AN APPROACH FOR DETERMINING THE INTERNAL FORCES IN A KNIFE BUCKET

Raina Vucheva[1], Violeta Trifonova – Genova[2]

[1] *University of Mining and Geology "St. Ivan Rilski", 1700 Sofia, r.wutschewa@abv.bg,*
[2] *University of Mining and Geology "St. Ivan Rilski", 1700 Sofia, violeta.trifonova@yahoo.com*

ABSTRACT: The article examines the internal forces in a knife bucket. It is modeled as a broken space frame. Its two ends are tilted. Furthermore it is supported by four-point rods. Most of the frame lies in one plane. The load on the teeth of the knife is asymmetric. It is located on the local axis of the section and is composed of two groups. The first group includes transvers forces and moments, lying in the plane. The second group load is composed of forces, lying in the plane and moments, perpendicular to it. A case is reviewed, in which the first group load is significantly higher than the second. This leads to solving the plane-space frame. A method of forces is used to determine the internal forces. The basic frame is obtained after removal of the unnecessary connections. For this frame linear equations and its relevant coefficients are described. By solution of the equations the reaction forces and the diagrams in the frame are obtained.
In the paper analytical expressions of bending and torsional moments in limited points of segments are given. These expressions are two groups. The first group of expressions is determined due to the action of the unit forces and moments, applied in excess ties, imposed on the system. The second group of expressions is obtained by external loads, composed of external forces and moments. The paper presents only a part of the developed methodology to determine the internal forces in a knife bucket of an excavator.

Keywords: knife bucket, method of forces, broken space frame

ЕДИН ПОДХОД ЗА ОПРЕДЕЛЯНЕ НА ВЪТРЕШНИТЕ СИЛИ В НОЖ НА КОФА НА БАГЕР

Райна Вучева[1] , Виолета Трифонова – Генова[2]
[1] *Минно-геоложки университет "Св. Иван Рилски", 1700 София, r.wutschewa@abv.bg,*
[2] *Минно-геоложки университет "Св. Иван Рилски ", 1700 София, violeta.trifonova@yahoo.com*

РЕЗЮМЕ: В статията се изследват вътрешните сили в конкретен нож на кофа на багер. Той е моделиран като начупена пространствена рамка. Двата й края са запънати. Освен това в четири точки тя е подпряна с прътове. По-голямата част от рамката лежи в една равнина. Натоварването върху зъбите на ножа е несиметрично. То е разположено върху локалните оси на рамката и се състои от две групи. Първата група включва напречни сили и моменти, лежащи в равнината. Втората група натоварване се състои от сили, лежащи в равнината, и моменти, перпендикулярни на нея. Тук се разглежда случай, при който първата група натоварване е значително по-голяма от втората. Това води до решаване на равнинно-пространствена рамка. За определяне на вътрешните сили в неопределимата рамка се използва силов метод. След отстраняване на излишните връзки е определена основната система. За нея са описани каноничните уравнения и съответните коефициенти. От решението на тези уравнения са получени реакциите и диаграмите в рамката.
В работата са дадени аналитичните изрази за огъващите и усукващите моменти в гранични точки на участъците. Тези изрази са две групи. Първата група изрази са определени вследствие на действието на единичните сили и моменти, приложени в излишните връзки, наложени на системата. Втората група изрази са получени от външното натоварване, състоящо се от външни сили и съсредоточени моменти. В настоящата работа са представени само част от етапите на разработена методика за определяне на вътрешните сили в нож на кофа.

Ключови думи: нож на кофа, силов метод, пространствена рамка.

Introduction

The finite element method is often used to solve mechanical problems. According to it, the continuum body is presented as discrete models, interacting with each other. This method is described in many books for beams, plates and shells. It has led to the development of many powerful software products which facilitate the engineers' work. The implementation requires on the one hand, time to get acquainted with it and on the other hand, good preparation of the constructors to analyse the results. The exact description of the model's geometry is also important. It concerns the knife bucket of an excavator SRS 4000 (Dinev, 2016).

