ON STRESSES AROUND A TUNNEL PASSING THROUGH TWO ROCK LAYERS

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ABSTRACT: The article deals with the issue of determining the stresses around a circular tunnel. It passes through two rock layers. Each of them has an arbitrarily located plane of isotropy. The boundary plane between them is inclined to the axis of the tunnel. Furthermore, the plane is parallel to the horizontal axis of the circular cross-section of the tunnel. The specified class of tasks is solved with the complex potential theory and an approach from the mechanics of layered media. The expressions for the stresses in the two layers have been derived.
The results are applied to a real rock mass consisting of two transversely isotropic layers. The stresses at points of the circular section of the tunnel in the two layers were obtained. Two tangential normal stress diagrams are given.

Key words: complex potential theory, mechanics of layered media, tunnel.

ВЪРХУ НАПРЕЖЕНИЯ ОКОЛО ТУНЕЛ ПРЕМИНАВАЩ ПРЕЗ ДВА СКАЛНИ ПЛАСТА

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РЕЗЮМЕ: В статията се разглежда въпроса за определяне на напреженията около кръгъл тунел. Той преминава през два скални пласта. Всеки от тях притежава произволно разположена равнина на изотропия. Граничната равнина между пластовете е наклонена спрямо оста на тунела. Освен това, равнината е успоредна на хоризонталната ос на кръговото напречно сечение на тунела. Указаният клас задачи се решава с комплексна потенциална теория и подход от механика на напластените среди. Получени са изразите за напреженията в двата пласта.
Резултатите са приложени за реален скален масив, състоящ се от два трансверзално изотропни пласта. Получени са напреженията в точки от кръговото сечение на тунела в двата пласта. Дадени са две диаграми на тангенциалните нормални напрежения.

Ключови думи: комплексна потенциална теория, механика на напластените среди, тунел.

Introduction

In general, the earth mass is an environment consisting of different layers. Each of them has different physical and technical properties. They are different in the layering plane and in a direction perpendicular to it. The stresses in the vicinity of a tunnel driven into the rock mass are determined by the methods of mechanics of layered media and the complex potential theory (Mishkelishvili, 1953; Lu et al., 2024).

Solutions to problems in which the boundary planes between layers are parallel to the tunnel axis are described in (Tifonova-Genova et al., 2023). These tasks are of two types. The first type covers tasks in which the rock mass consists of thin and parallel layers. In the second type of task, the layers around the tunnel are steep and thick. In the article cited above, a method for determining the stresses in a rock mass consisting of two isotropic layers is proposed. The boundary plane between them is inclined to the axis of the tunnel and parallel to the horizontal axis of its cross section.

The present work aims to extend the method described above. Each layer has a plane of isotropy arbitrarily located relative to the boundary plane.

Methods

1. Formulation of the problem

A horizontal circular tunnel with a radius R is driven at a great depth H . It goes through two layers.

Part of the rock massif around the tunnel is considered, which has the shape of a cube (Fig.1). The stresses on the walls of the cube are equal to the stresses in the undisturbed rock mass. The boundary plane between the layers is inclined to the axis of the tunnel. Moreover, it is parallel to the horizontal axis of the tunnel cross-section. This plane divides the cube into two parts. Each part has a plane of isotropy that is randomly located relative to the sides of the cube.

2. Method for determination of stresses

The following three-stage approach is used to determine the stresses in each layer:

During the *first stage*, layers with different characteristics in the plane of isotropy and in a direction perpendicular to it are replaced by layers that are isotropic.

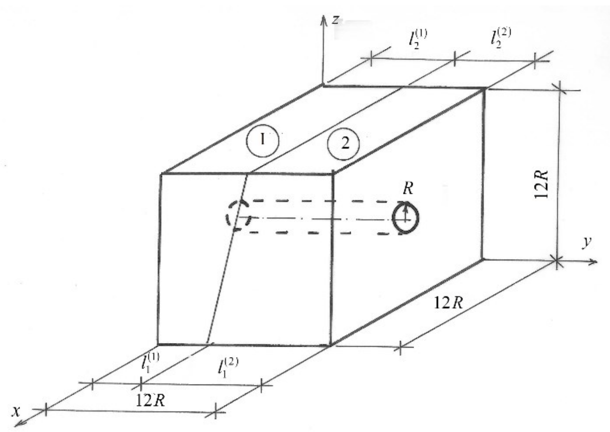

Fig. 1. A tunnel passing through two layers

The mechanical characteristics of these layers are determined in two ways. According to the first way (Kurleniya et al., 1983), these characteristics are:

$$E^{(m)} = \sqrt[3]{E_1^{(m)} E_1^{(m)} E_2^{(m)}} \; ; \mu^{(m)} = \sqrt[3]{\mu_1^{(m)} \mu_1^{(m)} \mu_2^{(m)}} \; . \quad (1)$$

