THE LIE GROUP ANALYSIS AND THE COEFFICIENT PROBLEM IN THE NON-LINEAR STOCHASTIC EARTH'S SUBSIDENCE MECHANICS

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ABSTRACT. The problem is in the field of applied geo-mechanics. The investigation is focused on strata and ground movement over mined out areas. Using the Lie group analysis of the main equation of the non-linear stochastic geo-mechanics, a transformation is obtained. The partial differential equation of the non-linear stochastic geo-mechanics is transformed into an ordinary one. Applying measurements data for the vertical displacements, the coefficient problem is solved. An algorithm for organizing the solving procedure is described. The obtained relations may be used to plan and manage mining operations.

Keywords: surface subsidence mechanics, nonlinear stochastic model, Lie group analysis, coefficient problem

ГРУПОВИЯ АНАЛИЗ НА ЛИИ ПРИ РЕШАВАНЕ НА КОЕФИЦИЕНТНАТА ЗАДАЧА В МЕХАНИКА НА МУЛДАТА

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РЕЗЮМЕ: Въз основа на груповия анализ на Лии, приложен към основното уравнение на нелинейната стохастична геомеханика, е решена коефициентната задачата в механика на минната мулда. Нейна цел е определянето на коефициента в основното уравнение на нелинейната стохастична геомеханика по данни от полеви измервания. Последният характеризира поведението на подработения скален масив по отношение на мулдообразуването. Представен е алгоритъм за провеждане на решението. Анализирани са предимствата и недостатъците на разгледания метод. Направени са изводи.

Ключови думи: механика на мулдата, групов анализ, коефициентна задача

Introduction

Underground mining of minerals or civil engineering works are the reasons for subsiding the overlying rock mass into the underground cavities. The studied result of that phenomenon is the subsidence of the earth's surface. It creates unfavorable conditions for the functioning of buildings, infrastructure, and equipment and natural objects on the earth's surface.

In order to analyze the subsiding process caused by underground mining, it is important to determine the rock mass properties connected with the studied phenomenon.

The functional coefficient $A(w)$ in the main equation of the non-linear stochastic geo-mechanics (Vulkov 1989, 2006) characterizes the heterogeneity of the rock mass and its behavior in the process of subsiding. In this paper, the problem of determining this functional coefficient is studied.

The plane problem of determining the equation of the mining subsidence trough, (Vulkov 1989, 2006) - Fig.1 - is considered using the Lie group analysis method for the main equation of the non-linear stochastic geo-mechanics.

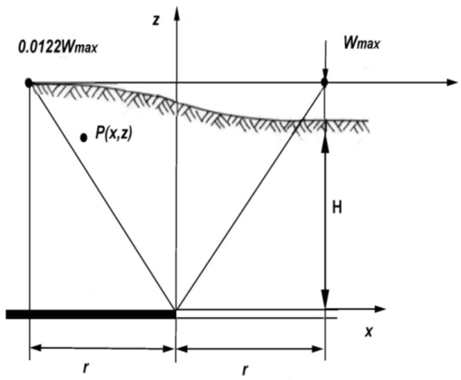

Fig. 1.

The coefficient problem

The problem of determining the non-linear coefficient can be reduced to the following problem for the quasilinear Fourier equation:

$$\left[A(w)\,w_x\right]_x = w_z \quad -\infty \le x \le \infty;\quad 0 \prec x \le H \qquad (1)$$

$$w(x,0)=\varphi_o(x) \qquad -\infty \le x \le \infty \quad z=0 \tag{2}$$

$$w(0,H)=0,5w_o,\ x=0,\ z=H, \tag{3}$$

where $w(x,y)$ is the vertical displacement of point P of the influence zone with coordinates (x,y); $A(w)$ is the studied coefficient; $\varphi=\varphi(x)$ is a function describing the subsidence of the immediate seam top; H is the depth of the coal seam; w_o is the maximal vertical displacement on the earth`s surface arising by the existing conditions.

Relationship (3) is experimentally determined.

The Lie group analysis of the main equation

L.V. Ovsyannikov (1959, 1978) made the Lee group analysis of the equation similar to the main equation of the non-linear stochastic geo-mechanics. The group invariant solutions to that equation may be classified after the type of the non-linear coefficient, which characterizes the heterogeneity of the rock mass:

- when the coefficient $A(w)$ is an arbitrary function, then equation (3) allows three independent operators:

$$\xi_1=\frac{\partial}{\partial x},\ \xi_2=\frac{\partial}{\partial z},\ \xi_3=x\frac{\partial}{\partial x}+2z\frac{\partial}{\partial z}; \tag{4}$$

- bigger number of operators may exist only when

$$A(w)=\exp(w) \text{ and } A(w)=w^{2n}. \tag{5}$$

By constructing the main equation of the non-linear stochastic geo-mechanics, the non-linear coefficient is obtained in a polynomial form (Vulkov 1989). So, the cases in (6) are not interesting for this study.

