DYNAMIC DETERMINISTIC FACTOR ANALYSIS OF LABOUR PRODUCTIVITY IN THE BULGARIAN MINING INDUSTRY

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ABSTRACT. This article presents a dynamic deterministic factor analysis of labour productivity in Bulgarian mining industry for the 2008 - 2021 period. The objective of this article is to reveal the quantitative impact of the factors of operating income and number of employed persons on the development of the indicator for labour productivity in the Bulgarian mining industry using the averaged chain substitution method. Quantitative impacts of net operating income and number of employed persons on the evolution of the indicator of labour productivity in the mining industry have been identified. The results of the deterministic factor analysis allowed us to draw reasonable conclusions and to reveal the trends of development of the factors of operating income and number of employed persons as well as of the factor of labour productivity in Bulgarian mining industry.

Key words: Bulgarian mining industry, labour productivity, dynamic deterministic factor analysis, averaged chain substitution method.

ДИНАМИЧЕН ДЕТЕРМИНИРАН ФАКТОРЕН АНАЛИЗ НА ПРОИЗВОДИТЕЛНОСТТА НА ТРУДА В БЪЛГАРСКАТА МИННАТА ИНДУСТРИЯ

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РЕЗЮМЕ. В доклада е представен динамичен детерминиран факторен анализ на производителността на труда в българската минната индустрия за периода 2008 – 2021 г. Целта на настоящия доклад е да се разкрият количествените влияния на факторите приходи от дейността и брой на заетите лица върху развитието на показателя производителност на труда в българската минна индустрия чрез усреднения метод на верижните замествания. Идентифицирани са количествените влияния на нетни приходи от дейността и брой на заетите лица върху изменението на показателя производителност на труда в минната индустрия. Резултатите от детерминираният факторен анализ позволяват да се изведат обосновани изводи и да се разкрият тенденциите в развитието на факторите приходи от дейността и брой на заетите лица, а също и на показателя производителност на труда в българската минна индустрия.

Ключови думи: българска минна индустрия, производителност на труда, динамичен детерминиран факторен анализ; усреднен метод на верижните замествания.

Introduction

In recent years, Bulgarian mining industry achieved labour productivity levels that were among the highest both at industrial sector scale and on national economy scale. The increased global production and consumption of metals and energy raw materials proved to be favourable to the development of extraction and processing of subsurface natural resources and, in turn, to the achieved labour productivity in the mining industry. The absence of specialised developments related to the analysis of labour productivity in the Bulgarian mining industry is an obstacle to finding practical solutions to specific complex issues, such as human resources, staff motivation, and factors affecting labour productivity in mining enterprises. It is necessary to clearly outline the trends in the development of the indicator of labour productivity in the Bulgarian mining industry against the background of the increased consumption of the products of mineral raw materials and the growing economic activity of economic entities. According to Kostadinova and Vladkova (2022), “in recent years, the topic of introducing flexible working hours and teleworking is increasingly finding its place in search of modern effective methods to increase productivity in the industrial sector”. This has become particularly relevant in the past two years when, due to the pandemic, a major part of staff had the opportunity to fulfil their duties from home.

At present, we have sufficient statistical information of a macro- and microeconomic nature on past processes and events, based on which we can build trends and outlooks on the future development of that indicator for the Bulgarian mining industry.

The purpose of this article is to present the results and to reveal the quantitative impact of the factors of operating income and number of employed persons of the enterprises from the Bulgarian mining industry on the performance indicator of labour productivity in the Bulgarian mining industry using the averaged chain substitution method for the period 2008-2021.

Research methods and methodology

According to Bartholomew (1984), “deterministic modelling of factor systems is a simple and effective means of formalising the relationship of economic indicators, which serves as a basis for quantifying the quantitative influence of individual factors in the change of the performance indicator”. If we may add, according to Joreskog and Sorbom (1979), “due to the fact that deterministic factor analysis is aimed at identifying the influence of the change in the factors on the change in the value of the resultative indicator of interest, excluding error, it is most relevant for practical application in market conditions”.

The indicator of labour productivity of the Bulgarian mining industry is characterised by the two-factor deterministic multiple model, which has the following form:

$$LP = \frac{R}{N}, \quad (1)$$

where:
R is the operating income of the mining industry sector, in BGN thousands;
N is the number of persons employed in the mining industry sector.

Deterministic factor analysis

Deterministic factor analysis (DFA) is a branch of economic analysis. Its aim is to directly, accurately, and unambiguously quantify the effect that the absolute changes of the participating factor variables (factors) exert on the absolute change of the performance (resultative) indicator in mathematically deterministic factor models.