Here $E_1^{(m)}$ is Young's modulus and $\mu_1^{(m)}$ is Poison's ratio in the plane of isotropy by layer m (m=1,2). In a plane perpendicular to this plane the characteristics are $E_2^{(m)}$ and $\mu_2^{(m)}$.

The second way to determine the mechanical characteristics in isotropic layers is given in (Marino et al., 1972):

$$E^{(m)} = \left[\frac{1}{2} \left(\frac{1}{E_1^{(m)}} + \frac{1}{E_2^{(m)}} \right) \right]^{-1} ; \; \mu^{(m)} = \mu_1^{(m)} . \qquad (2)$$

These formulae are applied for a specific rock mass in (Tonon et al., 2003).

In the *second stage*, the isotropic layers are replaced by an equivalent uniform homogeneous isotropic layer. It has the following characteristics (Trifonova-Genova, 1991):

$$E^{(o)} = \frac{\sum_{m=1}^{2} E^{(m)} V_m .}{\sum_{m=1}^{2} V_m} \; ; \; \mu^{(o)} = \frac{\sum_{m=1}^{2} \mu^{(m)} V_m E^{(m)}}{\sum_{m=1}^{2} V_m E^{(m)}} \; ;$$

$$\gamma^{(o)} = \frac{\sum_{m=1}^{2} \gamma^{(m)} V_m .}{\sum_{m=1}^{2} V_m} , \; \lambda^{(o)} = \frac{\mu^{(o)}}{1 - \mu^{(o)}} . \qquad (3)$$

The characteristics involved in these expressions are for layer m : $\gamma^{(m)}$ - volumetric weight;

$E^{(m)}$ - Young's modulus:

$\mu^{(m)}$ - Poison's ration;

in the uniform medium: $\gamma^{(o)}$ - volumetric weight;

$E^{(o)}$ - Young's modulus;

$\mu^{(o)}$ - Poison's ration;

$\lambda^{(o)}$ - side pressure coefficient.

In dependencies (3), the volumes of the layers are determined by the following expression:

$$V_m = 72 \left[l_1^{(m)} + l_2^{(m)} \right] r^2 \; ; \; m = 1 \div 2 \, . \qquad (4)$$

To determine the stresses in the uniform layer, the calculation scheme is drawn up. It is a square with a side $12R$ (Fig. 2). The vertical load on the square is $Q = \gamma^{(o)} H$, but $\lambda^{(o)} Q$ is the horizontal load. The stresses in the medium are plotted in a polar coordinate system ($Or\theta$). Angle θ is measured from axis z to axis x . The radius vector of a point from the center is r .

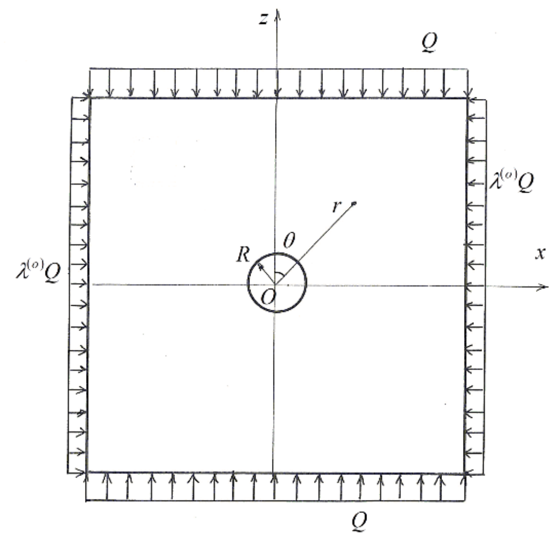

Fig. 2. Calculation scheme

In the *third stage*, the uniform isotropic medium is replaced by a layered medium. The stresses in each layer are determined by two conditions. The first condition describes the balance of forces, and the second equalises the relative deformations of the contact between the two layers. This results in a system of four equations (Trifonova-Genova et al., 2023). Along the contour of the opening, this system acquires the form:

