2$) horizontal stresses. Their analytical expressions are given in (Trifonova-Genova, 2017). The opening is in a circular form with radius r_o.

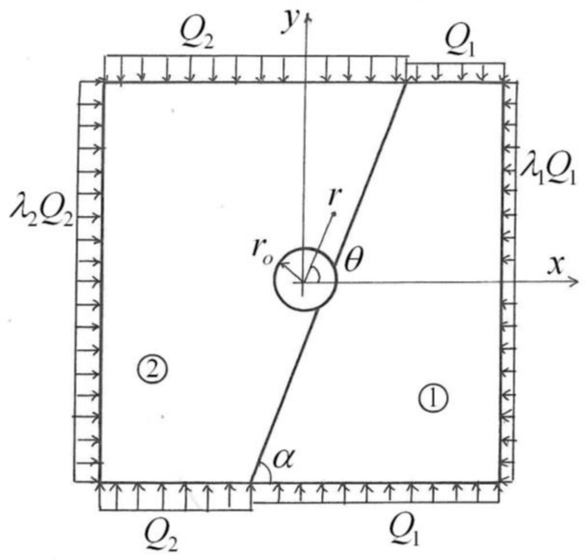

Fig.1 Computational scheme

2. Stresses in randomly located layers

According to the theory of layered environments, researchers replace the layered medium with an equivalent medium. For that, the stress state is obtained after the analytical method of Kolosov and Muskhelishvili (Muskhelishvili, 1966). This method of complex variable functions are further developed by Savin and Lekhnitski (Savin, 1961; Lekhnitski, 1963). The stresses in polar coordinates, volumetric weight, physical and mechanical characteristics of the medium are given in (Trifonova-Genova, 2018). The stresses in an equivalent medium participate in stresses in each layer.

3. Behavior of rock mass over time

The calculation of stresses and strains over time is of practical interest. The theory of hereditary creep is applied. It studies the mechanical constants of the medium and replaces them with temporary integral operators with a creep core. The difficulty in solving problems with this theory lies in the decipherment of operator expressions. These difficulties are circumvented when using the methods of variables modulus developed by Amusin and Linkov (Amusin et al., 1974). In this method, the integral operators of the Young’s modulus and Poisson’s ration are replaced by their own time functions (Trifonova-Genova, 2012; Dimitrov, 2009). For an isotropic medium, these functions have the form:

$$E_t^{(k)} = E^{(k)}\left(1+\Phi^{(k)}\right)^{-1}; \qquad k = 1{,}2\,; \tag{1}$$

$$\mu_t^{(k)} = 0{,}5 - \frac{0{,}5-\mu^{(k)}}{1+\Phi^{(k)}}; \qquad \Phi^{(k)} = \frac{\delta_k t^{1-\alpha_k}}{1-\alpha_k},$$

where

t is the moment of time, [s];

δ_k and α_k are creep parameters;

k is number of layer;

$E^{(k)}$ is Young’s modulus, [MPa];

$\mu^{(k)}$ is Poison’s ration;

$\Phi^{(k)}$ is the creep function.

These functions are used to determine the coefficients of deformations of Hooke’s generalised law. They also express the stresses in a generalised homogeneous medium and those in the individual layer.

4. Stresses in layers, functions of time

Stresses along the contour of the opening are of practical interest (Trifonova-Genova, 2018):

$$\sigma_{r,t}^{(k)} = 0\,;$$

$$\tau_{r\theta,t}^{(k)} = 0\,; \tag{2}$$

$$\sigma_{\theta,t}^{(k)} = \left\{\left[B_{1,t}^{(k)}c^2 + B_{2,t}^{(k)}s^2\right]s^2 + c^4 + 0{,}5s_2^2\right\}\sigma_{\theta,t}^{(o)},$$

where

$$c = \cos\beta\,; \qquad s = \sin\beta\,; \qquad s_2 = \sin(2\beta)\,;$$

$$\beta = \theta - \alpha\,; \qquad A_{*,t} = \left[A_1E_t^{(1)} + A_2E_t^{(2)}\right]^{-1};$$

$$A_{**,t} = \mu_t^{(1)}E_t^{(2)} - \mu_t^{(2)}E_t^{(1)}; \qquad k = 1{,}2\,;$$

$$B_{1,t}^{(k)} = -A_k.A_{**,t}.A_{*,t}\,; \qquad B_{2,t}^{(k)} = -A.A_{*,t}.E_t^{(k)}.$$

Here:

A is the total area of the square, [m2];

A_1 and A_2 are the areas of two layers.

