STRESSES AROUND A HORIZONTAL MINE GALLERY PASSING THROUGH A CRACKED ROCK MASS

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ABSTRACT: A horizontal mine gallery is driven into a rock mass with cracks. The cracks in it are located in horizontal parallel and closely spaced planes. The field between them is linear and isotropic. The cross-section of the gallery is an ellipse. The stresses at points around the gallery are determined by an approximate method. According to it, the rock mass is presented as an equivalent uniform transversely isotropic field. The expressions for the constants of this field include the normal and tangential stiffness of the cracks. The specified class of problems is solved with the complex potential theory. The expressions for the stresses are in a cylindrical coordinate system.
The constants for a real cracked rock mass are defined. The values of the tangential normal stresses at points of the ellipse are obtained. These stresses are compared to the stresses in an equivalent uniform isotropic rock mass.

Key words: complex potential theory, mine gallery, cracked rock mass

НАПРЕЖЕНИЯ ОКОЛО ХОРИЗОНТАЛНА МИННА ГАЛЕРИЯ, ПРЕМИНАВАЩА ПРЕЗ НАПУКАН СКАЛЕН МАСИВ

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РЕЗЮМЕ: Хоризонтална минна галерия е прокарана в скален масив с пукнатини. Пукнатините в него са разположени в хоризонтални успоредни и близко разположени равнини. Средата между тях е линейна и изотропна. Напречното сечение на галерията е елипса. Напреженията в точки около галерията се определят с приблизителен метод. Според него, скалния масив се представя като еквивалентна еднородна трансверзално изотропна среда. В изразите за константите на тази среда участват нормалната и тангенциална коравини на пукнатините. Указаният клас задачи се решава с комплексната потенциална теория. Изразите за напреженията са в цилиндрична координатна система.
За реален напукан скален масив са определени константите. Получени са стойностите на тангенциалните нормални напрежения в точки от елипсата. Тези напрежения са сравнени с напреженията в еквивалентен еднороден изотропен масив.

Ключови думи: комплексната потенциална теория, минна галерия, напукан скален масив.

Introduction

The distribution of stresses around the opening of a horizontal mine gallery is used in the assessment of rock strength. For this, it is necessary to match the stresses in the rock mass around the hole with the strength of the soil. When studying the stressed state of the rock massif around the gallery, a model is used that takes into account the continuity and uniformity of the environment. Analytical methods for this type of tasks have been developed in literature (Mushkhelishvili, 1953; Minchev, 1960).

But in nature, the properties of rocks are diverse, due to many natural factors. Among them, the existence of cracks has the greatest weight. If the rock massif has one or several cracks, the boundary element method is suitable (Crouch et al., 1983). It uses tension and compression boundary contact elements. For the case of a rock mass with very closely spaced cracks, this method is irrational. Therefore, an approximate approach is described in (Godman, 1976).

The aim of the present work is to investigate the influence of cracks in this array on the stresses around a horizontal mine gallery with an elliptical cross-section using this approach.

Methods

1. Formulation of the problem

At a great depth H, a mining gallery has been driven. It has an elliptical cross-section with dimensions $2a$ and $2b$. The Cartesian coordinate system has its origin at the center of the hole (fig.1).

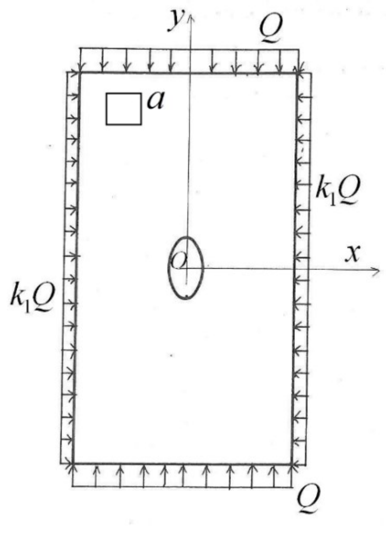

Fig. 1. Calculation scheme

The hole's influence extends into a rectangular area with dimensions $12a$ and $12b$. The vertical and horizontal load (Q and k_1Q) along the contour of the area (fig. 1) is equal to the stress components at a point of undisturbed rock array. The analytical expressions for these components are:

$$\sigma_y^o = \gamma y = Q\,;\ \sigma_x^o = k_1\sigma_y^o = k_1 Q\,;\ \sigma_z^o = k_2\sigma_y^o\,. \quad (1)$$

where

- k_1 and k_2 are the coefficients of lateral resistance;
- γ is the volumetric weight;
- y is the vertical coordinate of the point (fig.1).

