STRESSES IN A TRANSVERSAL ISOTROPIC ROCK MASS DUE TO DRIVING A MINE GALLERY

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ABSTRACT: A horizontal underground mining gallery is excavated in a transversely isotropic rock mass. It has a plane of isotropy inclined to the axis of production. The gallery has a circular cross-section. The stresses around the opening are determined by the complex potential theory. The behaviour of the rock mass is described by a partial differential equation as a function of the stresses. It depends on the roots of the characteristic equation. The coefficients in the characteristic equation are defined for a real rock mass. Its roots are the complex numbers. They are involved in the stress function variables and in the expressions for the stresses at points of the opening.

Key words: mine gallery, transversally isotropic rock mass, complex potential theory.

НАПРЕЖЕНИЯ В ТРАНСВЕРЗАЛНО-ИЗОТРОПЕН СКАЛЕН МАСИВ ВСЛЕДСТВИЕ ПРОКАРВАНЕ НА МИННА ГАЛЕРИЯ

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РЕЗЮМЕ: Хоризонтална подземна минна галерия е прокопана в трансверзално-изотропен скален масив. Той притежава наклонена спрямо оста на изработката равнина на изотропия. Галерията има кръгово напречно сечение. Напреженията около отвора се определят с комплексна потенциална теория. Поведението на скалния масив се описва с частно диференциално уравнение на функция на напреженията. Тя зависи от корените на характеристичното уравнение. За реален скален масив са определени коефициентите в характеристичното уравнение. Неговите корени са комплексните числа. Те участват в променливите на функция на напрежението и в изразите за напреженията в точки от отвора.

Ключови думи: минна галерия, трансверзално изотропен скален масив, комплексна потенциална теория.

Introduction

Determining the stresses in the rock mass as a result of driving a mine gallery or tunnel is a major task faced by engineers. Its solution depends on the manifestation of anisotropy and the shape of the cross section of the gallery. The behaviour of the rock mass is described by the partial quasi-bi-harmonic equation (Muskhelishvili, 1953; Savin, 1961; Lu et al., 2024). The solution of this equation depends on the roots of the characteristic equation.

Here, a rock mass having a plane of isotropy will be investigated. According to its location relative to the axis of the gallery, three types of development are known. The first type includes studies in which the plane of isotropy is horizontal or vertical (Vucheva et al., 2023). The second type covers developments in which the plane is inclined to the horizontal axis of the cross section (Tonnon et al., 2003; Vucheva et al., 2020; Trifonova-Genova et al.; 2022). The third type includes studies in which the plane is inclined to the axis of the work.

The aim of the present work is to compile the differential equation of the field for the third case. Furthermore, the steps to solve its characteristic equation are to be described.

Methods

1. Formulation of the problem

A mining gallery with a circular cross-section passes through a transversely isotropic rock mass. Its plane of isotropy is inclined to the axis of gallery.

2. Differential equation

The behaviour of the rock mass is described by several systems of equations. These are the condition of continuity of strains, the generalised Hooke's law giving the relationship between stresses and strains, the expressions for the stresses expressed by the function F(x,z). After uncomplicated mathematical operations, we arrive at a partial differential equation of the sixth order:

$$B_1 \frac{\partial^6 F}{\partial x^6} + B_2 \frac{\partial^6 F}{\partial x^4 \partial z^2} + B_3 \frac{\partial^6 F}{\partial x^2 \partial z^4} + B_4 \frac{\partial^6 F}{\partial z^6} = 0 , \quad (1)$$

where

$$B_1 = c_{22}c_{66} , \quad B_2 = c_{22}c_{55} + c_{10}c_{66} - c_{11}^2 ,$$
$$B_3 = c_{10}c_{55} + c_{33}c_{66} - 2c_{11} ; \quad B_4 = c_{33}c_{55} - c_{35}^2 ;$$
$$c_{10} = (2c_{23} + c_{44}); \quad c_{11} = (c_{25} + c_{46});$$
$$c_{ij} = a_{ij} - \frac{a_{i1}a_{j1}}{a_{11}} ; \; i, j = 2.3....6 .$$

Here, x and z are coordinates of a point from the environment around the opening. The coefficients a_{ij} in these formulae can be seen in the paper by Trifonova-Genova (2019). They are expressed by Young's modulus, Poisson's ratios in the plane of isotropy and in a direction perpendicular to it, and by the slope of the plane of isotropy φ (Fig.1).