The stresses in the homogeneous medium, due to the creep of rock mass, are:

$$\sigma_{\theta,t}^{(o)} = -2\gamma^{(o)}H\left(\lambda_{1,t} - 2\lambda_{2,t}\cos 2\theta\right), \tag{3}$$

where

$$\lambda_{1,t} = 0{,}5\left(1+\lambda_t^{(o)}\right);$$

$$\lambda_{2,t} = 0{,}5\left(1-\lambda_t^{(o)}\right);$$

$$\lambda_t^{(o)} = \frac{\mu_t^{(o)}}{1-\mu_t^{(o)}}\,;$$

$\gamma^{(o)}$ is volumetric weight of generalised medium, [MN/m3];

$\mu_t^{(o)}$ is Poison’s ration of generalised medium;

H is the depth of circular opening, [m2].

For this medium the functions of Young’s modulus and Poison’s rations are:

$$E_t^{(o)} = \frac{\sum_{k=1}^{2} E_t^{(k)} A_k}{\sum_{k=1}^{2} A_k}; \quad \mu_t^{(o)} = \frac{\sum_{k=1}^{2} \mu_t^{(k)} E_t^{(k)} A_k}{\sum_{k=1}^{2} A_k E_t^{(k)}}. \tag{4}$$

In moment $t_o = 0$, expressions (2), (3), and (4) acquire the form given in (Trifonova-Genova, 2016, 2018).

5. Numerical example

A circular opening with radius $r_o = 1.5m$ is passed to the depth of $H = 100m$. Young's modulus, Poisson's rations, and volumetric weights for two layers are given in table 1.

Table 1. *Physical and mechanical characteristics in layers*

k	$E^{(k)}$	$\mu^{(k)}$	$\gamma^{(k)}$
dimension	MPa		MN/m^3
multiplier	10^4		10^{-2}
1	0.595	0.237	2.8
2	0.148	0.15	2.5
o	0.3986	0.2228	2.668

The vertical stresses along the contour of considered area of fig. 1 are:

$Q_1 = 2.8MPa$,

$Q_2 = 2.5MPa$.

The coefficients of lateral pressure $\lambda_1 = 0.3106$ and $\lambda_2 = 0.1763$ participate in the expressions for the horizontal stresses.

The total area of the square area is: $A = 324\ m^2$. The area of layer 1 is $A_1 = 181.656\ m^2$, and of layer 2 is $A_2 = 142.314\ m^2$.

The creep parameters are listed in table 2.

Table 2. *Creep parameters*

k	α_k	δ_k
dimension	-	$s^{\alpha-1}$
multiplier	-	10^{-2}
1	0.726	0.5715
2	0.670	0.3277

A time interval is considered with limits:

$t_o = 0$,

$t_f = 352 days = 3.05.10^7 s$.

Two tasks are solved for these boundaries. For the first task, the physical and mechanical characteristics are given in table 1. The same characteristics are calculated for second task. The results are shown in table 3. The characteristics were obtained for two generalised environments. Their values are given in the last rows of Table 1 and 3. The volumetric weight of the media is given in table 1.

Table 3. *Physical and mechanical characteristics when crawling on rocks*

k	$E^{(k)}$	$\mu^{(k)}$
dimension	MPa	-
multiplier	10^4	-
1	0.1777	0.4215
2	0.0376	0.4112
o	0.1162	0.4200

Thirteen points are selected along the contour of the circular opening. Their angular coordinates are determined for them.