One of the criteria for reliability of the numerical method results is to compare them with their model tasks solved with classical methods. The approach of these methods to studying the behavior of structural elements consists in obtaining equilibrium equations for an infinite little element, establishing ratios between individual variables and solving these equations.

One such analytical method is the method of forces. The obtaining of (Dinev, 2016) the computational scheme of knifes will be explored. The load on the teeth consists of asymmetric forces. Typical of the frame is that it is for the most part plane.

One possible case of loading with forces is described here. It includes transverse concentrated forces and moments, lying in the plane of the frame. The purpose of the work is to describe with analytical expression the internal forces in boundary points of share in the frame.

Exposition

1. Application of the problem

Current computational scheme with dimensions according to Fig. 1 by (Dinev N., 2016) is considered. The load consists by force P_i^*, which is perpendicular to the plan $y_* z_*$, bending and torsional moments (M_{y_i} and M_{x_i}), which lie in the same plane and are applied in points A_i ($i = 1 \div 4$). In sections A_* and B_* the frame is fixed and is supported by four point rods.

2. Reactions forces

The load determines the approach to obtaining the internal forces. It is a broken and indefinable plane-space frame.

To build the diagrams it is necessary to determine the unnecessary connections imposed on the frame (fig.1). Their total number is 7. In this case, supports B_y, $M_{B,y}$, $M_{B,x}$, H_y, F_y, $A_{2,y}$, $A_{3,y}$ are suppressed and the basic frame is determined. There is a broken frame with hammock idler A.

The external loads and the reaction forces of the suppressed supports are applied. That's how the equivalent frame is obtained. The reaction forces must have such values, that the displacements in their directions of equivalent frame are zero. In this way a linear equation is obtained. This number is equal to the suppressed supports (Kisyov, 1978):

$$\sum_{j=1}^{7} \delta_{ij} X_j = \Delta_{i,P} \quad i = 1 \div 7 \ . \qquad (1)$$

Here δ_{ij} is the displacement to direction i of the basic frame by the unit force, loaded instead of and towards X_j and Δ_{iP} is the displacement to the direction of unit force X_i, in basic frame loaded with a given load (P).

Equation (1) can be presented as follows:

$$[A]\{X\} = \{B\} \ , \qquad (2)$$

Where:

$$\{X\}^T = \{X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6 \quad X_7\}^T \ ;$$
$$\{B\}^T = \{\Delta_{1,P} \quad \Delta_{2,P} \quad \Delta_{3,P} \quad \Delta_{4,P} \quad \Delta_{5,P} \quad \Delta_{6,P} \quad \Delta_{7,P}\}^T \ ;$$

The basic frame is loaded only by an external load and only unit forces are applied in the suppressed supports. The reaction forces and diagrams of M_x^I, M_x^{II}, M_y^I, M_y^{II} are determined for each scheme. They correspond to two cases of load.