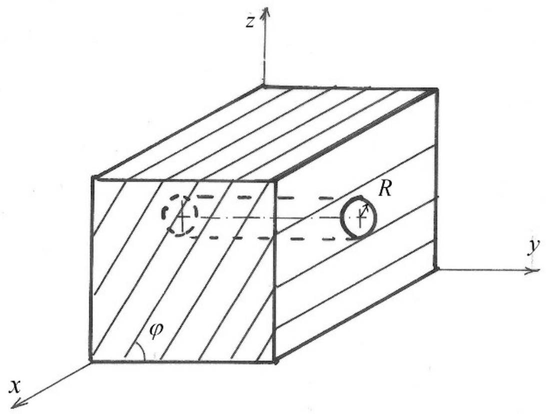

Fig.1. A horizontal circular gallery in a rock mass with an inclined plane of isotropy

3. Characteristic equation

The general integral of equation (1) depends on the roots of the characteristic equation:

$$B_1 s^6 + B_2 s^4 + B_3 s^2 + B_4 = 0 . \qquad (2)$$

This equation is an incomplete equation of the sixth degree. After putting $u = s^2$, equation (2) takes the form:

$$B_1 u^3 + B_2 u^2 + B_3 u + B_4 = 0 . \qquad (3)$$

4. Roots of a characteristic equation

Equation (3) is divided by B_1 and the resulting equation is:

$$u^3 + A_1 u^2 + A_2 u + A_3 = 0 , \qquad (4)$$

where

$$A_1 = \frac{B_2}{B_1}; \quad A_2 = \frac{B_3}{B_1}; \quad A_3 = \frac{B_4}{B_1} .$$

Cardano's formula (Cardano's method) is applied to determine the roots of equation (4):

$$u_1 = S + T - \frac{1}{3} A_1 ; \qquad (5)$$

$$u_2 = \frac{1}{2}(S+T) - \frac{1}{3} A_1 + \frac{1}{2} i\sqrt{3}(S-T);$$

$$u_2 = \frac{1}{2}(S+T) - \frac{1}{3} A_1 - \frac{1}{2} i\sqrt{3}(S-T),$$

where

$$Q = \frac{3a_2 - a_1^2}{9}; \quad R = \frac{9a_1 a_2 - 27a_3 - 2a_1^3}{9} ;$$

$$S = \sqrt[3]{R + \sqrt{Q^3 + R^2}} \ ; \ T = \sqrt[3]{R - \sqrt{Q^3 + R^2}} \ ;$$

$$D = Q^3 + R^2 .$$

If D is positive, then one root of the characteristic equation (4) is real, and both are complex conjugate numbers. The roots are obtained from the expression:

$$s_j = \sqrt{u_j} , \quad j = 1,2,3 . \qquad (6)$$

The remaining three roots are complex conjugated.

The resulting roots are involved in the expressions for the stress functions and their derivatives. Through them, the displacements and stresses in points of the opening of the mine gallery are expressed, which are the subject of further work.

5. Numerical example

The rock mass has a plane of isotropy tilted at an angle $\varphi = 70^\circ$ (Fig.1). For it, Young's modulus and Poisson's ratio are $E_1 = 14{,}5.10^3 N/m^2$ and $\mu_1 = 0{,}105$. In a direction perpendicular to this plane, the parameters are $E_2 = 41{,}5.10^3 N/m^2$ and $\mu_2 = 0{,}3$. The shear modulus in the same direction is $G_2 = 8{,}24.10^3 N/m^2$.

The strain coefficients are given in (Vucheva et al., 2020). The coefficients from (1) and the coefficients in the characteristic equation (4) were calculated. The latter are given in the following table:

Table 1. *Coefficients in equation (4)*

A_1	A_2	A_3
2.4436	2.1704	0.5974

From (5), D is calculated: $D = 0{,}002224$. The roots of equation (4) and the roots of equation (2) are given in Table 2.

Table 2. *Roots of equations (4) and (2)*

u_1	u_2	u_3
-0.4961	-0.6653+0.50605i	-0.6653-0.50605i
s_1	s_2	s_3
0.7043i	0,2938+0.8612i	0.2938-0.8612i
$\bar{s}_1$	$\bar{s}_2$	$\vec{s}_3$
-0.7043i	-0,2938+0.8612i	-0.2938-0.8612i

6. Key findings

The approach described in the work for solving the characteristic equation is applied to a rock mass with an isotropy plane inclined to the axis of the gallery.

Conclusion

The resulting roots are involved in the complex variables of the stress function F(x, z) from (1). It is a sum of two analytical functions of complex variables. Through them, the stresses surrounding crafting are expressed. Their expressions are a sum of a real and an imaginary part. The expressions for the stresses are obtained from this real part. When the stresses along the contour of the mining gallery are small, the engineer guarantees the stability of the gallery.

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