The normal tangential stresses in the two generalised media are calculated. These stresses are related to vertical stress in a generalised medium before digging the hole ($Q_1^o = \gamma^o H$). The relative normal tangential stresses at contour points are obtained. They are listed in table 4.

Table 4. *Relative normal tangential stresses in generalised media at both ends of a considered time interval*

point	θ	$\frac{\sigma_{\theta,0}^{(o)}}{Q_1^{(o)}}$	$\frac{\sigma_{\theta,t}^{(o)}}{Q_1^{(o)}}$
dimension	$[°]$	-	-
1	0	-2.7	-2.3
2	24˚47’	-2.3	-2.1
3	60	-0.6	-1.4
4	90	0.1	-1.1
5	120	-0.6	-1.4
6	150	-2.0	-2.0
7	180	-2.7	-2.3
8	210	-2.0	-2.0
9	240	-0.6	-1.4
10	270	0.1	-1.1
11	293˚07’	-0.3	-1.3
12	330	-2.0	-2.0
13	360	-2.7	-2.3

Table 4 shows a decrease in the maximum tangential stress at the end of the interval. In the vertical points, this stress from the tensile at the initial moment passes into the pressure at the end of the considered time interval.

The results of the table are illustrated in figure 2 with two curves.

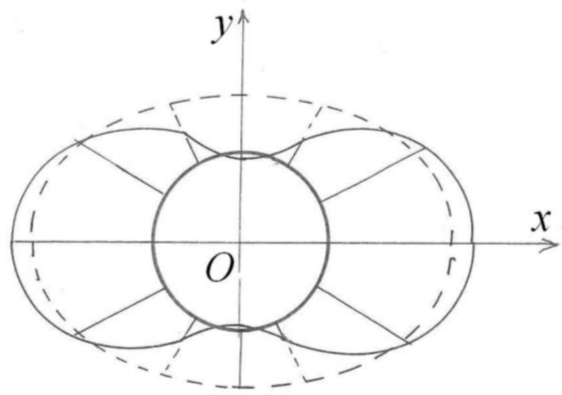

Fig.2. Diagrams of relative normal tangential stresses in generalised media at both ends of the considered time interval.

The solid line gives the relative normal tangential stress at the initial time. The other curve corresponds to the stresses at the end of the interval. Some authors use the solutions in table 4 in pre-design.

The relative normal tangential stresses in two layers, for two times interval limits, are given in table 5. The table shows a change in the maximum relative normal tangential stress in the layers. Thus, in layer 1, this stress decreases and in layer 2, it increases at the final moment.

Table 5. *Relative normal tangential stresses in layers at both ends of considered time interval*

Point	θ	$\frac{\sigma_{\theta,0}^{(1)}}{Q_1^{(o)}}$	$\frac{\sigma_{\theta,0}^{(2)}}{Q_1^{(o)}}$	$\frac{\sigma_{\theta,t}^{(1)}}{Q_1^{(o)}}$	$\frac{\sigma_{\theta,t}^{(2)}}{Q_1^{(o)}}$
dimension	$[^\circ]$	-	-	-	-
1	0	-3.7		-3.2	
2	24˚47’	-2.6	-2.0	-2.3	-1.9
3	60		-0.6		-1.5
4	90		+0.1		-1.2
5	120		-0.5		-1.2
6	150		-0.8		-0.7
7	180		-1.4		-1.1
8	210		-1.6		-1.7
9	240		-0.6		-1.5
10	270		+0.1		-1.2
11	293˚07’	-0.3	-0.3	-0.7	-1.2
12	330	-2.9		-3.0	
13	360	-3.7		-3.2	

It can be seen form the values in the table that there is a change in the stress jump at points on the boundary between the two layers. The jump in point 2 is a difference of 23% at the beginning. It decreases to 17% at the end. At point 11, there is no jump at time (t_0). The stress jump increases to 47% at the end of the interval (t_f).