The directions of these components are given in Figure 2.

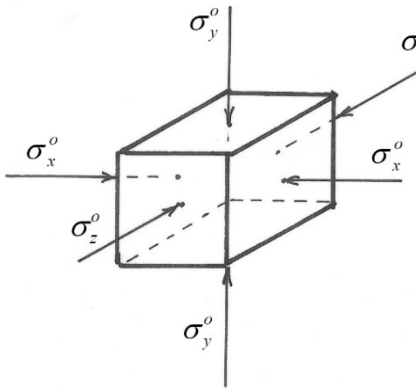

Fig. 2. Stresses in an undisturbed rock mass

For a rock mass with a horizontal plane of isotropy, the coefficients of (1) have the form:

$$k_1 = \frac{a_{13}a_{23} - a_{12}a_{33}}{a_{11}a_{33} - a_{13}^2}\,; \qquad k_2 = \frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{33} - a_{13}^2}\,. \quad (2)$$

The deformation coefficients in these expressions are expressed by the medium constants:

$$a_{11} = \frac{1}{E_{xx}}\,; \quad a_{33} = \frac{1}{E_{zz}}\,; \quad a_{12} = -\frac{\nu_{xy}}{E_{yy}}\,;$$

$$a_{13} = -\frac{\nu_{xz}}{E_{zz}}\,;\ a_{23} = -\frac{\nu_{zy}}{E_{yy}}\,, \quad (3)$$

where

- E_x, E_y, E_z are Young's modulus in the direction of the coordinate axes x, y and z;
- ν_{xy}, ν_{yx}, ν_{xz}, ν_{zx} are Poison's ratio characterising the transverse deformations in tension and compression in the direction of the coordinate axes.

If the medium is perfectly isotropic for the coefficients of lateral resistance k_1 and k_2, A. N. Dinnik's formula is obtained:

$$k = \frac{\nu}{1-\nu}\,. \quad (4)$$

2. Additional stresses

These tensions are caused by the presence of the gallery. They are determined by means of two unknown complex functions. The sum of these functions is a function of stresses F(x,y). It satisfies the quasi-bi harmonic equation (Muskhelichvili, 1953; Minchev, 1960):

$$b_{55}\frac{\partial^4 F}{\partial x^4} + b_{17}\frac{\partial^4 F}{\partial x^2 \partial y^2} + b_{11}\frac{\partial^4 F}{\partial y^4} = 0\,, \quad (5)$$

where $b_{17} = \left(2b_{13} + b_{55}\right)$.

Here, x and y are coordinates of a point midway around the hole. The coefficients in this equation are expressed by Young's modulus, Poisson's ratios in the plane of isotropy and in a direction perpendicular to it, and have the following form:

$$b_{11} = \frac{1 - \nu_{yx}\nu_{xy}}{E_y}\,;\ b_{33} = \frac{1 - \nu_{xz}^2}{E_x}\,;$$

$$b_{13} = -\frac{\nu_{yx} + \nu_{yx}\nu_{xy}}{E_y}\,;\ b_{55} = \frac{1}{G_{xy}}\,. \quad (6)$$

The stress function depends on variables in which the roots of the characteristic equation are involved (Ivanova, 2022; Minchev, 1960; Trifonova-Genova, 2019):

$$b_{11}s^4 + \left(2b_{13} + b_{55}\right)b_{17}s^2 + b_{33} = 0\,, \quad (7)$$

For the problem described above, the roots of equation (7) are:

$$s_1 = \beta_1 i\,;\ s_2 = \beta_2 i\,;\ s_3 = -\beta_1 i\,;\ s_4 = -\beta_2 i\,. \quad (8)$$

Here, i is an imaginary unit.

3. Total stresses at a point in the rock mass

Stresses in a transversely isotropic medium are defined as the sum of the principal stresses existing before the mining gallery was driven and the additional stresses caused by the presence of the hole. Then it goes from Cartesian to polar coordinates. In addition, the real part is separated from general complex expressions.

