AN APPROACH FOR DETERMINING THE NATURAL FREQUENCY OF A STEPPED SHAFT

Violeta Trifonova – Genova[1], Gergana Tonkova[2]

[1]*University of Mining and Geology "St. Ivan Rilski", 1700 Sofia, violeta.trifonova@yahoo.com*
[2] *University of Mining and Geology "St. Ivan Rilski", 1700 Sofia, g._pis @abv.bg*

ABSTRACT. The article discusses the question of the natural vibration in a stepped shaft with a transition segment. The test shaft consists of three segments. The second segment of the shaft is the transition between the first and the third, with a rounded radius. In the process of operation, the shaft acts by its natural vibration. For their determination, the approximate Reilly method is applied. According to the method a computational scheme is selected and then the load is calculated. A suitable method for determining displacements in points of the shaft axis is chosen and finally the frequency of the natural vibrations is calculated. According to the approximate method used, the shaft is modeled as a free beam loaded with vertical forces. Their values are equal to the weights of the individual portions to which each segment of the shaft is divided. These forces are applied across the widths of the portions selected by the package engineer. To determine the displacements from the computation scheme, the differential equation of the elastic line is used. The presence of many forces requires application of the method of numerical integration of the equation. For the transition segment of the stepped shaft, mathematical forms for determining the radii and weights of portions are derived. Accordingly, an algorithm for calculating their stiffness has been developed. Also, the mathematical forms that define the reaction forces and the bending moments in the origin of forces of the computational scheme are presented. The expressions for the displacement and the oscillations frequency are given. The presented solution supplements other existing solutions and helps to calculate more accurately the vibration of the shaft.

Key words: natural vibration, a stepped shaft, oscillation frequency, approximate method, differential equation of the elastic line

ЕДИН ПОДХОД ЗА ОПРЕДЕЛЯНЕ НА ЧЕСТОТАТА НА СОБСТВЕНИТЕ ТРЕПТЕНИЯ НА СТЪПАЛЕН ВАЛ

Виолета Трифонова – Генова[1], Гергана Тонкова[2]
[1] *Минно-геоложки университет "Св. Иван Рилски ", 1700 София, violeta.trifonova@yahoo.com*
[2] *Минно-геоложки университет "Св. Иван Рилски", 1700 София, g._pis @abv.bg*

РЕЗЮМЕ. В статията се разглежда въпросът за собствените трептения, възникващи в стъпален вал с преходен участък. Изследваният вал се състои от три участъка. Вторият участък от вала се явява преход между първия и третия, като е изпълнен със закръгление с определен радиус. В процеса на работа, върху вала действат собствени трептения. За тяхното определяне е приложен приблизителният метод на Рейли. Съгласно него е избрана изчислителна схема и след това е изчислено натоварването. Избран е подходящ метод за определяне на премествания в точки от оста на вала и накрая е извършено изчисляване на честотата на собствените трептения. Съгласно използвания приблизителен метод, валът се моделира като проста греда, натоварена с вертикални сили. Стойностите им са равни на теглата на отделните сегменти, на които е разделен всеки участък от вала. Тези сили са приложени в средите на избраните от конструктора ширини на сегментите. За определяне на преместванията от изчислителната схема се използва диференциалното уравнение на еластичната линия. Наличието на много сили изисква прилагане на метода на числено интегриране на това уравнение. За преходния участък на стъпалния вал са изведени аналитични изрази за определянето на радиусите и теглата на сегментите. Съобразно тях е разработен алгоритъм за изчисляване на коравините им. Изведени са и аналитичните изрази, с които са определени опорните реакции и огъващите моменти в приложните точки на силите от изчислителната схема. Дадени са изразите за преместванията и честотата на собствените трептения. Представеното решение допълва съществуващите решения и спомага за по-точното изчисляване на трептенията на вала.

Ключови думи: собствени трептения, стъпален вал, честота на собствени трептения, приблизителен метод, диференциално уравнение на еластична линия

Introduction

Most of the shafts used in the industry are stepped. In order to determine their reliability and continuous duty in field application, it is important to choose a method for their dimensioning under static and dynamic loads. That is why the improvement of the theoretical and numerical methods is a constant object of the authorities in this field.