$$\sigma_\theta^{(1)} V_1 + \sigma_\theta^{(2)} V_2 = \sigma_\theta^o V \; ; \qquad (5)$$
$$e_2 \sigma_\theta^{(2)} - e_1 \sigma_\theta^{(1)} = 0 \; ,$$

where

$$e_k = a_{11}^{(m)} - \frac{\left[a_{13}^{(m)} \right]^2}{a_{33}^{(m)}} \; ; a_{13}^{(m)} = \frac{\mu^{(m)}}{E^{(m)}} \; ; \; a_{11}^{(m)} = a_{33}^{(m)} = \frac{1}{E^{(m)}} \; ;$$

m=1,2; k=1,2.

From the solution of (5), the stresses in the two layers are obtained:

$$\sigma_\theta^{(1)} = \frac{e_2 V}{\Delta} \sigma_\theta^{(o)} ; \; \sigma_\theta^{(2)} = \frac{e_1 V}{\Delta} \sigma_\theta^{(o)} , \qquad (6)$$

where

$$\Delta = V_1 e_2 + V_2 e_1 \, .$$

The tangential normal stresses in the generalised isotropic medium have the form:

$$\sigma_\theta^o = -Q \left(\sigma_{\theta,1} - \sigma_{\theta,2} \cos 2\theta \right) , \qquad (7)$$

where

$$\sigma_{\theta,1} = 2\lambda_1\,;\ \sigma_{\theta,2} = 4\lambda_2\,;\ \lambda_1 = \frac{1+\lambda^{(o)}}{2}\,;\ \lambda_2 = \frac{1-\lambda^{(o)}}{2}\,.$$

3. Numerical example

A tunnel with a circular cross-section ($R = 1{,}5m$) crosses two transversely isotropic layers (Fig. 1). It was driven to a depth of $H = 300\,m$. The Young's modulus and the Poison's ratio in the layers are given in table 1.

Table 1. *Characteristics of transversely isotropic layers*

m	$E_1^{(m)}$	$E_2^{(m)}$	$\mu_1^{(m)}$	$\mu_2^{(m)}$
multiplier	10^3	10^3		
unit	MPa	MPa		
1	0.17	0.113	0.1091	0.2834
2	0.8601	0.2867	0.1623	0.502

Characteristics of the uniform isotropic layers according to (1) and characteristics of the uniform medium according to (3) were calculated. These parameters and bulk weights are listed in Table 2.

Table 2. *Characteristics of uniform layers and uniform media*

m	$E^{(m)}$	$\mu^{(m)}$	$\gamma^{(m)}$
multiplier	10^3		10^{-2}
unit	MPa		MN/m^3
1	0.148	0.15	0.28
2	0.595	0.237	0.25
o	0.416	0.202	0.262

The dimensions of the layers are given in Table 3.

Table 3. *Layer sizes*

i	l_1	l_2
multiplier		
Unit	m	m
1	5.4	9.0
2	12.6	9.0

The normal tangential stresses in two layers, $\sigma_\theta^{(1)}$ and $\sigma_\theta^{(2)}$, are determined according to (6). The diagrams of these stresses are symmetrical around the vertical and horizontal axis of the circular cross-section of the tunnel. The results are referred to the stress in the undisturbed medium Q . These stresses are calculated for seven first quadrant points and are listed in Table 4.

Figure 3 shows the normal tangential stress diagrams around the hole. Number 1 indicates the stresses diagram in layer 1, and number 2 - those in layer 2.

From the graphs obtained, the maximum difference between the stresses in the layers can be seen. For points on the horizontal axis, this difference is 2.955 or 3, and for points on the vertical axis, it is 0.143. This is related to the larger values of the modulus of linear deformation in the second layer compared to the first layer. The ratio of the Young's modulus of the layers in the plane of isotropy is 5.06, and in the direction perpendicular to it, it is 2.54.