The relative normal tangential stress, at the vertical points of the contour, changes. It goes from tensile in the initial moment to pressure in the final moment.

Using the data from table 5, the diagrams in figure 3 are constructed. The solid line shows the relative normal tangential stresses at time t_0 .The stresses at the end are given with a broken line.

A comparison the two curves in the figure shows a change in shape. The relative stresses at the horizontal points decrease and at the vertical points increase. Thus, the creep of the array needs to be taken into account.

6. Key finding

Analytical expressions of stresses in each layer are applied in layers whose slope is greater than half the right angle.

In this article, stress expressions are a summary of stress expressions in layers (Trifonova-Genova, 2018). The proposed expressions take into account the behaviour of the rock mass over time.

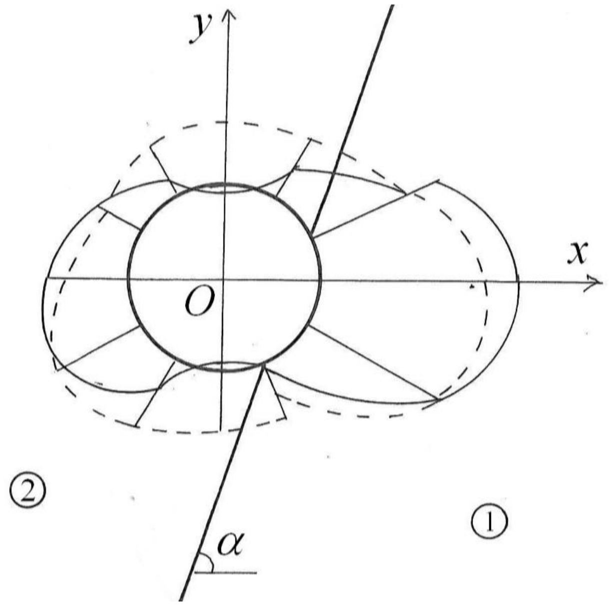

Fig.3. Diagrams of relative normal tangential stresses in the layers at both ends of the considered time interval.

Conclusion

The calculations make the following advantages in solving this class of tasks:

1. The method described in this article is very simple to be implemented.

2. The method makes it possible to summarise many layers. The solution described in the article is part of a generalised methodology for calculating stresses in an array consisting of many layers.

References

Amusin, B., A. Linkov. 1974. About using of variable modulus for solution of class of problems of linear creep. *Proceedings of Academy of Sciences USSR, Mechanics of Solid Body, Vol. 6*, 162-166.

Bulychev, N. S. 1982. *Underground construction mechanics*. Nedra, Moskow, 271p. (in Russian)

Dimitrov, U. 2009. Investigation of the rheological behaviour of an array around a structure with a direct exit to the surface. In: *Conference with international participation "Open and underwater mining“, Varna, Bulgaria*. (in Bulgarian with English abstract)

Lekhnitskii, S. G. 1963. *Theory of elasticity of an anisotropic elastic body*. Holden-Day, San Francisco.

Muskhelishvili, N. I. 1953. *Some basic problems of the Mathematical Theory of Elasticity*. Nordhoft, Gromingen, 706.

Savin, G. N. *Stress concentration around Hole*. Pergamon, London.

Trifonova-Genova, V. 2012. *Napregnato sustoyanie v naplasten masiv okolo krugla izrabotka*. I. K. “Sv. Ivan Rilski”, Sofia, 132p. (in Bulgarian)

Trifonova-Genova, V. 2016. One approach to determining the stresses in stratified field around opening. In: *Proceedings of the VII International Geomechanics Conference, 27 June-01 July, Varna, Bulgaria,* 246-251. (in Bulgarian with English abstract)

Trifonova-Genova, V. 2018. On the determination of stresses in stratified rock around a horizontal opening with a circular cross section. In: *Proceedings of the VII International Geomechanics Conference, 1-6 July, Varna, Bulgaria*, 59-64. (in Bulgarian with English abstract)