STRESSES AROUND A CIRCULAR HORIZONTAL OPENING DRIVEN IN CRACKED ROCK MASS

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ABSTRACT: The article examines the question of determining the stresses around a horizontal opening passing through a cracked array. The cracks are located in horizontal parallel and closely spaced planes. The medium between the planes is linear and isotropic. An approximate approach is proposed to determine the stresses in the rock mass. According to it, the rock mass is presented as an equivalent, uniform, transversely isotropic medium. The specified class of problems is solved by the complex potential theory.

The expressions for the physical constants of the transversely isotropic rock mass are determined by the normal and tangential stiffness of the cracks. The expressions for the stresses along the contour of a circular opening are in a cylindrical coordinate system.

For a real rock mass with cracks, the tangential normal stresses around the hole are determined. A comparison is made between the stresses in the real rock mass and in the equivalent transversely isotropic rock mass.

Key words: complex potential theory, mechanics of layered media.

НАПРЕЖЕНИЯ ОКОЛО КРЪГОВА ХОРИЗОНТАЛНА ИЗРАБОТКА, ПРОКАРАНА В НАПУКАН СКАЛЕН МАСИВ

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РЕЗЮМЕ: В статията се разглежда въпросът за определяне на напреженията около хоризонтална изработка, преминаваща напукан масив. Пукнатините са разположени в хоризонтални успоредни и близко разположени равнини. Средата между тях е линейна и изотропна. За определяне на напреженията в масива е предложен приблизителен подход. Според него масивът се представя като еквивалентна, еднородна, трансврзално изотропна среда. Указаният клас задачи се решава с комплексната потенциална теория.

Изразите за физическите константи на трансверзално изотропния масив са определени чрез нормалната и тангенциална коравини на пукнатините. Изразите за напреженията по контура на кръгов отвор са в цилиндрична координатна система.

За реален масив с пукнатини са определени тангенциалните нормални напрежения около отвора. Направено е сравнение между напреженията в този нееднороден масив и в еквивалентен еднороден изотропен масив.

Ключови думи: комплексната потенциална теория, механика на напластените среди.

Introduction

To assess the stability of the rock mass around the hole of a horizontal opening, it is necessary to know the distribution of stresses around the hole. The stability assessment is carried out by comparing the stresses in the array with the strength of the soil. Therefore, it is important to explore the tension surrounding the crafting. Analytical methods have been developed for the array model considering continuity and uniformity (Mushkhelishvili, 1953; Minchev, 1960).

But in nature, this uniformity is broken. The properties of rocks are varied due to many natural factors. Among them, the existence of cracks has the greatest weight. If the rock mass has one or several cracks, the boundary element method is suitable (Crouch et al., 1983). It uses tension and compression boundary contact elements. For the case of very closely spaced cracks, this method is irrational. Therefore, an approximate approach is described in Godman (1976).

The aim of the present work is to investigate the influence of the cracks in this rock mass on the stresses around a horizontal opening with this approach.

Methods

1. Formulation of the problem

At a great depth H, a horizontal opening in the form of a circle with a radius r_o has been driven. The Cartesian coordinate system has its origin at the center of the opening (Fig.1). The hole's influence extends over a square area of size $12r_o$. The loop load is equal to the stresses in the undisturbed array Q and λQ. The volumetric weight of the array is γ.

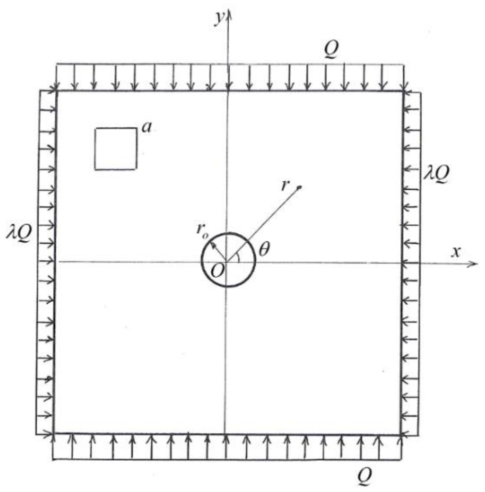

Fig. 1 Calculation scheme

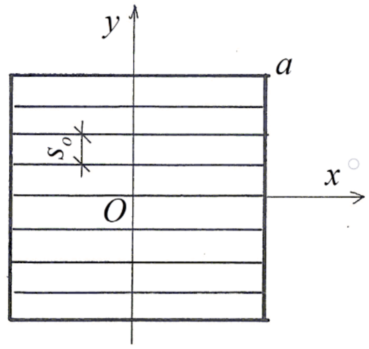

Fig.2 Cracked array

2. Rock mass characteristics

A rock mass is considered in which the cracks are located in parallel horizontal planes (Godman, 1976; Crouch et al., 1983; Kauch et al., 1987). The distance between the planes is s_o, and the scale between them is isotropic and linear (Fig.2). Cracks have normal stiffness K_n and tangential stiffness K_s. The rocks between the cracks are isotropic and linear.