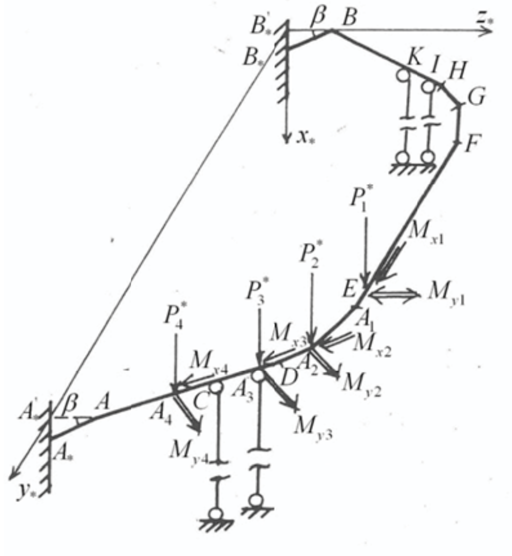

Fig. 1. Computational scheme

$$[A] = \begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \delta_{14} & \delta_{15} & \delta_{16} & \delta_{17} \\ \delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} & \delta_{25} & \delta_{26} & \delta_{27} \\ \delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} & \delta_{35} & \delta_{36} & \delta_{37} \\ \delta_{41} & \delta_{42} & \delta_{43} & \delta_{44} & \delta_{45} & \delta_{46} & \delta_{47} \\ \delta_{51} & \delta_{52} & \delta_{53} & \delta_{54} & \delta_{55} & \delta_{56} & \delta_{57} \\ \delta_{61} & \delta_{62} & \delta_{63} & \delta_{64} & \delta_{65} & \delta_{66} & \delta_{67} \\ \delta_{71} & \delta_{72} & \delta_{73} & \delta_{74} & \delta_{75} & \delta_{76} & \delta_{77} \end{bmatrix} .$$

Then the coefficients of the system (1) can be determined by the expression:

$$\delta_{ij} = \delta_{ij}^1 + \delta_{ij}^2 \ ; \qquad \Delta_{iP} = \Delta_{iP}^1 + \Delta_{iP}^2 \ , \qquad (3)$$

Where:

$$\delta_{ij}^1 = \sum_{k=1}^{m} \frac{1}{EJ_{y,k}} \int_{l_k} M_{y,i} M_{y,j} dx \ ;$$

$$\delta_{ij}^2 = \sum_{k=1}^{m} \frac{1}{EJ_{x,k}} \int_{l_k} M_{x,i} M_{x,j} dx \ ;$$

$$\Delta_{iP}^1 = \sum_{k=1}^{m} \frac{1}{EJ_{y,k}} \int_{l_k} M_{y,i} M_{y,P} dx \ ;$$

$$\Delta_{iP}^2 = \sum_{k=1}^{m} \frac{1}{EJ_{x,k}} \int_{l_k} M_{x,i} M_{x,P} dx \ .$$

Here m is the number of segment, i is the index of unknown support ($i = 1 \div 7$) and l_k is the length of segment.

Ready tables are constructed according to the rules, given by Vereshtchaguin (Trifonova-Genova, 2017). According to them two diagrams with same or different form are multiplied. These forms can be rectangular, triangular, trapezial and a combination of them.

The equations of equilibrium to the other reaction forces are described. They consist of: a sum of projected forces by axis x_* and equations for moments by axis y_* and z_*.

3. Diagrams of moments by load

3.1. Diagrams of unit forces

The segment i between points j and k has length (Fig. 2):

$$l_i = \sqrt{\left(\bar{y}_{ot,i}\right)^2 + \left(\bar{z}_{ot,i}\right)^2}\,, \tag{4}$$

Where:

$$\bar{y}_{ot,i} = y_*^k - y_*^j\,;\ \bar{z}_{ot,i} = z_*^k - z_*^j\,.$$

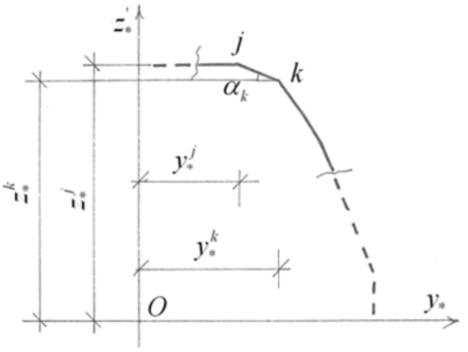

Fig. 2. The coordinates of points by the segment i

In equation (4) y_*^j and z_*^j are the coordinates of point j, but y_*^k and z_*^k are the same of point k.

The slope of section i with regard to the horizon is calculated by the expression:

$$\alpha_i = arctg\,\frac{\bar{y}_{ot,i}}{\bar{z}_{ot,i}}\,. \tag{5}$$

The arm of moment in point k by force applied in point j is determined by the expression (Fig.3a) (Valkov et al., 2013; Valkov, 2011) :

$$d_k^j = \sqrt{\left(\bar{y}^*_{ot,i}\right)^2 + \left(\bar{z}^*_{ot,i}\right)^2}\,. \tag{6}$$

Here $\bar{y}_{ot,i}$ is the width and $\bar{z}_{ot,i}$ is the height of the segment i.