Here, we will focus on the stresses along the hole contour, which have the form (Minchev, 1960):

$$\sigma_\theta = \frac{\sigma_y^o}{e_1}\left(e_2 + e_3\right); \quad (9)$$

$$\sigma_z = -\left[\frac{e_4\sigma_\theta}{a_{33}e_1} + k_2\sigma_y^o\right],$$

where

$$e_1 = \sin^2 \vartheta + m^2 \cos^2 \vartheta;\ m = \frac{b}{a};\ tg\vartheta = \frac{b}{a} tg\theta;$$
$$e_2 = k_1 \sin^2 \vartheta + m^2 \cos^2 \vartheta;\ e_3 = \frac{BC - AD}{C^2 + D^2};$$
$$e_4 = a_{13} \sin^2 \vartheta + a_{23} m^2 \cos^2 \vartheta;$$
$$A = A_1 A_4 + A_2 A_5 + A_3 A_6;$$
$$A_4 = \sin^3 \vartheta \cos \vartheta;\ A_5 = \sin \vartheta \cos^3 \vartheta;$$
$$A_6 = \sin^2 \vartheta \cos^2 \vartheta;\ \beta_3 = \beta_1 + \beta_2;\ \beta_4 = \beta_1 \beta_2;$$
$$A_1 = k_1 m \beta_3 - \beta_4;\ A_3 = k_1 m (2 + \beta_4);$$
$$A_2 = \beta_3 m^3 - (1 + 2\beta_4) m^3 - k_1 m^4;$$
$$A_7 = \sin^4 \vartheta;\ A_8 = \cos^4 \vartheta;$$
$$B = B_3 A_6 + B_1 A_7 + B_2 A_8;$$
$$B_1 = k_1 m \beta_3 - \beta_4;\ B_2 = k_1 m^4 - \beta_3 m^3;$$
$$B_3 = k_1 m^2 (2 + \beta_4) - (1 + 2\beta_4) m^2;$$
$$C = \sin^2 \vartheta - \beta_4 m^2 \cos^2 \vartheta;\ D = \beta_4 m \sin \vartheta \cos \vartheta.$$

If the rock mass is isotropic, the following formulae are used for the stresses:

$$\sigma_\theta = \frac{\sigma_y^o}{e_1} (e_2 + e_3), \tag{10}$$
$$\sigma_z = -\nu \left(\sigma_\theta + \frac{1}{1 - \nu} \right),$$

where

$$e_1 = \sin^2 \vartheta + m^2 \cos^2 \vartheta;\ m = \frac{b}{a};$$
$$e_2 = k_1 \sin^2 \vartheta + m^2 \cos^2 \vartheta;\ e_3 = \frac{BC - AD}{C^2 + D^2};$$
$$A = A_1 A_4 + A_2 A_5 + A_3 A_6;$$
$$A_4 = \sin^3 \vartheta \cos \vartheta;\ A_5 = \sin \vartheta \cos^3 \vartheta;$$
$$A_6 = \sin^2 \vartheta \cos^2 \vartheta;\ A_1 = 2k_1 - 1;\ A_3 = 3k_1;$$
$$A_2 = -1 - k_1;\ B = B_3 A_6 + B_1 A_7 + B_2 A_8;$$
$$B_1 = 2k_1 - 1;\ B_2 = k_1 - 2\beta_3;\ B_3 = 3k_1 - 3;$$
$$C = \sin^2 \vartheta - \cos^2 \vartheta;\ D = 2 \sin \vartheta \cos \vartheta;$$
$$C^2 + D^2 = 1.$$

4. Characteristics of a rock mass

The cracks in the rock massif (Figure 1 with square a) are located in horizontal planes. The distance between them is s_o. The material between the planes is isotropic (Fig.3).

Cracks have normal K_n and tangential K_s stiffness. These parameters are involved in the characteristics of the cracked array (Godman, 1976; Vucheva et al., 2023):

$$E_y = \frac{E}{1 + E / (s_o K_n)};\ G_{xy} = \frac{G}{1 + G / (s_o K_s)};$$

$$E_x = E_z = E;\ \nu_{xy} = \nu_{xz} = \nu;\ \nu_{yx} = \frac{E_y}{E_x} \nu_{xy};$$
$$\nu_{zx} = \frac{E_z}{E_x} \nu_{xz} = \nu. \tag{11}$$

Here, E is the Young's modulus, ν is Poison's ratio, and G is the shear modulus in an isotropic field.