3. Method for determining stresses

According to the approximate method proposed by Goodman, the rock mass is presented as a transversely isotropic medium (Godman, 1976). The plane of isotropy is horizontal. The axis z lies in the plane of isotropy and makes a right angle with the plane of the drawing (Fig.1). The medium has the following physical characteristics:

$$E_y = \frac{E}{1 + E/(s_o K_n)}\,;\ G_{xy} = \frac{G}{1 + G/(s_o K_s)}\,;$$

$$E_x = E_z = E\,;\ \nu_{xy} = \nu_{xz} = \nu\,;\ \nu_{yx} = \frac{E_y}{E_x}\nu_{xy}\,;$$

$$\nu_{zx} = \frac{E_z}{E_x}\nu_{xz} = \nu\,. \qquad (1)$$

Here, E is Young's modulus, ν is Poison's ratio, and G is a shearing module in an isotropic field. The independent constants for the transversely isotropic medium in (1) are as follows:

E_x, E_y, E_z are Young's modulus in the direction of the coordinate axes x, y и z;

ν_{xy}, ν_{yx}, ν_{xz}, ν_{zx} are Poison's ratios characterising the transverse deformations in tension and compression in the direction of the coordinate axes.

4. Characteristic equation

The behavior of the medium is described by a fourth-order differential equation. Its characteristic equation is (Ivanova, 2022; Minchev, 1960; Trifonova-Genova, 2019):

$$b_{11}s^4 + b_{17}s^2 + b_{33} = 0\,, \qquad (2)$$

where

$$b_{17} = 2b_{13} + b_{55}\,.$$

The coefficients in this equation have the following form (Ivanova et al., 2018):

$$b_{11} = \frac{1 - \nu_{yx}\nu_{xy}}{E_y}\ ;\ b_{33} = \frac{1 - \nu_{xz}^2}{E_x}\,;$$

$$b_{13} = -\frac{\nu_{yx} + \nu_{yx}\nu_{xy}}{E_y}\ ;\ b_{55} = \frac{1}{G_{xy}}\,. \qquad (3)$$

These coefficients are also expressed by the deformation coefficients given in Trifonova-Genova (2019). The roots of equation (2) are:

$$s_1 = \beta_1 i\,;\ s_2 = \beta_2 i\,;\ s_3 = -\beta_1 i\,;\ s_4 = -\beta_2 i\,. \qquad (4)$$

Here, i is an imaginary unit.

The obtained roots allow to determine the complex function of the stresses and through it to obtain the expressions for the stresses at an arbitrary point around the hole.

5. Stresses

A cylindrical coordinate system is selected ($r\theta z$) (Fig.1). The axis z is perpendicular to the drawing. An angle θ is measured from the horizontal axis x to y (Fig.1). There are no radial and tangential stresses along the contour of the hole, and the normal tangential stresses have the form (Vucheva et al., 2020; Ivanova, 2022; Minchev, 1960; Trifnonova-Genova, 2019):

$$\sigma_\theta = \frac{Q}{e_1}(e_2 + e_3 + e_4), \qquad (5)$$

where

$$e_1 = d_1 d_2 \beta_3\,; \quad e_2 = \beta_1 d_1 d_2 \beta_3\,;\ e_3 = d_3 d_2 \beta_4\,;$$

$$e_4 = d_4 d_1 \beta_5\,; \beta_3 = \beta_1 - \beta_2\,; \quad \beta_4 = \beta_2 - \lambda\,;$$

$\beta_5 = \lambda - \beta_1 ; \lambda = \dfrac{\nu}{1-\nu}$;

$d_3 = a_1 \sin^2\theta\cos^2\theta - \beta_1^2 \sin^4\theta - \beta_1 \cos^4\theta$;

$d_4 = b_1 \sin^2\theta\cos^2\theta - \beta_2^2 \sin^4\theta - \beta_2 \cos^4\theta$;

$d_1 = \sin^2\theta + \beta_1^2 \cos^2\theta ; d_2 = \sin^2\theta + \beta_2^2 \cos^2\theta$;

$a_1 = \beta_1^3 - 2\beta_1^2 - 2\beta_1 + 1 ;\ b_1 = \beta_2^3 - 2\beta_2^2 - 2\beta_2 + 1$.