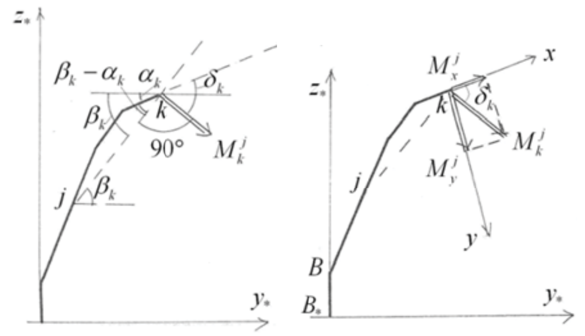

Fig. 3a Fig. 3б

Fig. 3. The components of moment in point k, by the unit force in point j (left part of frame).

The slope of arm by horizontal axis is written in:

$$\beta_k = arctg\,\frac{y^*_{ot,k}}{z^*_{ot,k}}\,. \tag{7}$$

The method of forces is applied in this analysis. The components of moments by unit force applied in point j and perpendicular to the plane of frame are determined by two steps. In the first step M_k^j is determined:

$$M_k^j = 1.d_k^j = d_k^j\,, \tag{8}$$

This moment is perpendicular by arm.

In the second step this moment is resolved by local axis of section (Fig.3b):

$$M_y^j = M_k^j \sin\delta_k\,;\ M_z^j = M_k^j \cos\delta_k\,, \tag{9}$$

In this expression the angle is determined by two methods. The angle to the points found on the left of the axis of symmetry, is calculated by (Fig.3a and Fig.3b)

$$\delta_k = 90 - \left(\alpha_k - \beta_k\right)\,. \tag{10}$$

When the angle to the points found on the right of the axis of symmetry, is calculated by

$$\delta_k = 90 - \left(\alpha_k + \beta_k\right). \tag{11}$$

In the points B_*, K, I, A_3, C unit forces are applied (Fig.1) and the diagrams $M_{x,i}^j$, $M_{y,i}^j$ are obtained. In these formulas i is the segment number, but the number $j = 1,4,5,6,7$. The numbering starts at segment B_*B ($i = 1$) and goes to segment AA_* ($i = 15$).

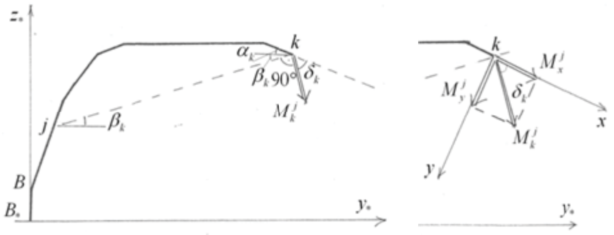

Fig. 4a Fig. 4b

Fig. 4. The components of moments in point k, by unit force applied in point j (right part of frame)

3.2. Diagrams of unit moments

The components of moment in the current segment, when the frame is loaded by unit moments by axis y_* (Fig.6) are given as follows:

$$M^2_{x,i} = \varsigma \sin \alpha_i ; \; M^2_{y,i} = \cos \alpha_i . \tag{12}$$

The coefficient ς has two values. For points located to the left of the axis of symmetry $\varsigma = -1$ (fig.6a). The second value is $\varsigma = 1$ (fig.6b).

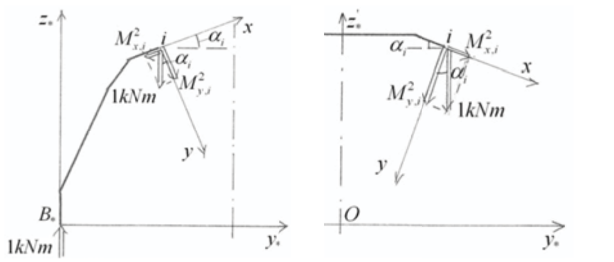

Fig.6a Fig.6b

Fig. 6. The Components of moment in point k, by unit vertical moment, applied in point B_*

In the case when the basic frame is loaded by unit moment by axis y_*, the components of moments are expressed:

$$M^3_{x,i} = \cos \alpha_i ; \; M^3_{y,i} = \varsigma \sin \alpha_i . \tag{13}$$

The coefficient ς has a point value 1 for points located to the left of the axis of symmetry (Fig.7a), and the value -1 for the others (Fig.7b).