Table 4. *Normal tangential stresses in layers*

n	$\theta[\circ]$	$\sigma_\theta^{(1)}/Q$	$\sigma_\theta^{(2)}/Q$
1	0	0.0452	0.1881
2	15	-0.0205	-0.0854
3	30	-0.1999	-0.8324
4	45	-0.4450	-1.8528
5	60	-0.6901	-2.8733
6	75	-0.8695	-3.6202
7	90	-0.9352	-3.8937

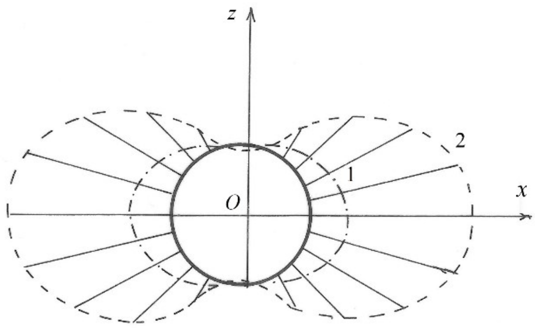

Fig. 3. Diagrams of normal tangential stresses for layers 1 and 2

4. Key findings

The approach described in the work is applied to rock mass with randomly located planes of isotropy relative to the boundary plane. The method is easy to apply and requires the use of popular computing tools such as spreadsheets (Harvey, 2018).

The analytical expressions for the stresses in each layer (6) are applied when the ratio of Young's moduli in the layers is less than or equal to 2. For a ratio greater than 2, it is recommended to use the finite element numerical method (Podgórski, 2018). The task is spatial and a tetrahedral element is used.

This method is recommended to be used by engineers in pre-project design.

Conclusion

The research carried out can be extended in the following directions:

1. The method for determining the stresses in an array consisting of two layers can be generalised for more layers
2. The solution for the described class of problems can be applied to layers with time-varying deformations.
3. This method can be applied to a tunnel with a square, rectangular, and non-circular cross-section passing through two layers. The analytical expressions for the stresses in each layer are expressed by the known expressions for the stresses at points in the vicinity of the tunnel with different cross-sections (Kargar et al., 2014; Lu et al., 2014; Zhao et al., 2015).

References

Harvey, G. (2018). *Microsoft Excel 2019 for Dummies*. Alex Soft, 496p.

Kargar, A. R., Rahmannejad. R., Hajabasi, M. A. (2014). A semi analytical elastic solution for stress field of lined non circular tunnels at depth using complex variable method. *Int. J. Solids. Struct*, vol. 51, 1475-1482.

Kuleniya, M. V., Popov, S. N. (1983). *Teoreticheskie osnovi opredeleniya v naprejenii v gornih porodah*. Izdatelstvo “Nauka”, Novosibirsk, 93p.

Lu, A., Wang, H., Zhang L. (2024). *Complex variable solution in the Mechanical Analysis of Tunnels*. Springer, 400p. ISBN-10 981971690X.

Lu, A. Z., Zhang, N., Kuang, L. (2014.) Analytical solution of stress and displacement for a non-circular tunnel at great depth, including Support delay. *Int. J. Rock Mech. Min. Sci.*, vol. 70;69-81.

Martino, B., Ribacchi. R. (1972). Observazioni su alcuni metodi di misura delle caratteristiche di racceo ammassi rocciosi con perticolare riferimento al problema dell’anisotropia. *L’Industria Mineraria Serie II*, Anno XXIII, 5:193-203.

Muskhelishvili, N. I. (1953). *Some basic problems of the mathematical Theory of Elasticity*. - Gromingen: Nordhoft, 706 p.

Podgórski, J. (2018). Finite element method. 182p. http://akropolis.pol.lublin.pl/users/jpkmb/Finite_Element_Method.pdf

Tonnon, F., Amadei, B. (2003). Stresses in anisotropic rock masses: an engineering perspective building on geological knowledge. *Int. J. Rock Mech. Min. Sci.* vol. 40.1099-1200.

Trifonova-Genova, V., Tonkova. G. (2023). Stress around circular opening passing through two isotropic layers. *Annual of the University of Mining and geology “St. Iv, Rilski” - Sofia*, Vol. 66, Section II: Mining and Mineral Processing, 74-77.

Zhao, C., Yang, S. (2015). Analytical solution for rock stress around square tunnels using complex variable theory, *Int. J. Rock Mech. Min Sci.*, Vol. 80., 302-307.