STRESSES IN A TRANSVERSELY ISOTROPIC ROCK MASS AROUND A CIRCULAR OPENING

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ABSTRACT: The article considers the issue of determining the stresses in the transversely isotropic rock mass. The isotropy plane is horizontal. A circular opening is made in the rock mass. The problem is solved by the method of complex variable functions.
The paper proposes to take into account the behaviour of the rocks within a time interval since the opening of the hole. For this purpose, the method of variable modules is applied. The stresses along the contour of the hole at both ends of the time interval are determined.
A numerical example of a real rock mass is given. The values of the stresses in the horizontal and vertical points of the contour of the opening are calculated.

Key words: method of complex variable functions, method of variable modules, stresses.

НАПРЕЖЕНИЯ В ТРАНСВЕРЗАЛНО-ИЗОТРОПЕН СКАЛЕН МАСИВ ОКОЛО КРЪГОВА ИЗРАБОТКА

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РЕЗЮМЕ: В статията се разглежда въпросът за определяне на напреженията в трансверзално-изотропен масив. Равнината на изотропия е хоризонтална. В масива е прокарана кръгова изработка. Задачата се решава с метода на функции на комплексна променлива.
В работата се предлага да се отчете поведението на скалите след определено време от прокарване на отвора. За това се прилага метода на променливите модули. Определени са напреженията по контура на отвора в двата края на интервала от време.
Даден е числен пример за реален масив. Изчислени са стойностите на напреженията в хоризонталните и вертикални точки от контура на отвора.

Ключови думи: метод на функции на комплексна променлива, метод на променливи модули, напрежения.

Introduction

A large number of studies have been published on the issue of stresses around a circular hole. Many of them apply a linear homogeneous medium as a model and use linear equations in the theory of elasticity. To solve this problem, the complex function method is applied (Muskhelishvili, 1953; Lekhnitskii, 1963; Guz et al., 2007; Lu et al., 2007). The model is complicated when the array has an isotropy plane (Minchev, 1960; Hefny, 1999).

The movement of rocks over time is reported with the theory of heredity. According to this, the relationship between stresses and strains is given by integral operator equations. In many problems of the mechanics of underground facilities, the method of variable modules, developed by Amusin (Amusin et al., 1974) can be applied. The method is applied to an array composed of parallel isotropic layers whose thickness is small (Trifonova-Genova, 2012).

The current work focuses on determining the analytical expressions for the stresses in time around a circular opening passing through a transversely isotropic array.

Methods

1. Formulation of the problem

At a depth of H, a work with a radius of r_o was drilled in a transversely isotropic array. The plane of isotropy is horizontal. For this class of problems, the behavior of the array is described by the partial differential equation of the stress function (Muskhelichvili, 1953; Lekhnitskii, 1963; Guz et al., 2007). This equation is of the fourth order.

2. Creep of the rock mass

The problem of the hereditary theory of elasticity can be considered as a problem of the theory of elasticity. Temporary functions are written in it instead of physical and mechanical constants. For an array with a horizontal plane of isotropy, these functions have the form (Trifonova-Genova, 2012):

$$E_{k,t} = E_k\left(1+\Phi_k\right)^{-1}\ ; \qquad G_{2,t} = G_2\left(1+\Phi_2\right)^{-1};$$
$$\Phi_k = \delta_k t^{1-\alpha_k}\left(1-\alpha_k\right)^{-1}\ ; \tag{1}$$

$$\mu_{2,t} = E_{1,t}\left(\frac{\mu_2}{E_1} - \frac{1}{2E_2} + \frac{1}{2E_{2,t}}\right); \qquad k = 1,2;$$

$$\mu_{1,t} = \mu_2\left(1 + \frac{E_2}{E_1}\right) + \mu_1 - \mu_{2,t}\left(1 + \frac{E_{2,t}}{E_{1,t}}\right),$$

where

t is the time,[s];

α_k ; δ_k are creep parameters;

$E_{1,t}$ is Young's modulus in the isotropy plane, [MPa];

$\mu_{1,t}$ is Poison’s ration in the the isotropy plane;

$E_{2,t}$ is Young's modulus in a plane perpendicular to the isotropy plane, [MPa];

μ_{2t} is Poison’s ration in a plane perpendicular to the isotropy plane;

$G_{2,t}$ is shear modulus in a plane perpendicular to the isotropy plane, [MPa].