The types of factor model depend on the type of mathematical dependence describing the relationship between the performance (resultant) indicator (*P*) and the participating factor variables ($a, b, c, \ldots$) in a mathematically deterministic factor model.

In the practice of deterministic factor analysis, the following types of factor model are known:

- *additive or different models* – in them, the resultative indicator is the sum or difference of the factor variables involved, namely: $P = a \pm b \pm c \pm \cdots$;
- *multiplicative models* – here, the resultative indicator is the product of the factor variables involved, namely: $P = a * b * \ldots$;
- *multiple model* – here, the resultative indicator is the quotient of the factor variables involved, namely: $P = \frac{a}{b}$;
- *mixed (combined) models* – these are combinations of additive or different, multiplicative or multiple factor models and can be as follows: multiplicative-multiple, additive or different-multiple, additive or different-multiplicative, and additive or different-multiplicative-multiple models.

In DFA, the following methods have been developed for quantifying the impact of the factors in mathematically deterministic factor models:

- the differential method;
- the coefficient method;
- the chain substitution method;
- the absolute differences method;
- the relative differences method;
- the equity participation method;
- the method of simple addition of an “indecomposable remainder”;
- the weighted finite difference method;
- the logarithmic method;
- the method of increment division into factors;
- the integral method;
- the index method.

Each of the DFA methods has its stages outlined, a specific application, limitations, capacities, advantages, and disadvantages. All these methods are presented in detail in the scientific and educational literature in the field of DFA. Unfortunately, none of the above methods achieves accuracy and unambiguity when it comes to the distribution of the so-called “indecomposable remainder” among the individual quantitative influences of factor variables on the absolute change of the resultative indicator. This “indecomposable remainder” is the result of the simultaneous (combined) change of factor variables in the presence of a multiplicative or multiple (relative) element in the type of factor model. The various DFA methods employ different means of distributing the “indecomposable remainder” among the individual factor influences.

Averaged chain substitution method

The averaged chain substitution method is a new DFA method characterised by absolute accuracy and unambiguousness of the results, and it is universally applicable to all factor models.

The stages, essence, advantages, disadvantages, and results of the approbation of the average method of chain substitutions is presented by Mitev (2020, 2021, 2022, 2023). Unlike the other DFA methods, the averaged chain substitution method has complete universality of application and absolute accuracy and clarity for all types of factor model when quantifying the individual factor influences of the involved factor variables on the absolute change of the performance indicator (ΔP).

For the purposes of dynamic deterministic factor analysis, the analysed period is divided into sub-periods.

The changes of the performance indicator (ΔLP) and of the participating factor variables operating income (ΔR) and number of employed persons (ΔN) during the analysed sub-periods can be represented by the following expressions:

$$\Delta LP_t = LP_t - LP_{t-1}; \quad (2)$$

$$\Delta R_t = R_t - R_{t-1}; \quad (3)$$

$$\Delta N_t = N_t - N_{t-1}; \quad (4)$$

where:
t is the index of the *t*th value of the performance indicator and of the participating factor variables over time, $t = 0, 1, 2, \ldots, T$;
t_0 and t_T are the beginning and the end of the whole analysed period respectively;
t_{t-1} and t_t are the beginning and the end of the *t*th sub-period respectively.

The index of the *t*th sub-period takes values between *0÷1* and *t-1÷T* ($t-1 \div t = 0 \div 1, 1 \div 2, 2 \div 3, \ldots$).

From here, it is easy to perform a dynamic DFA of the performance indicator for the whole period and for individual sub-periods.

The **methods applied** in the development of this study were: the methods of analysis and synthesis, a systematic approach, the method of comparison, and the averaged chain substitution method.

Dynamic deterministic factor analysis of operating income per employed person in the mining industry for the period 2008-2021

For multi-factor models, such as the labour productivity mathematical model, it is characteristic that the averaged chain substitution model is the only accurate among all DFA models. It was developed and published by the author between 2020 and 2023. In this model, individual quantitative impacts of changes in the factor variables on the change in the performance indicator, according to Mitev (2020, 2021, 2022, 2023) are determined by the following expressions:

$$\Delta LP_{(R)} = \frac{\Delta R}{2}\left(\frac{1}{N_0} + \frac{1}{N_1}\right); \quad (5)$$

$$\Delta LP_{(N)} = \frac{R_1 + R_0}{2}\left(\frac{1}{N_1} - \frac{1}{N_0}\right). \quad (6)$$

The necessary data for performing the dynamic deterministic factor analysis of labour productivity in the mining industry for the period 2008-2021 were taken from the INFOSTAT system of the National Statistical Institute of Bulgaria.