6. Stresses in an isotropic field

The elastic properties of the medium in different directions are the same. The following dependencies exist between the coefficients in equation (2):

$$b_{11} = b_{22} = 0{,}5(2b_{13} + b_{55}). \quad (6)$$

The roots of the characteristic equation (2) are:

$$s_1 = s_2 = i ;\ s_3 = s_4 = -i . \quad (7)$$

Along the contour of the circular opening, stresses are obtained from (5) and have the form:

$$\sigma_{r,o} = 0 ;\ \sigma_{\theta,o} = Q(\sigma_{\theta,5} + \lambda\sigma_{\theta,6} + \sigma_{\theta,7});$$
$$\tau_{r\theta,o} = 0 ;\ \sigma_{z,o} = \nu\sigma_{\theta,o} , \quad (8)$$

where

$$\sigma_{\theta,5} = \lambda \sin^2\theta + \cos^2\theta ;$$
$$\sigma_{\theta,6} = 3\sin^4\theta\cos^2\theta + 2\sin^6\theta - \cos^6\theta ;$$
$$\sigma_{\theta,7} = 3\sin^2\theta\cos^4\theta + 2\cos^6\theta - \sin^6\theta .$$

The stresses in equations (5) and (8) are symmetric about the vertical and horizontal axes of the hole cross-section (Fig.1).

7. Numerical example

The characteristics of an isotropic medium between the planes in which the cracks are located are given: $E = 10^4 MPa$, $\nu = 0{,}2$, and $G = 0{,}4167.10^4 MPa$. The crack stiffness and plane spacing are: $K_n = \infty$, $s_o = 2m$ and $K_s = 0{,}2315.10^3 MPa/m$. Then, the physical characteristics of the cracked medium are: $E_x = E_z = E_y = E$, $\nu_{xy} = \nu_{xz} = \nu$, $\nu_{yx} = \nu_{zx} = \nu$, $s_o K_s = \dfrac{G}{9}$ and $G_{xy} = \dfrac{G}{10}$.

The coefficients from equation (3) are: $b_{11} = 0{,}96.10^{-7}$, $b_{33} = 0{,}96.10^{-7}$, $b_{13} = -0{,}24.10^{-7}$ and $b_{55} = 24.10^{-7}$. The roots of equation (2) have the form (4), where: $\beta_1 = 0{,}202$, $\beta_2 = 4{,}945$. The coefficient from the equation (5) are: $\lambda = 0{,}25$, $H = 100m$, $\gamma = 2{,}5.10^{-3} MN/m^3$, $\beta_3 = -4{,}743$, $\beta_4 = 4{,}695$, $\beta_5 = 0{,}048$, $a_1 = 0{,}522$, $b_1 = 63{,}145$.

The tangential normal stresses around the hole were calculated in both types of rock mass. Equations (5) are used in the cracked array, and equations (8) - in the isotropic rock mass. These stresses are referred to the stress in an undisturbed rock mass Q . Due to the symmetry about the two axes, calculations were made for first quadrant points. The results are listed in Table 1.

Table 1. *Normal tangential stresses in both types of rock mass*

n	$[^\circ]$	σ_θ / Q	$\sigma_{\theta,o} / Q$
1	0	-4.891	-2.75
2	15	-1.512	-2.546
3	30	-0.338	-2.000
4	45	-0.236	-1.249
5	60	-0.386	-0.500
6	75	-0.598	+0.049
7	90	-0.836	+0.25

The diagrams of the normal tangential stresses are given in Figure 3. The solid line indicates the normal tangential stresses in the cracked rock mass, and the dashed line indicates those in the isotropic rock mass.

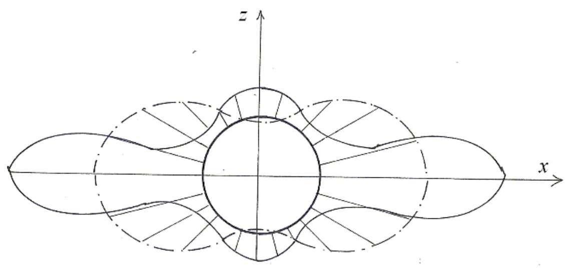

Fig. 3. Diagrams of the normal tangential stresses in the two fields

The following conclusions can be drawn from Table 1 and Figure 3:

In both environments, the maximum values of the tangential normal stresses occur at points on the hole contour through which the horizontal axis passes. Stresses in the cracked rock mass are 1.8 times greater than the corresponding stresses in the isotropic rock mass.

Along the vertical axis, the stresses in the two environments are of different signs. In the cracked rock mass, the stresses are negative and in the isotropic rock mass – positive. The stress values in the studied environment are 5.8 times smaller than the same along the axis x .

8. Key findings

From the results obtained above, it can be concluded that the stresses in a cracked medium are greater than the same in an isotropic field. Therefore, it is necessary to take into account this non-uniformity and apply the environment model proposed in the work.

The method described in the article is easy to implement. It does not require the application of software packages, but the use of popular computing tools, such as spreadsheets (Harvey, 2018).

Conclusion

The development can be used by qualified engineers in the field of construction of underground facilities and shafts.

The method can be extended by applying it to a rock mass with a system of inclined parallel planes in which there are cracks. This non-uniformity can be accounted for when determining the stress state of the medium around the horizontal fabrication with a different cross-sectional shape.

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