A time interval (t_0, t_f) is considered to study creep. Two tasks are solved for these boundaries. They use the physical and mechanical characteristics corresponding to the specific moment.

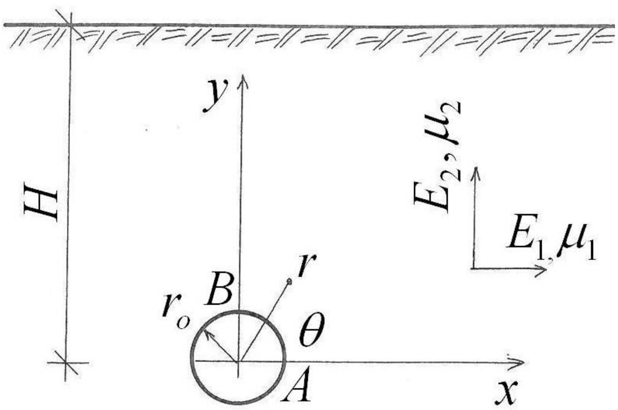

Fig.1. Calculation scheme

3. Stress

The characteristics of equation (1) participate in the expressions for the coefficients of the partial differential equation. This equation is a function of stress. To determine it, the characteristic equation is written (Vucheva et al., 2020):

$$b_{11,t}s^4 + b_{17,t}s^2 + b_{33,t} = 0, \tag{2}$$

where

$b_{17,t} = 2b_{13,t} + b_{55,t}$.

The coefficients in this equation are a function of time and have the following form (Ivanova et al., 2018):

$$b_{jl,t} = a_{jl,t} - \frac{a_{j2,t}a_{l2,t}}{a_{22,t}}; \; j,l = 1,3,5. \tag{3}$$

For the selected array, the strain coefficients of equation (3) have the form (Trifonova-Genova, 2019):

$$a_{11,t} = (E_{1,t})^{-1};\; a_{22,t} = (E_{1,t})^{-1};\; a_{33,t} = (E_{2,t})^{-1};$$
$$a_{55,t} = (G_{2,t})^{-1}; \tag{4}$$
$$a_{12,t} = -\frac{\mu_{1,t}}{E_{1,t}};\; a_{13,t} = -\frac{\mu_{2,t}}{E_{1,t}};\; a_{23,t} = -\frac{\mu_{2,t}}{E_{2,t}}.$$

The roots of equation (2) are:

$$s_{1,t} = \beta_{1,t}i;\; s_{2,t} = \beta_{2,t}i;\; s_{3,t} = -\beta_{1,t}i;$$
$$s_{4,t} = -\beta_{2,t}i. \tag{5}$$

Here, i is an imaginary unit.

The type of stress function depends on these roots. The detailed type of this function can be found in the special literature (Minchev, 1960; Lekhnitskii, 1963). This function expresses the stresses around the opening.

There are no radial and tangential stresses along the contour of the hole. Normal tangential stresses along the contour of the circular fabrication at any moment t have a form analogous to that of (Minchev, 1960):

$$\sigma_{\theta,t} = \frac{Q}{e_1}(e_2 + e_3 + e_4), \tag{6}$$

where

$$e_1 = d_{1,t}d_{2,t}\beta_{3,t}; \qquad e_2 = \beta_{1,t}d_{1,t}d_{2,t}\beta_{3,t};$$
$$e_3 = d_{3,t}d_{2,t}\beta_{4,t}; \qquad e_4 = d_{4,t}d_{1,t}\beta_{5,t};$$
$$\beta_{3,t} = \beta_{1,t} - \beta_{2,t}; \qquad \beta_{4,t} = \beta_{2,t} - \lambda;$$
$$\beta_{5,t} = \lambda - \beta_{1,t}; \qquad \lambda = \frac{\mu_2}{1-\mu_1}; \qquad Q = \gamma H.$$

The other coefficients in the above equation depend on the angle. For $\theta = 0, \pi$, the coefficients of (6) are:

$$d_{1,t} = \beta_{1,t}^2;\; d_{2,t} = \beta_{2,t}^2;\; d_{3,t} = -\beta_{1,t};$$
$$d_{4,t} = -\beta_{2,t}. \tag{7}$$