The input data and the results obtained from the dynamic deterministic factor analysis of the indicator operating income per employed person in the mining industry using the averaged chain substitution method by year and for the entire 2008 - 2021 period are shown in Table 1.

A check that the absolute change in the performance indicator is equal to the sum of the two factor influences shows some minor errors far behind the decimal point. This confirms the high accuracy of the averaged chain substitution method.

Conclusion

For the entire analysed period 2008-2021, the operating income per employed person in the mining industry increased by BGN 72.081 thousand, i.e. by 53.52%. The decrease in operating income by BGN 8.325 thousand (0.21%) has led to decrease in labour productivity by BGN 0.354 thousand/employee (0.26%). The decrease in the number of employed persons by 10.448 thousand employees, i.e. by 35.00% improved labour productivity by BGN 72.434 thousand/employed person or 53.78%.

Hence, the decrease in the number of employees in the mining sub-sector plays a key role in the increase in labour productivity. The decrease in operating income over the analysed period has an insignificant negative effect on the change in labour productivity, namely by only BGN 354 per employee. The lack of growth of operating income does not help to improve this key indicator.

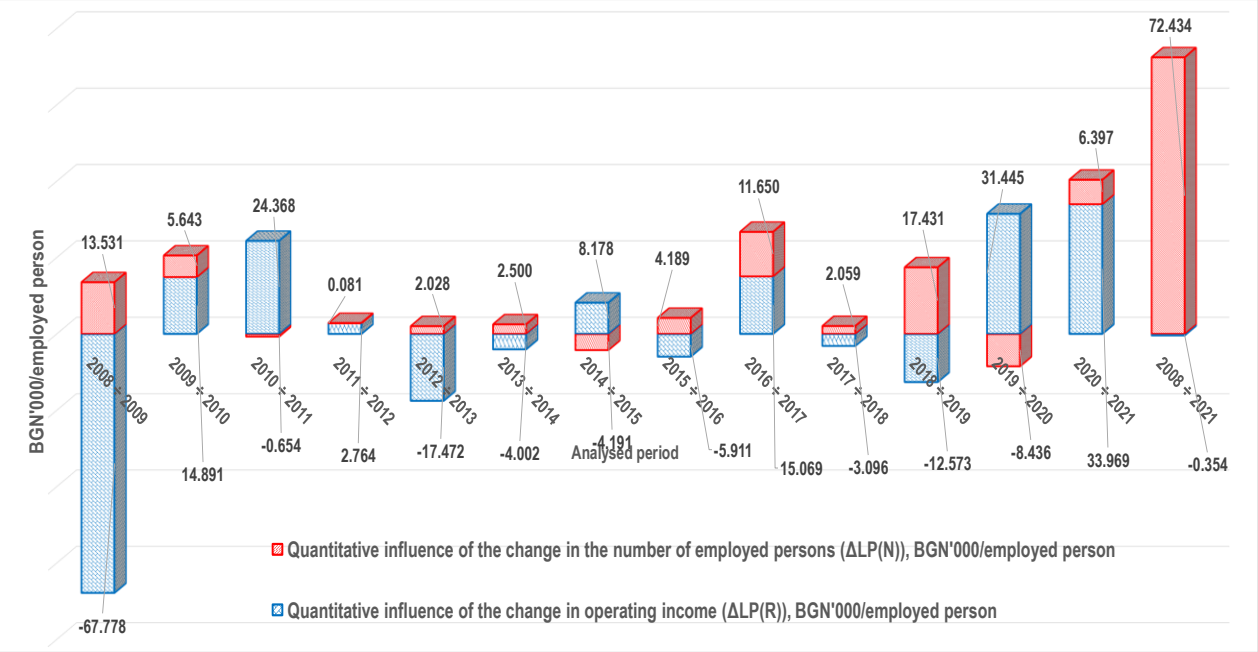

Fig. 1. Quantitative impacts of the factors operating income and number of employed persons on changes in the indicator operating income per employed person in the mining industry by sub-periods calculated using the averaged chain substitution method

Table 1. *Input data and results obtained from the dynamic deterministic factor analysis of the indicator of operating income per employed person in the mining industry using the averaged chain substitution method by sub-periods and for the entire 2008 - 2021 period*

Input data

Indicators	Year													
	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Operating income (Rt), BGN'000	4 020 688	2 122 084	2 502 713	3 108 505	3 177 390	2 745 745	2 648 808	2 848 348	2 704 084	3 048 475	2 981 356	2 727 440	3 341 833	4 012 363
Number of persons employed (N_t), , num.	29 854	26 384	24 788	24 932	24 916	24 498	23 954	24 862	23 963	21 843	21 522	19 022	20 084	19 406
Income from the activity per one employed person (LPt), BGN'000/employed person	134.678	80.431	100.965	124.679	127.524	112.080	110.579	114.566	112.844	139.563	138.526	143.383	166.393	206.759

Source: INFOSTAT (NSI, 2023), compiled by the authors.