Classical methods are also applied to the stepped shafts. To determine the frequency of its natural vibrations on a stepped shaft, an approximate method based on the method of Reilly is known (Feodosiev, 1965). A full study of this method in a shaft with a curved section is of interest. After a study, a solution has been found in the present work to determine the frequency of the natural vibrations, occurring in a stepped shaft with a transition section. It should be borne in mind that this area is of small size.

Exposition

The main objective of the article is to develop an approximate method and describe the approach road for its application to the stepped shaft with a transitional curvilinear segment.

1. Formulation of the problem

A stepped cylindrical shaft with step and geometric parameters according to Figure 1 (Anchev, 2011) is investigated. Here l_1, l_2 and l_3 are the lengths of the three sections, r is the radius of the transition zone, D and d are the diameters of the first and third segment of the relevant shaft respectively.

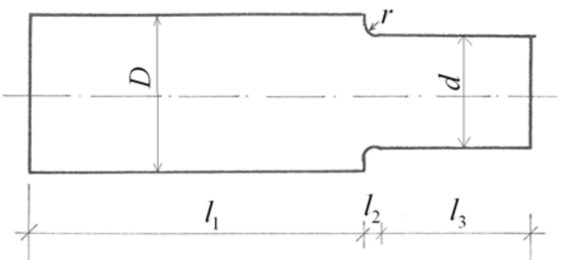

Fig. 1. Stepped shaft with a curved segment

2. Method for determination of internal forces

a) Weights of the portions in the individual segments of the shaft

In order to solve the problem a method is applied (Kisyov, 1965) according to which the shaft is divided into portions of sized widths $\Delta x_i, (i = 1, 2, 3)$ as shown in Figure 2. For each portion weights P_i are determined, as in the first and third segment are equal:

$$P_1 = V_1\gamma = S_1\Delta x_1\gamma = \pi R_1^2 \Delta x_1 \gamma \,,$$
$$P_3 = V_3\gamma = S_3\Delta x_3\gamma = \pi R_3^2 \Delta x_3 \gamma \,, \qquad (1)$$

where V_1 and V_3 are the volumes of portions in the first and third segments, S_1 and S_3 are the areas of their cross sections, expressed by the radii R_1 and R_3, and γ is the volumetric weight of the material of the shaft.

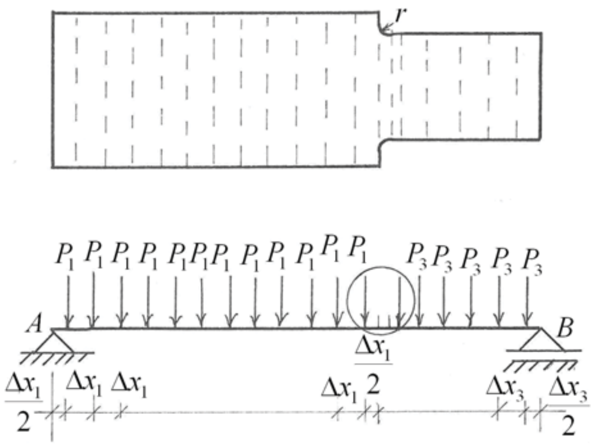

Fig. 2. Computational scheme

For the second curvilinear segment the weights of the portions are variables:

$$P_{2,i} = V_{2,i}\gamma = S_{2,i}\Delta x_2\gamma = \pi R_{2,i}^2 \Delta x_2 \gamma \,, \qquad (2)$$

where: $V_{2,i}$ are the volumes of the portions in the second segment; $S_{2,i}$ are the areas of the cross portions of the segments expressed by the radii $R_{2,i}$.