Generally, the different solutions of the equation (2) of ground of one parametric under groups are shown in table 1 (Ovsyannikov, 1959).

Table 1

№	Case	$A_{11}(P)$ - arbitrary function
1	I	ξ_1
2	II	ξ_2
3	III	ξ_3
4	IV	$\xi_1+\xi_2$

The invariant solutions in cases I, II, and IV are not interesting to use in solving geo-mechanical problems connected with the formation of the mining trough.
The attention in this study is concentrated on case III with invariants

$$\eta=\frac{x^2}{z} \quad \text{and} \quad P(x,z)=V(\eta). \tag{6}$$

Solution to the coefficient problem

The current investigation is concerned with the problem of obtaining the coefficient *A(w)* using the results of in-situ measurements of the earth's surface displacements.

The coefficient problem for (1) – (3) can be simplified by applying transformations (6).

They transform the quasilinear parabolic partial equation into an ordinary differential equation.

After introducing the following dimensionless variables in equation (8):

$$V=\frac{w}{w_o};\ A(V)=\frac{A(w)}{A(w_o)} \tag{7}$$

and substituting (6) and (11) in problems (1) – (3) it is obtained:

$$-V_\eta=4\left[A(V)V_\eta\right]_\eta \tag{8}$$

$$V(0)=0,5; \tag{9}$$

$$V(\infty)=0, \tag{10}$$

where $V_\eta=\frac{dV}{d\eta}$.

To solve the studied problem, the ordinary equation (12) is integrated once:

$$V-C=-4A(V)V_\eta. \tag{11}$$

From (11), the non-linear coefficient is obtained in the form:

$$A(V)=\frac{C-V}{4V_\eta}. \tag{12}$$

The constant in (12) is

$$C=4A(V_0)V_\eta(V_0)-V_0 \qquad 0\le V_0\le 1. \tag{13}$$

where $V(0)=0,5$ is the measured value of the vertical displacement above the border of the mined-out area (Fig. 1).

Finally, for *A(w)* is obtained:

$$A(V)=\frac{A(V_0)V_\eta(V_0)-V_0-0,25V}{V_\eta} \tag{14}$$

In order to apply relation (14), the measurements in some points of the horizontal u_0 and vertical w_0 displacements in-situ are needed. Using these values and the Avershin's relationship (1947)

$$u(x,y)=-A(w)\frac{\partial w}{\partial x},$$

the values of the functional coefficient at these points can be calculated:

$$A(V_0)=-\frac{u_0}{(w_0)_x}.$$

Result analysis and discussion

The characteristic of the geo-material $A(V_0)$ characterizing the heterogeneity of the rock mass and its behavior in the process of subsiding is determined using the Lee group analysis of the main equation of the non-linear stochastic geo-mechanics and some measurements data. Such information is available in the survey departments of the mines. To adapt relationship (14) to the specific conditions of a mining field, it is necessary to measure the values of the vertical and the horizontal displacements in some points in the zone of subsidence trough. Instead of this, one can use empirical knowledge such as $w(0,H) \approx 0,5w_o$ in fig.1.

By creating the model (1)-(3), it is assumed after S.Knothe (1956) that the following axioms are fulfilled:

- The mining subsidence is caused by the mining out of a single horizontal coal seam;
- The dynamic phase of the process of subsiding is over;
- The mining front is linear and long enough;
- The depth of the seam is $H \geq 150m$;
- The mined-out part of the coal seam is big enough.

The established results show that once the non-linear coefficient is obtained for some zone of the mining field, the same relationship can be used for the whole influence zone of the mining operations, and in mining fields with analogical conditions.

Conclusion

The considerations presented in this paper lead to a new mathematical model which allows the determination of rock mass properties by subsiding, caused by underground excavation of geo-material.

In conclusion, it can be noted that the considerations presented above correspond to a new mechanical method for describing the medium behavior properties by subsiding. The essence of such a model is its adaptation possibility for the specific conditions in a mining field by using measurements data in some points of the influence zone.

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This article was reviewed by Assoc. Prof. Dr. Stanislav Topalov and Assoc. Prof. Dr. Violeta Trifonova-Genova.