Analysis results

Indicators	Analysed period													
	2008 ÷ 2009	2009 ÷ 2010	2010 ÷ 2011	2011 ÷ 2012	2012 ÷ 2013	2013 ÷ 2014	2014 ÷ 2015	2015 ÷ 2016	2016 ÷ 2017	2017 ÷ 2018	2018 ÷ 2019	2019 ÷ 2020	2020 ÷ 2021	2008 ÷ 2021
Absolute change in operating income ($\Delta R=R_t - R_{t-1}$), BGN'000	-1 898 604	380 629	605 792	68 885	-431 645	-96 937	199 540	-144 264	344 391	-67 119	-253 916	614 393	670 530	-8 325
Relative change in operating income ($\%R=\Delta R*100/R_{t-1}$), %	-47.22%	17.94%	24.21%	2.22%	-13.58%	-3.53%	7.53%	-5.06%	12.74%	-2.20%	-8.52%	22.53%	20.06%	-0.21%
Absolute change in the number of employed persons ($\Delta N=N_t - N_{t-1}$), бр.	-3 470	-1 596	144	-16	-418	-544	908	-899	-2 120	-321	-2 500	1 062	-678	-10 448
Relative change in the number of employed persons ($\%N=\Delta N*100/N_{t-1}$), %	-11.62%	-6.05%	0.58%	-0.06%	-1.68%	-2.22%	3.79%	-3.62%	-8.85%	-1.47%	-11.62%	5.58%	-3.38%	-35.00%
Absolute change in Income from the activity per one employed person ($\Delta LP=LP_t - LP_{t-1}$), BGN'000/employed person	-54.248	20.534	23.715	2.845	-15.444	-1.501	3.987	-1.722	26.719	-1.037	4.857	23.009	40.366	72.081
Relative change in Income from the activity per one employed person (%LP=ΔLP*100/LPt-1), %	-40.28%	25.53%	23.49%	2.28%	-12.11%	-1.34%	3.61%	-1.50%	23.68%	-0.74%	3.51%	16.05%	24.26%	53.52%
Quantitative influence of the change in operating income ($\Delta LP_{(R)}$), BGN'000/employed person	-67.778	14.891	24.368	2.764	-17.472	-4.002	8.178	-5.911	15.069	-3.096	-12.573	31.445	33.969	-0.354
Quantitative influence of the change in operating income ($\%LP=\Delta LP(R)*100/LP_{t-1}$), %	-50.33%	18.51%	24.14%	2.22%	-13.70%	-3.57%	7.40%	-5.16%	13.35%	-2.22%	-9.08%	21.93%	20.42%	-0.26%
Quantitative influence of the change in he number of employed persons ($\Delta LP_{(N)}$), BGN'000/employed person	13.531	5.643	-0.654	0.081	2.028	2.500	-4.191	4.189	11.650	2.059	17.431	-8.436	6.397	72.434
Relative influence of the change in he number of employed persons ($\%LP=\Delta LP(N)*100/LP_{t-1}$), %	10.05%	7.02%	-0.65%	0.06%	1.59%	2.23%	-3.79%	3.66%	10.32%	1.48%	12.58%	-5.88%	3.84%	53.78%
Complex influence: $\Delta LP = \Delta LP_{(R)} + \Delta LP_{(N)}$, BGN'000/employed person	-54.248	20.534	23.715	2.845	-15.444	-1.501	3.987	-1.722	26.719	-1.037	4.857	23.009	40.366	72.081
Verification: $\Delta LP=\Delta LP_{(R)} + \Delta LP_{(N)}$	True	True	True	False	True	False	True	False	True	True	False	True	True	True
Error value, BGN'000/employed person	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Проверка: $\%LP=\%LP_{(R)}+\%LP_{(N)}$	False	False	False	False	False	False	False	False	False	False	False	False	False	False
Relative error value, %	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%	0.000000%

The results of the dynamic deterministic factor analysis show that the mining industry did not manage to achieve the level of operating income before the onset of the financial and economic crisis of 2008, the decrease in the number of employed persons in the industry by 10.448 thousand employees or 35.00% is the main factor for the improvement of labour productivity in the mining industry by 72.081 or 53.52%.

The main efforts to improve labour productivity in the Bulgarian mining industry should be aimed at increasing revenue from the sale of products by looking for new domestic and foreign markets.

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