4. Diagrams by external load

The diagrams of forces (P_i^{*I}), applied in point A_i ($i = 1 \div 4$), are described in paragraph 3. The external moments of Figure 8 are projected on the local axis of each segment.

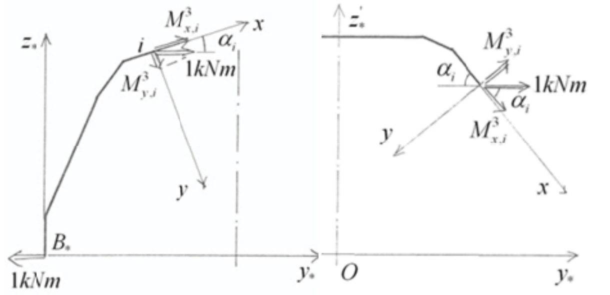

Fig. 7a Fig. 7b

Fig. 7. Component of moment in point k, by unit horizontal moment, applied in point B_*

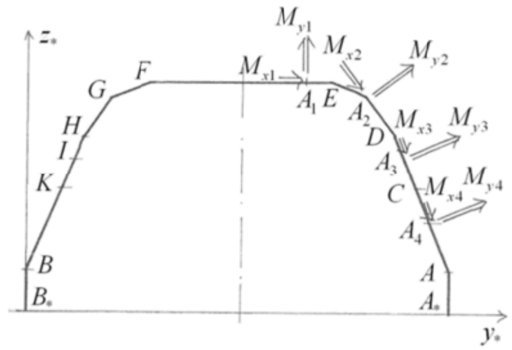

Fig. 8. External load in the plane of the frame

Then, their components can be written as:

$$M_{x,i} = \sum_{l=1}^{m_1} M^l_{x,i} ; \qquad M_{y,i} = \sum_{l=1}^{m_1} M^l_{y,i} , \tag{14}$$

where

$$M^l_{x,i} = -M_{xl} \cos \alpha_{ol,i} + M_{yl} \sin \alpha_{ol,i} ;$$
$$M^l_{y,i} = M_{xl} \sin \alpha_{ol,i} + M_{yl} \cos \alpha_{ol,i} .$$

In expressions (14) m_1 takes values, given in Table 1, depending on the number of segment.

Table 1
Values of constant m_1

m_1	1	2	3	4
section	8 и 9	10 и 11	12 и 13	14 и 15

The angle in expression (14) has the appearance (Fig.9):

$$\alpha_{o1,i} = \alpha_i \quad \rightarrow \quad 8 \le i \le 15 ; \tag{15}$$
$$\alpha_{o2,i} = \alpha_i - \alpha_{10} \quad \rightarrow \quad 10 \le i \le 15 ;$$
$$\alpha_{o3,i} = \alpha_i - \alpha_{11} \quad \rightarrow \quad 11 \le i \le 15 ;$$
$$\alpha_{o4,i} = \alpha_i - \alpha_{11} \quad \rightarrow \quad 14 \le i \le 15 .$$

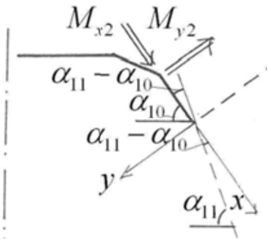

Fig. 9. External load in segment 10 and 11

4. The reactions of forces in indefinable frame

After solving the system (1) the reaction forces are determined. The principle of independent action by the forces is used (Kisyov, 1978):

$$R_j = R_{j,P} + \sum_{i=1}^{7} R_{j,i}^{l} X_i \,. \qquad (16)$$

In this expression R_j is the reaction force in the direction j of arbitrary support from the equivalent frame;

$R_{j,P}$ is the reaction force in direction j of support by the external load in basic frame only from the specified load;

$R_{j,i}$ is the reaction force in direction j of the same support by unit force in basic frame.