b) Current radius $R_{2,i}$ in second segment

The initial value of the radius in the second segment $R_{2,o}$ is the sum of the radius of the third segment and the bending radii (Fig. 3):

$$R_{2,o} = R_3 + r \,. \qquad (3)$$

For a certain value of the arrow f_i dimensions central angle is determined, in degrees (Tsikunov, 1970):

$$\frac{n_i^o}{4} = \arcsin\sqrt{\frac{f_i}{2r}} \,. \qquad (4)$$

On the other hand, the chord is expressed by the arrow of:

$$a_i = 2 f_i \cot g\left(\frac{n_i^o}{4}\right). \qquad (5)$$

The current radius is expressed by the difference between the initial radius and the current chord:

$$R_{2,i} = R_{2,o} - a_i \,. \qquad (6)$$

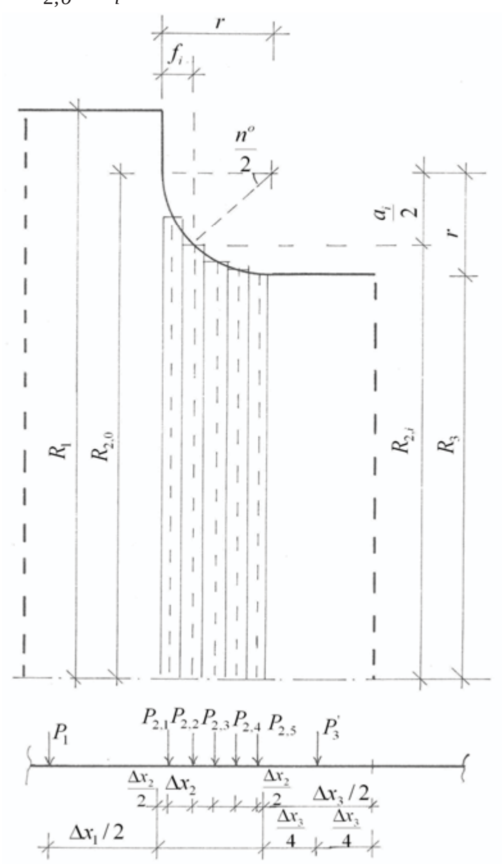

Fig. 3. Computational scheme for a second segment

The diameters and moments of inertia are calculated according to the expressions:

$$D_{2,i} = 2R_{2,i}\,; \qquad J_{C_{2},i} = \frac{\pi D_{2,i}^{2}}{32}\,, \tag{7}$$

and then the stiffness $EJ_{C_2 i}$.

In the next portion, the arrow accrues with the width:

$$f_i = f_{i-1} + \Delta x_2\,. \tag{8}$$

Formulas (3), (4), (5), (6), (7) and (8) are used to calculate the stiffness of the second segment.

c) Algorithm for determining the stiffness of individual portions in a second segment

The algorithm for determining the stiffness of individual portions in the second segment consists of the following eleven steps:

Step 1 – Enter the values of f_1, r , R_3 and Δx_2 .

Step 2 – A counter value is set i ($i = 1$).

Step 3 – Calculated $R_{2,o}$ from equation (3).

Step 4 – Calculated n_i^o from equation (4).

Step 5 – Calculated a_i from equation (5).

Step 6 – The current radius $R_{2,i}$ is calculated from equation (6).

Step 7 – Check that the current radius is smaller than the radius in the third section R_3 and if this is the case, go out of the cycle. Otherwise, move to the next step.

Step 8 – Calculate the diameter and the moment of inertia of (7), and then the stiffness EJ_{C2i} .

Step 9 – The counter is incremented by one.

Step 10 – The new arrow is calculated f_i from (8).

Step 11 – Check that the current arrow is larger than the bending radii. If this is the case, it goes out of the cycle. Otherwise, go to step 4.

An algorithm description is illustrated with a block diagram (Figure 4).

d) Determination of the supporting reactions

Consider the partial load computational scheme (Figure 5). The reaction forces are calculated (Valkov, 2004; Valkov et al., 2013):

$$A = \frac{1}{l}\left[P_1 \sum_{i=1}^{n_1} a_{i1} + \sum_{i=1}^{n_2} P_{2i} a_{i2} + P_3 \sum_{i=1}^{n_3} a_{i3} \right];$$

$$B = \frac{1}{l}\left[P_1 \sum_{i=1}^{n_1} a_{i4} + \sum_{i=1}^{n_2} P_{2i} a_{i5} + P_3 \sum_{i=1}^{n_3} a_{i6} \right]. \tag{9}$$