For $\theta = \pi/2, 3\pi/2$, the coefficients from (6) have the form:

$$d_{1,t} = 1;\; d_{2,t} = 1;\; d_{3,t} = -\beta_{1,t}^2;\; d_{4,t} = -\beta_{2,t}^2. \tag{8}$$

4. Numerical example

A circular opening with radius r_o is drawn to a depth of H. The array is transversely isotropic. Its characteristics are (Trifonova-Genova, 2012):

$E_{1,0} = 0.397.10^4 MPa$; $E_{2,0} = 0.1523.10^4 MPa$;
$\mu_{1,0} = 0.143$; $\mu_{2,0} = 0.198$;
$G_{2,0} = 0.0805.10^4 MPa$; $\gamma = 2.47.10^{-2} MN / m^3$.

The characteristics of the time interval are $t_0 = 0$ and $t_f = 5days \approx 1.296.10^7 s$.

Creep parameters are:

$\alpha_1 = \alpha_2 = 0.67$; $\delta_1 = 0.33.10^{-2} s^{\alpha-1}$;
$\delta_1 = 0.307.10^{-2} s^{\alpha-1}$.

The given characteristics of the scales are used to determine the stresses at the initial moment (t_0). The coefficients of equations (4), (3), and (2) are calculated. The roots of the last equation are given in Table 1 at time t_f .

Table 1. *Roots of a characteristic equation in two moments*

t	β_{1t}	β_{2t}
t_0	0.7565	1.9207
t_f	0.2034	1.8559

According to the method of variable modules, the characteristics of the array for the moment t_f are calculated:

$E_{1,f} = 0.0943.10^4 MPa$;
$E_{2,f} = 0.0496.10^4 MPa$; $\mu_{1,f} = -0.3613$;
$\mu_{2,f} = 0.7014$; $G_{2,f} = 0.0266.10^4 MPa$.

The coefficients of equations (4), (3) and (2) are calculated with the obtained characteristics. The roots of the last equation are placed in the last row of Table 1.

The vertical stress in the undisturbed array is $Q = 2.47 MPa$ and the coefficient of lateral resistance is $\lambda = 0.3375$.

For moment m1, the $\beta_{j,0}$ ($j = 3,4,5$) coefficients from equation (6) are calculated first. The normal tangential stresses are then determined at points A and B of Figure 1. For the first point, the coefficients $d_{k,0}$ ($k = 1 \div 4$) of equation (7), e_k , and the stress $\sigma^A_{\theta,0}$ of (6) are calculated sequentially. For the second point, $d_{k,0}$ from (8), e_k and the stress $\sigma^B_{\theta,0}$ from (6) are determined. The results obtained for both points are given in Table 2.

The described steps are repeated for moment t_f . The results are given in the last row of Table 2.

Table 2. *The normal tangential stresses in two moments*

t	$\sigma^A_{\theta,t}$	$\sigma^B_{\theta,t}$
t_0	-5.8403	-0.5099
t_f	-11.3163	-1.2867

The table shows the values of the normal tangential stresses at two points in time. It is noticed that the values of the stresses at the final moment are several times higher than those at the initial moment. Thus, this increase along the horizontal axis is 1.94 times, and along the vertical axis is 2.52 times.

5. Key finding

The article describes an algorithm for determining the stresses along the contour of a circular opening.

The stresses were received at two points in time: Immediately after digging the hole and after 150 days. The increase in stresses along the vertical axis is greater than those along the horizontal axis.

Conclusion

The article discusses the problem of the mechanics of underground facilities. The boundary conditions of the task are constant. The hereditary theory of elasticity is used to determine stresses.

The presented solution is a supplement to the existing analytical methods, which determine the stresses along the contour of a circular hole. This solution can be extended to other types of openings.

For the application of analytical methods in practice, it is important to determine the values of the physical and mechanical characteristics of the rocks. This is done according to certain standards, which are constantly evolving and enriching. These developments allow the determination of the characteristics in a transverse and isotropic array (Hakala et al., 2007). Thus, the final solution takes into account more precisely the real properties of the rocks around the opening.

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