For the other reaction forces (B_x, B_{z*}, M_{Bz*}) equilibrium equations are recorded.

5. Diagrams in indefinable frame

The same principle is applied for determining the diagrams of moments. They are a sum of the algebraic ordinates of the diagrams by external load (P) with the ordinates of all single diagrams, multiplied by the respective unknown values X_j in each segment. Following this rule, in an arbitrary section of the system, the diagrams M_x and M_y are built by (Kisyov, 1978):

$$M_x = M_{x,P}^{**} + \sum_{i=1}^{7} M_{x,i} X_i \,;$$

$$M_y = M_{y,P}^{**} + \sum_{i=1}^{7} M_{y,i} X_i \,. \qquad (17)$$

In these expressions $M_{x,P}^{**}$ and $M_{y,P}^{**}$ are the ordinates from the corresponding diagrams in an arbitrary segment, but $M_{x,i}$ and $M_{y,i}$ are the ordinates in a diagram unit with number i in the same segment.

6. Key findings

The expressions for internal forces are used to develop algorithms and programs for their automated determination. These results represent an extension of the solution given in (Kisyov, 1978) for a rectangular plane space frame.

Conclusion

A part of the developed methodology and algorithm for resolving the undefined plan-space frames is presented in this article. They are part of a comprehensive study of stresses in a knife bucket of an excavator.

The computational results from the external load on a specific frame will be described in future studies.

A description of the decision is forthcoming, by forces, lying in the plane and moments perpendicular to it.

References

Вълков М. Съпротивление на материалите – част 1 - еластостатика , Изд. Къща „Св. Ив. Рилски“, 2011, 300с. (Valkov M., Saprotivlenie na materialite – chast 1- elastostatika, Izd. Kashta “Sv. Iv. Rilski”, 2011, 300s.)

Вълков М. Теоретична механика – част 1 -Статика , Изд. Къща „Св. Ив. Рилски“, 2004, 204s. (Valkov M., Teoretichna mehanika – chast 1- Statika, Izd. Kashta “Sv. Iv. Rilski”, 2004, 204s.)

Вълков М., Ст.Пулев Ръководството за решаване на задачи по теоретична механика. Част I. Статика, Изд. къща „Св. Ив. Рилски”, С., 2013, 133 с. (Valkov M., St. Pulev, Rakovodstvo za reshavane na zadachi po teoretichna mehanika. Chast I. Statika, Izd. kashta “Sv. Iv. Rilski”, 2013, 133s.)

Динев Н., Р.Вучева, В.Трифонова-Генова, Изследване върху напрегнатото състояние на нож на кофа на роторен багер SRS 4000, *Годишник на МГУ „Св. Ив. Рилски”*, т. 59, св.I, 2016, 165 - 108. (Dinev N., R. Vucheva, V. Trifonova-Genova, Izsledvane varhu napregnatoto sastoyanie na nozh na kofa na rotoren bager SRS 4000, *Godishnik na MGU “Sv. Iv. Rilski”*, t.59, sv. I, 2016, 165 – 108.)

Кисьов И., Съпротивление на материалите, Д. И. “Техника”, С., 1978, 594с. (Kisyov I., Saprotivlenie na materialite, D. I. “Tehnika”, S., 1978, 594s.)

Трифонова-Генова В., М. Вълков, А. Стоянов, С. Сезонов, Съпротивление на материалите, Сборник от задачи и методични указания, Изд. Къща „Св. Ив. Рилски“, 2017. (Trifonova-Genova V., M. Vulkov, A. Stoyanov, S. Sezonov, Saprotivlenie na materialite, Sbornik ot zadachi i metodichni ukazania, Izd. Kashta “Sv. Iv. Rilski”, 2017.)

The article is reviewed by Prof. Dr. Svetlana Lilkova-Marinova and Assoc. Prof. Dr. Chona Koseva.