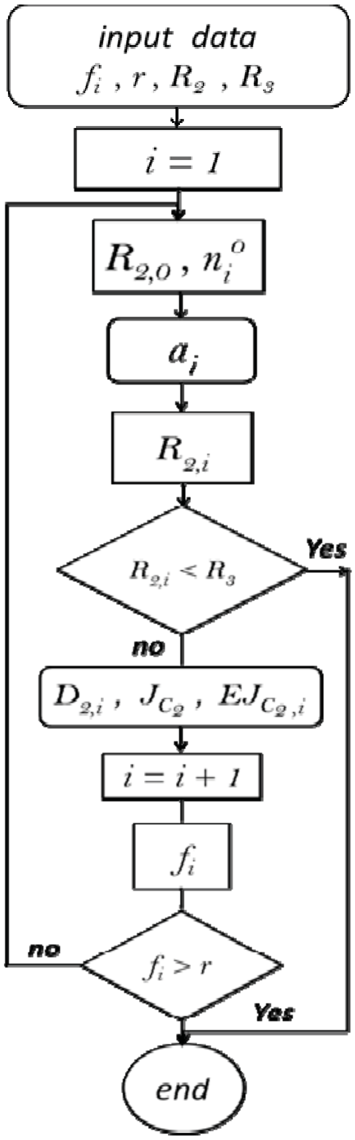

Fig. 4. Block diagram of the algorithm for determining the radius $R_{2,i}$

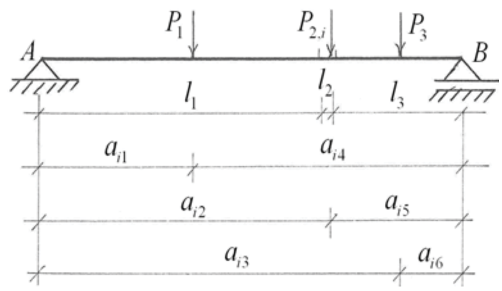

Fig. 5. Computational scheme with partial load

In expressions (9) the arms of the forces a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5} and a_{i6} are determined by the expressions:

$$a_{i1} = \frac{\Delta x_1}{2} + \Delta x_1(i-1); \quad 0 \le i \le n_1;$$
$$a_{i2} = a_I + \frac{\Delta x_2}{2} + \Delta x_2(i-1); \quad 0 \le i \le n_2;$$
$$a_{i3} = a_I + a_{II} + \Delta x_3(i-1); \quad 0 \le i \le n_3; \tag{10}$$
$$a_{i4} = a_{III} + a_{II} + \frac{\Delta x_1}{2} + \Delta x_3(i-1); \quad 0 \le i \le n_1;$$
$$a_{i5} = a_{II} + \frac{\Delta x_2}{2} + \Delta x_2(i-1); \quad 0 \le i \le n_2;$$
$$a_{i6} = \Delta x_3(i-1); \quad 0 \le i \le n_3,$$

where

$$a_I = n_1 \Delta x_1; \; a_{II} = n_2 \Delta x_2; \; a_{III} = n_3 \Delta x_3.$$

e) Bending moment diagram

The moments in the individual points are determined with the following expressions (Kisyov, 1978; Valkov, 2011):

$$M_i = -A\bar{b}_{1i} + P_1 \Delta x_1 S_1; \qquad 0 \le i \le n_1;$$
$$M_i = -B\bar{b}_{3i} + P_3 \Delta x_3 S_1; \qquad 0 \le i \le n_3; \tag{11}$$
$$M_i = -A\bar{b}_{2i} + \Delta x_2 S_2; \qquad 0 \le i \le n_2;$$

where

$$S_1 = \sum_{j=1}^{i-1} (i-j); \; S_2 = \sum_{j=1}^{i-1} P_{2,j}(i-j),$$

$$b_k = \frac{\Delta x_k}{2}; \qquad b_{jk} = b_k + \Delta x_k(i-1); \tag{12}$$
$$j = 1,2,3; \qquad k = 1,2,3.$$

3. Natural vibrations

The slope of the elastic line is determined by integration, expressed by the sum of (Feodosiev, 1965):

$$\theta_i^* = \sum_{k=1}^{i} \frac{M_k}{EJ_k} \Delta x_k + C_1; \; i = 1,..n. \tag{13}$$