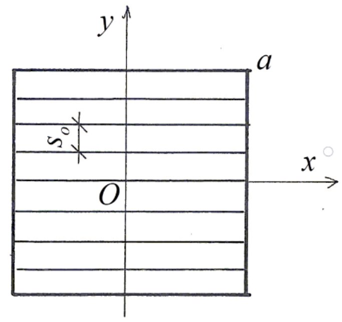

Fig. 3. Cracked rock mass

The constants from (11) are substituted into (3). The dependences are obtained for the deformation coefficients:

$$a_{11} = \frac{1}{E};\quad a_{33} = \frac{1}{E};\quad a_{12} = -\frac{\nu}{E_y};\quad a_{13} = -\frac{\nu}{E};$$
$$a_{23} = -\frac{\nu}{E_y}. \tag{12}$$

These coefficients are involved in expressions (1) for the stresses before the excavation of the gallery.

5. Numerical example

In a rock mass, the cracks are located in horizontal planes. The distance between them is $s_o = 2m$, and the normal and tangential stiffness are $K_n = \infty$ and $K_s = 0{,}2315 . 10^3 MPa / m$. Between the planes, the medium is isotropic with the following characteristics: $E = 10^4 MPa$, $\nu = 0{,}2$; and $G = 0{,}4167 . 10^4 MPa$. The mining gallery driven through the rock mass has an elliptical cross-section. The mining gallery has a width of $2b$ and a height of $2a$. The ratio of the major to the minor axis of the section is 1.5.

According to (11), the characteristics of the cracked medium are determined. The coefficients in (6) are determined and equation (7) is solved. The tangential normal stresses along the contour of the hole were calculated for two types of rock mass. Equations (5) are used in the cracked rock mass, and equations (8) are used in the isotropic rock mass. These stresses are referred to the stress Q in the undisturbed medium. Due to the symmetry about the two axes, calculations are made for first square points. The results are listed in Table 1.

The normal tangential stress diagrams are given in Figure 3. The solid line indicates the normal tangential stresses in the cracked rock mass, and the dotted line those in the isotropic rock mass.

Table 1. *Normal tangential stresses in two types of rocks*

n	$[\circ]$	σ_θ / Q	$\sigma_{\theta,o} / Q$
1	0	-4.891	-2.75
2	15	-1.512	-2.546
3	30	-0.338	-2.000
4	45	-0.236	-1.249
5	60	-0.386	-0.500
6	75	-0.598	+0.049
7	90	-0.836	+0.25

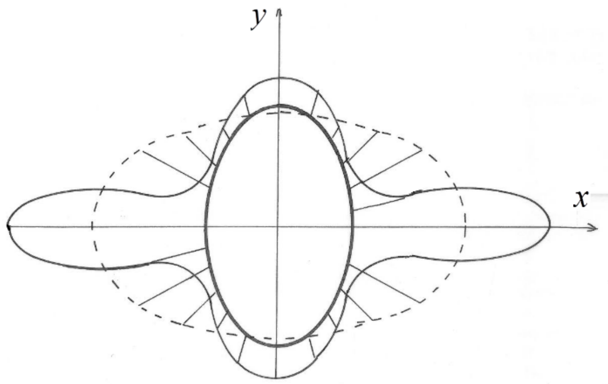

Fig. 4. Diagrams of the normal tangential stresses in the two media

The following conclusions can be drawn from Table 1 and Figure 4:

In both environments, the maximum values of the tangential normal stresses occur at points on the hole contour through which the horizontal axis passes. Stresses in the cracked rock mass are 1.8 times greater than the corresponding stresses in the isotropic mass.

Along the vertical axis, the stresses in the two media are of different signs. The stresses in the cracked rock mass are negative, and in the isotropic rock mass - positive. The values of stresses in the studied environment are 5.8 times smaller than the same along axis x .

6. Key findings

From the results obtained above, it can be concluded that the stresses in a cracked medium are greater than the same in an isotropic medium. Therefore, it is necessary to take into account this non-uniformity and apply the environment model proposed in the work.

The method described in the article is easy to implement and does not require the application of any software packages, but the use of popular calculation tools, such as spreadsheets (Harvey, 2018).

Conclusion

The described method can be applied to a rock mass having cracks located in inclined parallel planes. The opening of the gallery may be of a different shape.

The analytical relationships in this work can be used by engineers when investigating stresses around underground facilities.

When cracks are not located in horizontal planes, Goodman et al. (Goodman et al., 1963) proposed a contact element in the numerical finite element method (Whintely, 2017; Jing et al., 2002). The element has a rectangular shape with four nodes. The stresses in it are proportional to the deformations. The stiffness matrix included in the ANSYS software package (Reference manual, 2020) is proposed for it.

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