After integrating the expression (13) the displacement to point i is determined:

$$w_i = \sum_{k=1}^{i} \theta_i^* \Delta x_k + C_1 x_i + C_2 \tag{14}$$

In expression (14) C_1 and C_2 are coefficients which are determined by the boundary conditions $w_1(0) = 0$ and $w_n(l) = 0$, such as:

$$C_2 = 0 \text{ or } C_1 = -\frac{\sum_{k=1}^{n} \theta_k^* \Delta x_k}{l}. \tag{15}$$

The frequency of natural vibration has the form (Kisyov, 1978):

$$\left(\omega^I\right)^2 = \frac{\sum_{k=1}^{n} P_k^I w_k}{\sum_{k=1}^{n} P_k^I w_k^2}. \tag{16}$$

This determines the frequency of the first iteration of the approximate method. The forces are calculated according to:

$$P_i^{II} = P_i^I \frac{w_i \left(\omega^I\right)^2}{g}. \tag{17}$$

Here P_i^I is the value of the force, and w_i is the displacement from the first iteration.

Proceed with calculating the other parameters of the first iteration. The resulting solution is compared with that of the first iteration. The process continues until the results of two consecutive solutions differ with a predefined error.

4. Key findings

An approach for the application of the approximate method for a stepped shaft with curved transition segment is described in the article.

Analytical expressions for the radii and weights of the portions in the studied transition segment in the stepped shaft are obtained. According to them an algorithm for calculating the stiffness of the portions is developed. Typical of this is the choice of portion width. At a small width, a more accurate solution is reached. The derived expressions are the displacements and frequency of the natural vibrations.

The resulting solution is a summary described in (Feodosiev, 1965) approach to the natural frequency of the shaft.

Conclusion

The main advantage of the proposed algorithm is that it gives an accurate solution to the problem of determining the frequency of the natural vibrations of a stepped shaft with a curvilinear segment.

The disadvantage of the method under consideration is the possibility of a slight influence of the curvilinear segment on the value of the natural vibrations. This should be checked and tested on a real shaft, which is the subject of the next team work.

References

Анчев А, М. Ичкова, Определяне на коефициента на концентрация на напреженията при опън - натиск по МКЕ - известия на ТУ Габрово, том 42, 2011, 22-24с. (Anchev A., M. Ichkova, Opredelyane na koefitsienta na kontsentratsiya na naprezheniyata pri opan – natisk po MKE – Izvestiya na TU Gabrovo, tom 42, 2011, 22-24s.)

Вълков М. Съпротивление на материалите – част 1 - еластостатика , Изд. Къща „Св. Ив. Рилски“, 2011, 300с. (Valkov M., Saprotivlenie na materialite – chast 1 - elastostatika, Izd. Kashta “Sv. Iv. Rilski”, 2011, 300s.)

Вълков М. Теоретична механика – част 1 - Статика , Изд. Къща „Св. Ив. Рилски“, 2004, 204с. (Valkov M., Teoretichna mehanika – chast 1- Statika, Izd. Kashta “Sv. Iv. Rilski”, 2004, 204s.)

Вълков М., Ст.Пулев Ръководството за решаване на задачи по теоретична механика. Част I. Статика, Изд. къща „Св. Ив. Рилски”, С., 2013, 133 с. (Valkov M., S. Pulev, Rakovodstvo za reshavane na zadachi po teoretichna mehanika. Chast I. Statika, Izd. kashta “Sv. Iv. Rilski”, 2013, 133s.)

Кисьов И., Съпротивление на материалите, Д. И. “Техника”, С., 1978, 594с. (Kisyov I., Saprotivlenie na materialite, D. I. “Tehnika”, S., 1978, 594s.)

Феодосиев В. И., Съпротивление на материалите, “Техника”, София, 1965, 547с. (Feodosiev V. I., Saprotivlenie na materialite, “Tehnika”, S., 1965, 547s.)

Цикунов А. Е. Сборник математических формул, Изд. „Вышэйшая школа“, Минск, 1970, 203с. (Tsikunov A. E., Sbornik matematicheskih formul, Izd. “Vaishaya shkola” Minsk, 1970, 203s.)

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